Leader-following consensus for high-order stochastic multi-agent systems via dynamic output feedback control ⋆

Xiu You [a], Chang-Chun Hua [b], Hong-Nian Yu [c], Xin-Ping Guan [d]

[a] *Institute of Mathematical Sciences, Shanxi University, Taiyuan, 030006, China*

[b] *Institute of Electrical Engineering, Yanshan University, Qinhuangdao, 066004, China*

[c] *Faculty of Science and Technology, Bournemouth University, Bournemouth, BH92QB, UK*

[d] *School of Electronic and Electric Engineering, Shanghai Jiaotong University, Shanghai, 200240, China*

Abstract

This paper studies the leader-following consensus problem for high-order stochastic nonlinear multi-agent systems. Each agent is in the strict feedback form with nonlinear functions in drift and diffusion terms and admitting time-varying incremental rates. A novel distributed observer-type consensus protocol is proposed based only on the relative output measurements of neighboring agents. The dynamic gains in controller are designed to compensate for the time-varying coefficients of nonlinear functions. By using appropriate state transformation, it is proved that the 1th moment exponential leader-following consensus is guaranteed by the proposed protocol. Different from some existing approaches, the proposed method only requires the output information of neighboring agents to be shared and does not need the observers' full state variables to be transmitted, thus the network bandwidth is saved. In addition, the controller is constructed without using the traditional backstepping method, which makes the design procedure simple and convenient for use. Finally, a simulation example is given to illustrate the effectiveness of the theoretical results.

Key words: Multi-agent systems; High-order stochastic system; Leader-following consensus; Dynamic output feedback control.

1 Introduction

In the past decades, consensus problem has been extensively studied for various multi-agent systems (MASs), such as integrator-type systems in Li et al. (2014); Wang & Song (2018); Abdessameud & Tayebi (2018), general linear systems in Zhang & Hu (2018); Zhao et al. (2017), nonlinear systems in Meng et al. (2018); Wang et al. (2017) and strict feedback systems in Wang & Ji (2012); Zhang et al. (2015). It is well known that many physical systems are not only nonlinear but also always affected by various random disturbances and uncertainties, such as noise from the unpredictable environmental conditions. Therefore, stochastic nonlinear systems are ubiquitous in practice and it is necessary and beneficial to study the distributed consensus problem for networked stochastic nonlinear dynamical systems. The consensus problems have been studied in Zheng et al. (2011) and Zhao & Jia (2015) for first- and second-order stochastic MASs, respectively. The consensus tracking problem for high-order stochastic nonlinear MASs has been investigated in Li & Zhang (2014) and Tang et al. (2015). Although some progress has been made towards consensus control for stochastic MASs, the existing literatures often require that all the state variables of agent are available. To our best knowledge, there are no results focusing on output feedback consensus for high-order stochastic nonlinear MASs up to now, especially on the case that only output of agent can be transmitted to its neighbors.

It is known that the state variables of each agent could not always be obtained and only the output information is available. Thus the output feedback consensus is more realistic than state feedback consensus in practical scenario. Recently, the output feedback consensus problem has been studied for high-order linear MASs (Gao et al., 2015; Han et al., 2018) and nonlinear MASs (Li et al.,

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Email addresses: youxiu@sxu.edu.cn (Xiu You), cch@ysu.edu.cn (Chang-Chun Hua), yuh@bournemouth.ac.uk (Hong-Nian Yu), xpguan@sjtu.edu.cn (Xin-Ping Guan).

2018), respectively. However, all state variables of the observer for each agent in pre-mentioned literatures need to be transmitted to it's neighbors, which makes that its' quantities of transmitted information is the same as that in state feedback case. To save the communication costs, a dynamic output approach using only the output information (rather than the full observer states) from the neighboring agents has been proposed in You et al. (2018); Wen et al. (2016); You et al. (2018). Nevertheless, the model of MASs considered in these literatures are deterministic and the proposed results can not be easily extended to the consensus control for high-order MASs with stochastic dynamics. All of these motivate us for the study of this paper.

This paper investigates the leader-following consensus problem for high-order stochastic nonlinear MASs in strict feedback form. The main contributions of this paper are threefold. (i) General high-order stochastic nonlinear MASs is considered. To the authors' best knowledge, there is still no results reported on this model. Existing literatures present some good results for the agent model with deterministic low-order dynamics, such as Zheng et al. (2011); Zhao & Jia (2015); Wen et al. (2016) and You et al. (2018). The high-order stochastic nonlinear MASs are not discussed. The method proposed for the deterministic low-order dynamics case could not be used for the high-order stochastic nonlinear MASs. (ii) The output feedback consensus control problem is investigated. We design a novel dynamic consensus protocol requiring only neighboring agents' output information to be transmitted, and the observer state information is not needed to be transmitted. Compared to exiting methods in Gao et al. (2015); Han et al. (2018); Li et al. (2018), less information is transmitted and the communication bandwidth is saved. (iii) A simple control design method is proposed. The traditional step by step (backstepping) method is not needed any more. Compared with backstepping approach in Li & Zhang (2014), the construction of consensus protocol in this paper requires no recursive design and is simple and convenient for practical use.

2 Problem formulation and preliminaries

Consider the MASs composed of $N+1$ agents, having nth order nonlinear and stochastic dynamics. The dynamics of the ith $(i=0,1,\cdots,N)$ agent is described as:

$$\begin{aligned} dx_{i,m} &= (x_{i,m+1} + f_m(\overline{\boldsymbol{x}}_{i,m},t))\,dt + \boldsymbol{h}_m^T(\overline{\boldsymbol{x}}_{i,m},t)d\boldsymbol{\omega} \\ dx_{i,n} &= (u_i + f_n(\boldsymbol{x}_i,t))\,dt + \boldsymbol{h}_n^T(\boldsymbol{x}_i,t)d\boldsymbol{\omega} \\ y_i &= x_{i,1}, \end{aligned} \tag{1}$$

where $m=1,\cdots,n-1$, $\boldsymbol{x}_i=(x_{i,1},\cdots,x_{i,n})^T\in\mathbb{R}^n$, $u_i\in\mathbb{R}$ and $y_i\in\mathbb{R}$ are the state, input, and measured output of system, respectively, and $\boldsymbol{\omega}$ is an ρ-dimensional independent standard Wiener process (or Brownian motion). Note that $\overline{\boldsymbol{x}}_{i,m}=(x_{i,1},\cdots,x_{i,m})^T\in\mathbb{R}^m$. In our formulation, the agent indexed by 0 is referred as leader and the agents indexed by $1,\cdots,N$ are called followers. The followers connected to leader can receive the information from leader. For $m=1,\cdots,n$, the uncertain nonlinear functions $f_m(\cdot)$: $\mathbb{R}^m\times\mathbb{R}\to\mathbb{R}$ and $\boldsymbol{h}_m(\cdot)$: $\mathbb{R}^m\times\mathbb{R}\to\mathbb{R}^\rho$ are Borel measurable and satisfy the following Lipschitz assumption:

Assumption 1. For $i=0,1,\cdots,N$ and $m=1,\cdots,n$, $|f_m(\overline{\boldsymbol{x}}_{i,m},t)-f_m(\overline{\boldsymbol{x}}_{0,m},t)| \le \alpha_1(t)\sum_{l=1}^m|x_{i,l}-x_{0,l}|$, $\|\boldsymbol{h}_m(\overline{\boldsymbol{x}}_{i,m},t)-\boldsymbol{h}_m(\overline{\boldsymbol{x}}_{0,m},t)\| \le \alpha_2(t)\sum_{l=1}^m|x_{i,l}-x_{0,l}|$, for all $(\overline{\boldsymbol{x}}_{i,m},t),(\overline{\boldsymbol{x}}_{0,m},t)\in\mathbb{R}^m\times\mathbb{R}^+$, where $\alpha_1(t)\le c_1e^{c_2t}$ and $\alpha_2(t)\le\tilde{c}_1e^{\tilde{c}_2t}$ are strictly positive time-varying functions, in which c_1, c_2, $\tilde{c}_1$ and $\tilde{c}_2$ are known nonnegative constants.

In this paper, we use a graph $\mathcal{G}$ to describe the information exchanging among agents. The augmented graph $\bar{\mathcal{G}}$ contains $\mathcal{G}$ and a leader with edges between some followers and leader. The corresponding connectivity matrix $\boldsymbol{A}=[a_{ij}]$, in-degree matrix $\boldsymbol{D}=\text{diag}\{d_1,\cdots,d_N\}$, Laplacian matrix $\boldsymbol{L}=\boldsymbol{D}-\boldsymbol{A}$, pinning matrix $\boldsymbol{B}=\text{diag}\{b_1,\cdots,b_N\}$ are defined the same as that in Yang et al. (2014). Note that the details are not presented here for the sake of saving space. Then, we have the following assumption.

Assumption 2. $\mathcal{G}$ is fixed, undirected and the augmented graph $\bar{\mathcal{G}}$ contains a spanning tree with leader node 0 as root.

Based on the idea of the pth moment exponential practical stability in Cui et al. (2013), the following definition is reasonably introduced for consensus analysis.

Definition 1. For $i=1,\cdots,N$ and $m=1,\cdots,n$, the agents are said to reach 1th moment exponential leader-following consensus if there exist positive constants ϱ and ν such that $E\,|x_{i,m}-x_{0,m}|\le\varrho e^{-\nu t},\ t>0$.

To this end, a useful Lemma which is important to derive the main results of this paper is presented as follows:

Lemma 1. Let $\boldsymbol{\mathcal{B}}_1,\boldsymbol{\mathcal{B}}_2\in\mathbb{R}^n$ and $\boldsymbol{\mathcal{A}}\in\mathbb{R}^{n\times n}$, $\boldsymbol{\Xi}\in\mathbb{R}^{2n\times 2n}$ be the vectors and matrices, respectively, defined as $\boldsymbol{\mathcal{B}}_1=(0,\cdots,0,1)^T$, $\boldsymbol{\mathcal{B}}_2=(1,0,\cdots,0)^T$, $\boldsymbol{\Xi}=\text{diag}\left(\widetilde{\Xi},\widetilde{\Xi}\right)$ with $\widetilde{\Xi}=\text{diag}\left(0,\cdots,n-1\right)$ and $\boldsymbol{\mathcal{A}}=\begin{pmatrix}0&1&\cdots&0\\ \vdots&\vdots&\ddots&\vdots\\ 0&0&\cdots&1\\ 0&0&\cdots&0\end{pmatrix}$. Then, column vectors $\boldsymbol{k}_g=(g_1,\cdots,g_n)^T$, $\bar{\boldsymbol{k}}_g=(\bar{g}_1,\cdots,\bar{g}_n)^T$ and $\boldsymbol{\kappa}=$

$(\kappa_1, \cdots, \kappa_n)^T$ can be found such that $\boldsymbol{M} = \left(\boldsymbol{I}_N \otimes \widetilde{\boldsymbol{M}}\right) - \left((\boldsymbol{L}+\boldsymbol{B}) \otimes \left(\bar{\boldsymbol{k}}\boldsymbol{\mathcal{B}}_3^T\right)\right)$ is a Hurwitz matrix, where $\boldsymbol{I}_N$ is the N-dimensional identity matrix, $\boldsymbol{L}$ and $\boldsymbol{B}$ are graph matrices, $\boldsymbol{\mathcal{B}}_3 = \left(\boldsymbol{\mathcal{B}}_2^T, \boldsymbol{\mathcal{B}}_2^T\right)^T$ and $\widetilde{\boldsymbol{M}} = \begin{pmatrix} \boldsymbol{\mathcal{A}} + \boldsymbol{k}_g\boldsymbol{\mathcal{B}}_2^T + \boldsymbol{\mathcal{B}}_1\boldsymbol{\kappa}^T & \boldsymbol{0} \\ -\boldsymbol{k}_g\boldsymbol{\mathcal{B}}_2^T & \boldsymbol{\mathcal{A}} \end{pmatrix}$, $\bar{\boldsymbol{k}} = \left(\bar{\boldsymbol{k}}_g^T, -\bar{\boldsymbol{k}}_g^T\right)^T$. Furthermore, there exist positive definite matrix $\boldsymbol{P} \in \mathbb{R}^{2Nn\times 2Nn}$ and positive constants $\tilde{\lambda}_1$, $\tilde{\lambda}_2$, β, h satisfying $\tilde{\lambda}_1 \boldsymbol{I}_{2Nn} \leq \boldsymbol{P} \leq \tilde{\lambda}_2 \boldsymbol{I}_{2Nn}$, $\boldsymbol{M}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{M} \leq -\beta \boldsymbol{P}$ and $-h\boldsymbol{P} \leq (\boldsymbol{I}_N \otimes \Xi)\boldsymbol{P} + \boldsymbol{P}(\boldsymbol{I}_N \otimes \Xi) \leq h\boldsymbol{P}$.

Proof. Denote $\widehat{\boldsymbol{M}} = \widetilde{\boldsymbol{M}} - \bar{\boldsymbol{k}}\boldsymbol{\mathcal{B}}_3^T$, we have $\widehat{\boldsymbol{M}} = \begin{pmatrix} \boldsymbol{\mathcal{A}} + \boldsymbol{k}_g\boldsymbol{\mathcal{B}}_2^T + \boldsymbol{\mathcal{B}}_1\boldsymbol{\kappa}^T - \bar{\boldsymbol{k}}_g\boldsymbol{\mathcal{B}}_2^T & -\bar{\boldsymbol{k}}_g\boldsymbol{\mathcal{B}}_2^T \\ -\boldsymbol{k}_g\boldsymbol{\mathcal{B}}_2^T + \bar{\boldsymbol{k}}_g\boldsymbol{\mathcal{B}}_2^T & \boldsymbol{\mathcal{A}} + \bar{\boldsymbol{k}}_g\boldsymbol{\mathcal{B}}_2^T \end{pmatrix}$. From Lemma 2 and Lemma 3 in Wang & Ji (2012), it is obvious that there exist column vectors $\bar{\boldsymbol{k}}_g$, $\boldsymbol{\kappa}$ such that matrix $\overline{\boldsymbol{M}} = \begin{pmatrix} \boldsymbol{\mathcal{A}} + \boldsymbol{\mathcal{B}}_1\boldsymbol{\kappa}^T & -\bar{\boldsymbol{k}}_g\boldsymbol{\mathcal{B}}_2^T \\ \boldsymbol{0} & \boldsymbol{\mathcal{A}} + \bar{\boldsymbol{k}}_g\boldsymbol{\mathcal{B}}_2^T \end{pmatrix}$ is Hurwitz. Then, choosing $\bar{\boldsymbol{k}}_g = \boldsymbol{k}_g$, we get that $\widehat{\boldsymbol{M}} = \overline{\boldsymbol{M}}$ is also Hurwitz. Under Assumption 2, we know that $\boldsymbol{L}+\boldsymbol{B}$ is a positive definite matrix (Wang & Ji, 2012) and its eigenvalues $\lambda_i > 0, i = 1, \cdots, N$. Thus, following the proof process of Lemma 3 in Wang et al. (2008), we have that matrices $\left(\widetilde{\boldsymbol{M}} - \lambda_i \bar{\boldsymbol{k}}\boldsymbol{\mathcal{B}}_3^T\right), i = 1, \cdots, N$ are also Hurwitz by choosing $\bar{\boldsymbol{k}}$ appropriately. It follows that matrix $\boldsymbol{M}$ is Hurwitz (Wang & Ji, 2012). Then, the matrix inequalities in Lemma 1 are easily obtained and the proofs are omitted here for brevity.

Remark 1. As shown in Mazenc et al. (1994), for systems with growth nonlinearities relevant to unmeasured state components, the output feedback stabilization problem may not be solvable. Therefore, it is necessary to assume some growth conditions to deal with nonlinearities depending on unmeasured states. In many existing works (Zhang et al. (2015) and the references therein), the nonlinearities are assumed to admit an incremental rate depending on measured output or time t. Thus, in this paper, we assume that the incremental rate of $f_m(\overline{\boldsymbol{x}}_{i,m}, t)$ and $\boldsymbol{h}_m(\overline{\boldsymbol{x}}_{i,m}, t)$ all depends on time t, which is given as Assumption 1. Note that conditions $\alpha_1(t) \leq c_1 e^{c_2 t}$, $\alpha_2(t) \leq \tilde{c}_1 e^{\tilde{c}_2 t}$ in Assumption 1 are easy to be satisfied for nonlinear and stochastic terms. It is easy to check that the constants and many time-varying functions satisfy above conditions, such as linear function, quadratic function, trigonometric function, logarithmic function and their combinations with respect to t. Therefore, system (1) satisfying Assumption 1 includes many types of linear and nonlinear systems as special cases. The linear system considered in Wang & Song (2018); Abdessameud & Tayebi (2018) satisfies (1) with $c_1 = c_2 = \tilde{c}_1 = \tilde{c}_2 = 0$. The nonlinear system considered in Wang & Ji (2012) satisfies (1) with $c_2 = \tilde{c}_1 = \tilde{c}_2 = 0$. The time-varying system considered in Zhang et al. (2015) satisfies (1) with $\tilde{c}_1 = \tilde{c}_2 = 0$ and the second-order stochastic system in Zhao & Jia (2015) satisfies (1) with $\tilde{c}_1$, $\tilde{c}_2$ being nonnegative constants.

Remark 2. In Lemma 1, vectors $\boldsymbol{k}_g$, $\bar{\boldsymbol{k}}_g$, $\boldsymbol{\kappa}$, positive definite matrix $\boldsymbol{P}$ and positive constants $\tilde{\lambda}_1$, $\tilde{\lambda}_2$, β, h can be obtained by the following procedure. Firstly, find out the eigenvalues $\lambda_i > 0, i = 1, \cdots, N$ of $\boldsymbol{L}+\boldsymbol{B}$. Secondly, find out vectors $\boldsymbol{k}_g$, $\bar{\boldsymbol{k}}_g$ and $\boldsymbol{\kappa}$ such that $\left(\widetilde{\boldsymbol{M}} - \lambda_i \bar{\boldsymbol{k}}\boldsymbol{\mathcal{B}}_3^T\right)$ are all Hurwitz. Finally, by solving matrix inequalities in Lemma 1, we can obtain positive definite matrix $\boldsymbol{P}$ and positive constants $\tilde{\lambda}_1$, $\tilde{\lambda}_2$, β, h.

3 Main result

In this section, a distributed controller is designed to guarantee the 1th moment exponential leader-following consensus. Since only relative output information is available, the local output consensus error for the ith agent is defined as $\sigma_{i,1} = \sum_{j\in\mathcal{N}_i} a_{ij}(y_j - y_i) + b_i(y_0 - y_i)$. Based on this, we design the distributed dynamic output feedback controller for the ith agent as

$$\begin{aligned} dz_{i,m} &= (z_{i,m+1} + g_m F^m z_{i,1} + \bar{g}_m F^m \sigma_{i,1})\, dt \\ dz_{i,n} &= \left(\sum_{m=1}^{n} \kappa_m F^{n-m+1} z_{i,m} + g_n F^n z_{i,1}\right) dt \\ &\quad + \bar{g}_n F^n \sigma_{i,1} dt \\ dF &= \upsilon(F,t)dt, \quad u_i = \sum_{m=1}^{n} \kappa_m F^{n-m+1} z_{i,m} + u_0 \end{aligned} \tag{2}$$

where $z_{i,m} \in \mathbb{R}, m = 1, \cdots, n$ are the state variables of the $\boldsymbol{z}_i-$system. g_m, $\bar{g}_m$, κ_m are the coefficients provided in Lemma 1, and u_0 is a time-varying function only respect to time t. $F(t)$ is an extra state satisfying $F(0) > 1$ and $\upsilon(F,t)$ is a function to be determined below. Then, we establish the following result.

Theorem 1. Suppose that Assumptions 1-2 are satisfied. Then, the 1th moment exponential leader-following consensus problem of MAS given by (1) is solved by the distributed consensus protocol (2) with

$$\begin{aligned} &\upsilon(F,t) \\ &= -\frac{F}{h}\left(\frac{\beta(F-1)}{3} - \gamma - \frac{4Nn\alpha_1(t)}{\sqrt{\tilde{\lambda}_1}} - \frac{4\tilde{\lambda}_2 n^2 \alpha_2^2(t)}{\tilde{\lambda}_1}\right) \end{aligned} \tag{3}$$

in which h, β, $\tilde{\lambda}_1$, $\tilde{\lambda}_2$ are provided in Lemma 1, n is the order of agent system and $\gamma = 2c(n+h)$ with $c = \max\{c_2, 2\tilde{c}_2\}$.

Proof. For $m = 1, \cdots, n$, define consensus errors for the ith agent as $\tilde{x}_{i,m} = x_{i,m} - x_{0,m}$ and let $r_{i,m} = \tilde{x}_{i,m} - z_{i,m}$.

It follows from (1) and (2) that

$$dr_{i,m} = (r_{i,m+1} + \phi_{i,m} - g_m F^m z_{i,1} - \bar{g}_m F^m \sigma_{i,1})\, dt + \boldsymbol{\varsigma}_{i,m}^T d\boldsymbol{\omega}$$
$$dr_{i,n} = (\phi_{i,n} - g_n F^n z_{i,1} - \bar{g}_n F^n \sigma_{i,1})\, dt + \boldsymbol{\varsigma}_{i,n}^T d\boldsymbol{\omega} \quad (4)$$

where $\phi_{i,m} = f_m(\overline{\boldsymbol{x}}_{i,m}, t) - f_m(\overline{\boldsymbol{x}}_{0,m}, t)$ and $\boldsymbol{\varsigma}_{i,m} = \boldsymbol{h}_m(\overline{\boldsymbol{x}}_{i,m}, t) - \boldsymbol{h}_m(\overline{\boldsymbol{x}}_{0,m}, t)$ for $m = 1, \cdots, n$.

To go further, we first make sure that $F(t)$ in (2) is larger than 1. From (3), it is known that $v(1,t) > 0$ for all $t > 0$. Thus we can choose the initial condition of $F(t)$ strictly larger than 1 to ensure $F(t) \geq 1$. Then, as by now routine in the analysis of stabilization for single nonlinear system, we introduce the following state transformation

$$\eta_{i,m} = z_{i,m}/F^{m-1+h}, \; \zeta_{i,m} = r_{i,m}/F^{m-1+h} \quad (5)$$

The novelty here is that h is not taken as 0 or 1 as usual. Instead, it is a positive constant chosen to satisfy $-h\boldsymbol{P} \leq (\boldsymbol{I}_N \otimes \boldsymbol{\Xi})\, \boldsymbol{P} + \boldsymbol{P}\, (\boldsymbol{I}_N \otimes \boldsymbol{\Xi}) \leq h\boldsymbol{P}$ shown in Lemma 1.

Then, $\boldsymbol{z}_i$-system in (2) and system (4) is rewritten as

$$\begin{aligned}
d\eta_{i,m} &= \Big(F\eta_{i,m+1} + g_m F\eta_{i,1} + \bar{g}_m F^{1-h}\sigma_{i,1} \\
&\quad -(m-1+h)\frac{dF/dt}{F}\eta_{i,m}\Big)dt \\
d\eta_{i,n} &= \Big(F\sum_{m=1}^{n}\kappa_m \eta_{i,m} + g_n F\eta_{i,1} + \bar{g}_n F^{1-h}\sigma_{i,1} \\
&\quad -(n-1+h)\frac{dF/dt}{F}\eta_{i,n}\Big)dt \\
d\zeta_{i,m} &= \Big(F\zeta_{i,m+1} + \varphi_{i,m} - g_m F\eta_{i,1} - \bar{g}_m F^{1-h}\sigma_{i,1} \\
&\quad -(m-1+h)\frac{dF/dt}{F}\zeta_{i,m}\Big)dt + \boldsymbol{\psi}_{i,m}^T d\boldsymbol{\omega} \\
d\zeta_{i,n} &= \Big(\varphi_{i,n} - g_n F\eta_{i,1} - \bar{g}_n F^{1-h}\sigma_{i,1} \\
&\quad -(n-1+h)\frac{dF/dt}{F}\zeta_{i,n}\Big)dt + \boldsymbol{\psi}_{i,n}^T d\boldsymbol{\omega} \quad (6)
\end{aligned}$$

where $\varphi_{i,m} = \phi_{i,m}/F^{m-1+h}$, $\boldsymbol{\psi}_{i,m} = \boldsymbol{\varsigma}_{i,m}/F^{m-1+h}$ for $m = 1, \cdots, n$. Letting $\boldsymbol{\eta}_i = (\eta_{i,1}, \cdots, \eta_{i,n})^T$ and $\boldsymbol{\zeta}_i = (\zeta_{i,1}, \cdots, \zeta_{i,n})^T$, then (6) can be rewritten as $d\boldsymbol{\eta}_i = \left(F\left(\boldsymbol{\mathcal{A}} + \boldsymbol{k}_g\boldsymbol{\mathcal{B}}_2^T + \boldsymbol{\mathcal{B}}_1\boldsymbol{\kappa}^T\right)\boldsymbol{\eta}_i + F^{1-h}\bar{\boldsymbol{k}}_g\sigma_{i,1} - \frac{dF/dt}{F}\widetilde{\boldsymbol{\Pi}}\boldsymbol{\eta}_i\right)dt$, $d\boldsymbol{\zeta}_i = \left(F\boldsymbol{\mathcal{A}}\boldsymbol{\zeta}_i + \boldsymbol{\varphi}_i - F\boldsymbol{k}_g\boldsymbol{\mathcal{B}}_2^T\boldsymbol{\eta}_i - F^{1-h}\bar{\boldsymbol{k}}_g\sigma_{i,1} - \frac{dF/dt}{F}\widetilde{\boldsymbol{\Pi}}\boldsymbol{\zeta}_i\right)dt + \boldsymbol{\psi}_i^T d\boldsymbol{\omega}$, where matrix $\boldsymbol{\mathcal{A}}$, vectors $\boldsymbol{\mathcal{B}}_1, \boldsymbol{\mathcal{B}}_2, \boldsymbol{k}_g, \bar{\boldsymbol{k}}_g, \boldsymbol{\kappa}$ are defined in Lemma 1 and $\boldsymbol{\varphi}_i = (\varphi_{i,1}, \cdots, \varphi_{i,n})^T$, $\boldsymbol{\psi}_i = (\boldsymbol{\psi}_{i,1}, \cdots, \boldsymbol{\psi}_{i,n})$, $\widetilde{\boldsymbol{\Pi}} = \operatorname{diag}(h, 1+h, \cdots, n-1+h)$. Further defining $\boldsymbol{\delta}_i = \left(\boldsymbol{\eta}_i^T, \boldsymbol{\zeta}_i^T\right)^T$, one has that $d\boldsymbol{\delta}_i = \widetilde{\boldsymbol{\psi}}_i^T d\boldsymbol{\omega} + \left(F\widetilde{\boldsymbol{M}}\boldsymbol{\delta}_i + \widetilde{\boldsymbol{\varphi}}_i + F^{1-h}\bar{\boldsymbol{k}}\sigma_{i,1} - \frac{dF/dt}{F}\boldsymbol{\Pi}\boldsymbol{\delta}_i\right)dt$, where $\widetilde{\boldsymbol{\varphi}}_i = \left(\boldsymbol{0}_{n\times 1}^T, \boldsymbol{\varphi}_i^T\right)^T$, $\widetilde{\boldsymbol{\psi}}_i = (\boldsymbol{0}_{\rho\times n}, \boldsymbol{\psi}_i)$, $\boldsymbol{\Pi} = \operatorname{diag}\{\widetilde{\boldsymbol{\Pi}}, \widetilde{\boldsymbol{\Pi}}\}$ and $\bar{\boldsymbol{k}}$, $\widetilde{\boldsymbol{M}}$ are provided in Lemma 1. From (4) and (5), we have $F^{-h}\tilde{x}_{i,1} = \eta_{i,1} + \zeta_{i,1}$. This together with $y_i = x_{i,1}$ results in $F^{-h}\sigma_{i,1} = F^{-h}\left(\sum_{j\in\mathcal{N}_i} a_{ij}\left(\tilde{x}_{j,1} - \tilde{x}_{i,1}\right) - b_i\tilde{x}_{i,1}\right) = \boldsymbol{\mathcal{B}}_3^T\left(\sum_{j\in\mathcal{N}_i} a_{ij}\left(\boldsymbol{\delta}_j - \boldsymbol{\delta}_i\right) - b_i\boldsymbol{\delta}_i\right)$, in which $\boldsymbol{\mathcal{B}}_3$ is defined in Lemma 1. Thus, $\boldsymbol{\delta}_i$-system can be rewritten in global form as

$$d\boldsymbol{\delta} = \left(F\boldsymbol{M}\boldsymbol{\delta} - \frac{dF/dt}{F}\left(\boldsymbol{I}_N \otimes \boldsymbol{\Pi}\right)\boldsymbol{\delta} + \widetilde{\boldsymbol{\varphi}}\right)dt + \widetilde{\boldsymbol{\psi}}^T d\boldsymbol{\omega} \quad (7)$$

where $\boldsymbol{\delta} = \left(\boldsymbol{\delta}_1^T, \cdots, \boldsymbol{\delta}_N^T\right)^T$, $\widetilde{\boldsymbol{\varphi}} = \left(\widetilde{\boldsymbol{\varphi}}_1^T, \cdots, \widetilde{\boldsymbol{\varphi}}_N^T\right)^T$, $\widetilde{\boldsymbol{\psi}} = \left(\widetilde{\boldsymbol{\psi}}_1, \cdots, \widetilde{\boldsymbol{\psi}}_N\right)$ and $\boldsymbol{M} = \boldsymbol{I}_N \otimes \widetilde{\boldsymbol{M}} - (\boldsymbol{L} + \boldsymbol{B}) \otimes \left(\bar{\boldsymbol{k}}\boldsymbol{\mathcal{B}}_3^T\right)$.

Based on the discussions above, we construct the Lyapunov candidate function as $V = \boldsymbol{\delta}^T\boldsymbol{P}\boldsymbol{\delta}$, where positive definite matrix $\boldsymbol{P}$ is defined in Lemma 1. Then, using $\boldsymbol{\Pi} = \boldsymbol{\Xi} + h\boldsymbol{I}_{2n}$ with $\boldsymbol{\Xi}$ shown in Lemma 1 and taking differential operator $\mathcal{L}$ of V (see the definition in Zhao & Jia (2015)) along system (7), one has

$$\begin{aligned}
\mathcal{L}V &= 2F\boldsymbol{\delta}^T\boldsymbol{P}\boldsymbol{M}\boldsymbol{\delta} + 2\boldsymbol{\delta}^T\boldsymbol{P}\widetilde{\boldsymbol{\varphi}} + \operatorname{Tr}\left\{\widetilde{\boldsymbol{\psi}}\boldsymbol{P}\widetilde{\boldsymbol{\psi}}^T\right\} \\
&\quad -2\frac{dF/dt}{F}\boldsymbol{\delta}^T\boldsymbol{P}\left(\boldsymbol{I}_N \otimes \boldsymbol{\Xi}\right)\boldsymbol{\delta} - 2h\frac{dF/dt}{F}\boldsymbol{\delta}^T\boldsymbol{P}\boldsymbol{\delta} \quad (8)
\end{aligned}$$

In the following, we will estimate the terms on the right-hand side of (8). By utilizing Lemma 1, we obtain

$$2F\boldsymbol{\delta}^T\boldsymbol{P}\boldsymbol{M}\boldsymbol{\delta} \leq -\beta F\boldsymbol{\delta}^T\boldsymbol{P}\boldsymbol{\delta} \quad (9)$$

Using Assumption 1, definition (5) and the fact that $F \geq 1$, $\boldsymbol{\delta}_i = \left(\boldsymbol{\eta}_i^T, \boldsymbol{\zeta}_i^T\right)^T$ and $\boldsymbol{\delta}_i = (\delta_{i,1}, \cdots, \delta_{i,2n})^T$, we have $|\varphi_{i,m}| = \frac{|\phi_{i,m}|}{F^{m-1+h}} = \frac{|f_m(\overline{\boldsymbol{x}}_{i,m},t) - f_m(\overline{\boldsymbol{x}}_{0,m},t)|}{F^{m-1+h}} \leq \alpha_1(t)\sum_{l=1}^{m}|\delta_{i,l} + \delta_{i,n+l}|$. Define $|\boldsymbol{x}|_\Gamma = (|x_1|, \cdots, |x_N|)^T$ for any vector $\boldsymbol{x} = (x_1, \cdots, x_N)^T$. Then, since $\widetilde{\boldsymbol{\varphi}} = \left(\widetilde{\boldsymbol{\varphi}}_1^T, \cdots, \widetilde{\boldsymbol{\varphi}}_N^T\right)^T$ with $\widetilde{\boldsymbol{\varphi}}_i = \left(\boldsymbol{0}_{n\times 1}^T, \boldsymbol{\varphi}_i^T\right)^T$, $\boldsymbol{\varphi}_i = (\varphi_{i,1}, \cdots, \varphi_{i,n})^T$, we get

$$\begin{aligned}
2\boldsymbol{\delta}^T\boldsymbol{P}\widetilde{\boldsymbol{\varphi}} &\leq 2\sum_{i=1}^{N}\sum_{m=1}^{n}\left|\boldsymbol{\delta}^T\boldsymbol{P}_{(2i-1)n+m}\right||\varphi_{i,m}| \\
&\leq 2\alpha_1(t)\sum_{i=1}^{N}\sum_{m=1}^{n}\left|\boldsymbol{\delta}^T\boldsymbol{P}_{(2i-1)n+m}\right|\sum_{l=1}^{m}\left(|\delta_{i,l}| + |\delta_{i,n+l}|\right) \\
&\leq 2\alpha_1(t)\sum_{i=1}^{N}\sum_{m=1}^{n}\left|\boldsymbol{\delta}^T\boldsymbol{P}_{(2i-1)n+m}\right|
\end{aligned}$$

$$\times \sum_{l=1}^{m} \left(\left|\bar{\delta}_{(2i-2)n+l}\right| + \left|\bar{\delta}_{(2i-1)n+l}\right|\right)$$
$$\leq 2\alpha_1(t) \sum_{i=1}^{2Nn} \left|\boldsymbol{\delta}^T \boldsymbol{P}_i\right| \sum_{j=1}^{i} \left|\bar{\delta}_j\right| \leq 4Nn\alpha_1(t) \left|\boldsymbol{\delta}^T \boldsymbol{P}\right|_\Gamma |\boldsymbol{\delta}|_\Gamma$$
$$\leq \left(4Nn\alpha_1(t)/\sqrt{\tilde{\lambda}_1}\right) \boldsymbol{\delta}^T \boldsymbol{P} \boldsymbol{\delta} \tag{10}$$

where $\boldsymbol{P}_i$ is the ith column of matrix $\boldsymbol{P}$. $\bar{\delta}_j$ is the jth element of column vector $\boldsymbol{\delta}$ and $\tilde{\lambda}_1$ is given in Lemma 1. Similarly, from Assumption 1 and $\widetilde{\boldsymbol{\psi}} = \left(\widetilde{\boldsymbol{\psi}}_1, \cdots, \widetilde{\boldsymbol{\psi}}_N\right)$ with $\widetilde{\boldsymbol{\psi}}_i = (\mathbf{0}_{\rho\times n}, \boldsymbol{\psi}_i)$, $\boldsymbol{\psi}_i = \left(\boldsymbol{\psi}_{i,1}, \cdots, \boldsymbol{\psi}_{i,n}\right)$ and $\boldsymbol{\psi}_{i,m} = \varsigma_{i,m}/F^{m-1+h}$, we obtain $\left\|\widetilde{\boldsymbol{\psi}}\right\| \leq 2n\alpha_2(t) \|\boldsymbol{\delta}\|$. Then, using Lemma 1, we get

$$\mathrm{Tr}\left\{\widetilde{\boldsymbol{\psi}} \boldsymbol{P} \widetilde{\boldsymbol{\psi}}^T\right\} \leq 4n^2\alpha_2^2(t) \left(\tilde{\lambda}_2/\tilde{\lambda}_1\right) \boldsymbol{\delta}^T \boldsymbol{P} \boldsymbol{\delta} \tag{11}$$

Moreover, using $-h\boldsymbol{P} \leq (\boldsymbol{I}_N \otimes \Xi) \boldsymbol{P} + \boldsymbol{P} (\boldsymbol{I}_N \otimes \Xi) \leq h\boldsymbol{P}$ in Lemma 1, we have

$$-2\frac{dF/dt}{F} \boldsymbol{\delta}^T \boldsymbol{P} (\boldsymbol{I}_N \otimes \Xi) \boldsymbol{\delta} \leq h\frac{|dF/dt|}{F} \boldsymbol{\delta}^T \boldsymbol{P} \boldsymbol{\delta} \tag{12}$$

Then, substituting (9)-(12) into (8) gives

$$\mathcal{L}V \leq -\chi \boldsymbol{\delta}^T \boldsymbol{P} \boldsymbol{\delta} \tag{13}$$

where $\chi = \beta F + h\left(2\frac{dF/dt}{F} - \frac{|dF/dt|}{F}\right) - 4Nn\alpha_1(t)/\sqrt{\tilde{\lambda}_1} - 4n^2\alpha_2^2(t)\left(\tilde{\lambda}_2/\tilde{\lambda}_1\right)$. Then, using (3), if $dF/dt \geq 0$ $(F > 1)$, we have $\chi = \frac{\beta}{3}(2F+1) + \gamma \geq \beta + \gamma$. Else, if $dF/dt \leq 0$ $(F > 1)$, we obtain $\chi = \beta + 8Nn\alpha_1(t)/\sqrt{\tilde{\lambda}_1} + 8n^2\alpha_2^2(t)\left(\tilde{\lambda}_2/\tilde{\lambda}_1\right) + 3\gamma \geq \beta + \gamma$. This together with inequality (13) leads to

$$\mathcal{L}V \leq -(\beta+\gamma)\boldsymbol{\delta}^T \boldsymbol{P} \boldsymbol{\delta} \leq -(\beta+\gamma)V \tag{14}$$

Therefore, using Lemma 1 in Cui et al. (2013) and Lemma 1 in Wang et al. (2014), there exists a unique solution of system (7) and system (7) is exponentially stable in probability. Furthermore, from (14), the inequality $d(E[V(t)])/dt \leq -(\beta+\gamma)E[V(t)]$ can be obtained directly by Theorem 4.1 in Deng et al. (2001), which implies that $E[V(t)] \leq e^{-(\beta+\gamma)t}[V(0)]$, i.e., $E\left[\boldsymbol{\delta}^T(t)\boldsymbol{P}\boldsymbol{\delta}(t)\right] \leq \left[\boldsymbol{\delta}^T(0)\boldsymbol{P}\boldsymbol{\delta}(0)\right] e^{-(\beta+\gamma)t}$. This together with $\tilde{\lambda}_1 \boldsymbol{I}_{2nN} \leq \boldsymbol{P} \leq \tilde{\lambda}_2 \boldsymbol{I}_{2nN}$ in Lemma 1 and equality (5) follows that $E|z_{i,m}(t)| \leq c_\delta F^{m-1+h} e^{-\frac{(\beta+\gamma)}{2}t}$, where $c_\delta = \sqrt{\left(\tilde{\lambda}_2/\tilde{\lambda}_1\right)\boldsymbol{\delta}^T(0)\boldsymbol{\delta}(0)}$. According to (3), one knows that $F(t)$ is upper-bounded by function $\tilde{\alpha}(t) = \left(12Nn\alpha_1(t)/\sqrt{\tilde{\lambda}_1} + 12n^2\alpha_2^2(t)\left(\tilde{\lambda}_2/\tilde{\lambda}_1\right) + 3\gamma + \beta\right)/\beta$. Then, from $\alpha_1(t) \leq c_1 e^{c_2 t}$ and $\alpha_2(t) \leq \tilde{c}_1 e^{\tilde{c}_2 t}$ in Assumption 1, we obtain

$$\begin{aligned} E|z_{i,m}(t)| &\leq c_\delta (\tilde{\alpha}(t))^{m-1+h} e^{-\frac{(\beta+\gamma)}{2}t} \\ &\leq c_\delta \theta^{m-1+h} e^{-\frac{\beta}{2}t} \end{aligned} \tag{15}$$

where $\theta = \left(12n^2\tilde{c}_1^2\left(\frac{\tilde{\lambda}_2}{\tilde{\lambda}_1}\right) + 12Nnc_1/\sqrt{\tilde{\lambda}_1} + 3\gamma + \beta\right)/\beta$. Similarly, we get $E|r_{i,m}(t)| \leq c_\delta\theta^{m-1+h}e^{-\frac{\beta}{2}t}$. As $\tilde{x}_{i,m}(t) = z_{i,m}(t) + r_{i,m}(t)$, we have $E|x_{i,m}(t) - x_{0,m}(t)| \leq E|z_{i,m}(t)| + E|r_{i,m}(t)| \leq 2c_\delta\theta^{m-1+h}e^{-\frac{\beta}{2}t}$. Therefore, from Definition 1, the 1th moment exponential leader-following consensus of MAS (1) is achieved. This completes the proof.

Corollary 1. Suppose that Assumptions 1-2 are satisfied and $\alpha_1(t)$, $\alpha_2(t)$ in Assumption 1 are given as $\alpha_1(t) = c_1 e^{c_2 t}$ and $\alpha_2(t) = \tilde{c}_1 e^{\tilde{c}_2 t}$. Then, the 1th moment exponential leader-following consensus problem of MAS given by (1) is solved by the distributed consensus protocol (2) with $F(t) = \left(\left(4Nnc_1/\sqrt{\tilde{\lambda}_1}\right)e^{c_2 t} + 4n^2\tilde{c}_1^2 e^{2\tilde{c}_2 t}\left(\tilde{\lambda}_2/\tilde{\lambda}_1\right) + \gamma + \beta\right)/\beta$, where β, γ, $\tilde{\lambda}_1$, $\tilde{\lambda}_2$, n are the same as that in Theorem 1.

Proof. The proof can be completed by following similar steps in the proof of Theorem 1, except the derivation from (13) to (14). Since substituting $F(t)$ into (13) gives $\chi = \beta + \gamma$, which also leads to the establishment of inequality (14), the 1th moment exponential leader-following consensus of MAS (1) is achieved. The detailed proof is omitted here for brevity.

Remark 3. In controller (2), u_0 is a time-varying function only dependent on time t or equal to zero. Therefore, when u_0 is not zero, it can be obtained directly without communication transmission, and thus be regarded as a pre-known information for the followers. For example, choosing $u_0 = \sin(t)$, since there is no state information of any neighbouring agents, each follower can pre-obtain this information without any communication. It also means that function u_0 will not affect the distributedness of designed controller (2). Note that in the works of Wang & Ji (2012), Zhang et al. (2015) and some other papers, u_0 is also assumed to be a time-varying function which pre-known for all followers. Additionally, in the work of Wu et al. (2016), u_0 is actually equal to zero and in the works of Hu & Zheng (2014); Hu et al. (2015), u_0 is essentially equal to the time-varying function $\alpha_0(t)$.

Remark 4. For designed controller (2), since the number of networked agents is known, we can obtain a solution of g_m, κ_m, h, β including all connected situations but independent of global communication information. For example, if $N+1=3$ (two followers with a leader), under Assumption 2, all cases of topology matrix and corresponding eigenvalues are shown as $(\boldsymbol{L}+\boldsymbol{B})_s$ and $\lambda_{s,i}, s=1,2,3,4, i=1,2$, respectively. Following the procedure in Remark 2, we can obtain g_m, $\bar{g}_m$, κ_m such that $\left(\widetilde{\boldsymbol{M}}-\lambda_{s,i}\bar{\boldsymbol{k}}\boldsymbol{\mathcal{B}}_3^T\right)$ are all Hurwitz. Then, based on pre-known g_m, $\bar{g}_m$, κ_m, the parameters h, β can be obtained by solving the inequalities in Lemma 1. Thus, a solution of g_m, $\bar{g}_m$, κ_m, h, β, which covers all the communication cases with $N+1=3$, is obtained.

Remark 5. In this paper, controller (2) is designed with the term $F^{n-m+1}z_{i,m}$ rather than just $F(t)$. Then, similar to the proof of (15), since $\gamma=2c(n+h)$ and $c=\max\{c_2, 2\tilde{c}_2\}$, we have $E\left|F^{n-m+1}z_{i,m}(t)\right| \leq c_\delta\left(\tilde{\alpha}(t)\right)^{n+h}e^{-\frac{(\beta+\gamma)}{2}t} \leq c_\delta\theta^{n+h}e^{-\frac{\beta}{2}t}$. It implies that the term $F^{n-m+1}z_{i,m}(t)$ in (2) will converge to zero with 1th moment exponential practical stability performance. Therefore, even though parameter $F(t)$ is unstable, the designed controller (2) with term $F^{n-m+1}z_{i,m}$ is 1th moment exponential practical stable. Thus, controller (2) can be realized in practice and a small error of $z_{i,j}(t)$ will not render serious control errors.

Remark 6. You et al. (2018) solved the consensus problem for deterministic nonlinear MASs with nonlinear part satisfying linear Lipschitz conditions with constant gain. Compared which, there are two differences: (i) The system in You et al. (2018) is a deterministic system whose nonlinear dynamics satisfies the traditional Lipschitz conditions with constant gain. The consensus problem is solved by the linear dynamic output feedback control method. In contrast, the agent system considered in this paper is a stochastic system whose nonlinear dynamics admitting time-varying incremental rates, and we have to develop specific nonlinear dynamic output control techniques to deal with the complex couplings among agents' time-varying nonlinearities and randomness. (ii) Due to the exiting of nonlinear stochastic dynamics, the stability theory for deterministic system in You et al. (2018) can't be used for the consensus analysis of this paper, and thus we have to apply more complicated stochastic differential equations for the analysis of stochastic MASs.

Remark 7. Compared to the work in You et al. (2018), the contributions of this paper are characterized as follows: (i) More general high-order stochastic nonlinear system is considered with nonlinear dynamics admitting time-varying incremental rates, which is more challenging than that in You et al. (2018) whose dynamics are with Lipschitz conditions and without diffusion term. (ii) In the controller design, we propose a novel dynamic parameter $F(t)$ with adaptive law (3) to deal with the nonlinearities rather than a constant F in You et al. (2018) to guarantee the consensus of considered stochastic MASs. (iii) The stochastic stability is proved for the consensus error system. We prove that the 1th moment leader-following consensus is achieved with the designed controller (2).

Remark 8. It is known that the backstepping control method usually needs to introduce the virtual controller step by step and requires the repeated differential of virtual controller. Thus, it may lead to calculating explosion. To avoid the repeated differential actions, the dynamic surface control (DSC) method is proposed. But the complexity of control still increases rapidly as the number of system orders increases. In this paper, by using dynamic gain method, the output feedback consensus controller can be obtained directly rather than step by step, and thus the pre-mentioned shortcomings of backstepping control can be avoided. Therefore, the constructed controller in this paper makes the design procedure simple and convenient for use. Additionally, the drawbacks of dynamic output feedback control method is that it can only deal with the nonlinearities admitting time-varying incremental rates. Namely, if the incremental rates in Assumption 1 is a function not only dependent on time t but also rely on agents' state variables, the proposed dynamic gain approach is not applicable.

Remark 9. The design ideals of this paper is summarised as follows. First, based on the dynamic output control method, we design a observer-type system ($\boldsymbol{z}_i$-system) to compensate for the controlled system (agents' system (1)). It should be noted that the observer-type system is designed based only on the relative output measurements, which will reduce the quantities of information to be transmitted. Second, a dynamic parameter $F(t)$ is proposed to deal with the couplings among agents' nonlinearities and randomness, and thus the consensus performance is improved. Third, by using the state information of $\boldsymbol{z}_i$-system and the dynamic parameter $F(t)$ in combination, a novel consensus controller is designed. Note that by designing controller as (2) and choosing adaptive law $v(F,t)$ as (3), the differential operator $\mathcal{L}$ of V along system (7) is negative, and hence guarantees the 1th moment exponential consensus according to the stochastic stability theory.

4 Numerical Examples

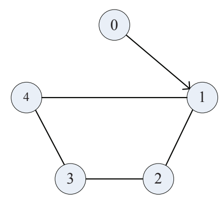

Fig. 1 Topology of augmented graph $\overline{\mathcal{G}}$

In this section, an actual simulation example is provided to validate our theoretical results. Consider a MAS consisting of five pendulum systems with stochastic noises, in which leader is labeled by 0 and four followers are labeled from 1 to 4. Suppose that the information exchanging among agents is shown in Fig. 1. Therefore, the ith agent system is described by (1) with $f_1(x_{i,1}, t) = 0$, $f_2(\boldsymbol{x}_i, t) = -\sin(x_{i,1})/l(t) - q(t)x_{i,2}$, $h_1(x_{i,1}, t) = 0.1tx_{i,1}$, $h_2(\boldsymbol{x}_i, t) = e^{0.1t}x_{i,1} + 0.1tx_{i,2}$ and ω be a scalar Gaussian white noise with zero-mean and variance of 2. For the pendulum system, its length $l(t)$ plays the role of an unknown time-varying parameter satisfying $l(t) \geq 5, t \geq 0$ and its resistance due to pivot and surrounding air is governed by the law $q(t) := t$ (see Zhang et al. (2015)). It is obvious that the nonlinearities $f_m(\cdot)$ and $h_m(\cdot)$ satisfy Assumption 1 with $\alpha_1(t) = t + 0.2$ and $\alpha_2(t) = e^{0.1t} + 0.1t$. According to the proof of Theorem 1, we construct the consensus protocol as $dz_{i,1} = (z_{i,2} - 2Fz_{i,1} - 2F\sigma_{i,1})\,dt$, $dz_{i,2} = \left(-F^2z_{i,1} - 2Fz_{i,2} - F^2z_{i,1} - F^2\sigma_{i,1}\right)dt$, $u_i = -F^2z_{i,1} - 2Fz_{i,2} + u_0$ with $u_0 = \sin(0.1t)$ and $\upsilon(F, t) = -(\frac{1}{400})F\left((\frac{0.01}{3})(F-1) - 804 - 32\alpha_1(t) - 80\alpha_2^2(t)\right)$.

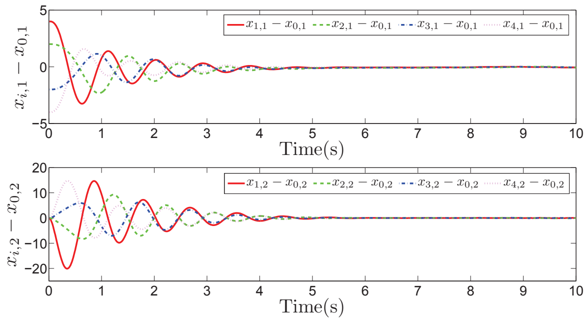

Fig. 2 Leader-following consensus results

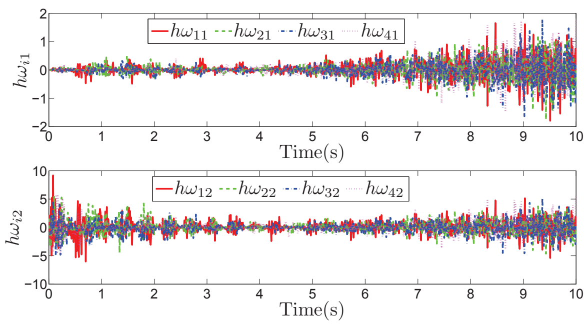

Fig. 3 Comprehensive stochastic noises imposed on agents

Setting the initial values $x_{0,1} = 0$, $x_{1,1} = 4$, $x_{2,1} = 2$, $x_{3,1} = -2$, $x_{4,1} = -4$ and $x_{i,2} = z_{i,1} = z_{i,2} = 0, i = 1, 2, 3, 4$, we obtain Fig. 2, which depicts the 1th moment exponential leader-following consensus results and shows the efficiency of the distributed consensus controllers. Let $h\omega_{i1} = h_1(x_{i,1}, t)\,(d\omega/dt)$ and $h\omega_{i2} = h_2(\boldsymbol{x}_i, t)\,(d\omega/dt)$. Then, the Fig. 3, which depicts $h\omega_{i1}$ and $h\omega_{i2}$, shows comprehensive stochastic noises imposed on agents. Furthermore, the time varying parameter $F(t)$ is shown in Fig. 4 with $F(0) = 3$.

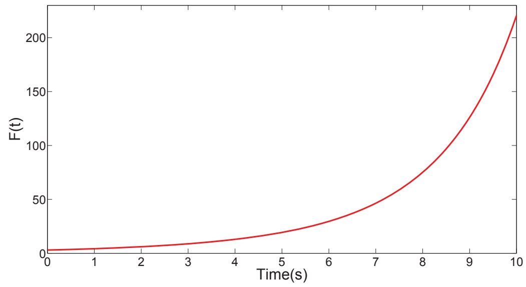

Fig. 4 Time varying parameter $F(t)$

5 Conclusion

In this paper, the leader-following consensus problem for a class of high-order stochastic MASs in strict-feedback form have been studied. The nonlinear terms in each agent are assumed to satisfy Lipschitz conditions with time-varying gains. By introducing dynamic output feedback approach together with an adaptive parameter, a novel distributed consensus protocol has been proposed. Based on appropriate state transformation, it has been proved that all the agents in network $\bar{\mathcal{G}}$ reach the 1th moment exponential leader-following consensus. The proposed dynamic output feedback controller only requires the relative output information of neighboring agents to be transmitted, and then the communication cost is reduced greatly. A simulation example has been given to illustrate the effectiveness of the proposed theoretical result.

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