Analysis of accident injury-severity outcomes: The zero-inflated hierarchical ordered probit model with correlated disturbances

by

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Abstract

In accident injury-severity analysis, an inherent limitation of the traditional ordered probit approach arises from the *a priori* consideration of a homogeneous source for the accidents that result in a no-injury (or zero-injury) outcome. Conceptually, no-injury accidents may be subject to the effect of two underlying injury-severity states, which are more likely to be observed in accident datasets with excessive amounts of no-injury accident observations. To account for this possibility along with the possibility of heterogeneity stemming from the fixed nature of the ordered probability thresholds, a zero-inflated hierarchical ordered probit approach with correlated disturbances is employed, for the first time – to the authors' knowledge – in accident research. The latter consists of a binary probit and an ordered probit component that are simultaneously modeled in order to identify the influential factors for each underlying injury-severity state. At the same time, the model formulation accounts for possible correlation between the disturbance terms of the two model components, and allows for the ordered thresholds to vary as a function of threshold-specific explanatory variables. Using injury-severity data from single-vehicle accidents that occurred in the State of Washington, from 2011 to 2013, the implementation potential of the proposed approach is demonstrated. The comparative assessment between the zero-inflated hierarchical ordered probit approach with correlated disturbances and its lower-order counterparts highlights the potential of the proposed approach to account for the effect of underlying states on injury-severity outcome probabilities and to explain more with the same amount of information.

Keywords

Accident injury-severities; Zero-inflated hierarchical ordered probit; Correlated disturbances; Injury-severity states; Threshold decomposition

Introduction

In contemporary accident research, the investigation of the determinants of accident injury-severity outcomes is largely based on high-scale datasets consisting of police-reported accidents that occurred throughout a specific period of time. A structural characteristic of such datasets originates from the preponderance of accident observations that are associated with a no-injury outcome (Jiang et al., 2013). Even though the high percentage of no-injury accidents does not imply traffic safety improvements in the roadway network, such excessive amount of observations is typically anticipated, especially when information about urban settings or highly congested networks is collected. In this context, accounting for such a common pattern in the injury-severity data constitutes an imminent statistical challenge with possible effect on parameters' efficiency and inferences' accuracy.

Recent advances on statistical and econometric approaches have addressed several issues arising from the restrictive formulations of the traditional ordered and discrete outcome frameworks, which are typically used in the analysis of accident injury-severity data (for a detailed review, see: Lord and Mannering, 2010; Savolainen et al., 2011; Mannering and Bhat, 2014, Mannering et al., 2016; Mannering, 2018). Such issues include various patterns of unobserved heterogeneity (Russo et al., 2014; Yasmin et al., 2014; Behnood et al., 2014; Eluru and Yasmin, 2015; Bogue at al., 2017; Seraneeprakarn et al., 2017; Behnood and Mannering, 2016; Fountas and Anastasopoulos, 2017;Osman et al., 2017; Fountas et al., 2018a), endogeneity (Rana et al., 2010; Abay et al., 2013; Sarwar and Anastasopoulos, 2017), temporal and spatial correlation (Castro et al., 2012; Chiou and Fu, 2015; Behnood and Mannering, 2015; Bhat et al., 2017; Zeng et al., 2018; Osama and Sayed, 2017; Paleti et al., 2017; Mannering, 2018), and under-reporting of accidents (Savolainen et al., 2011; Mannering et al., 2014). Despite the development of more flexible estimation structures (such as random parameters and latent class techniques, multivariate formulations, and so on), the theoretical causality for the preponderance of no-injury accidents and its possible implications on model estimation are not effectively accounted for in such structures.

The latter is important, especially when consideration is given to the sources of no-injury observations (or zero-injury observations, according to the typical numbering of the injury-severity outcomes in an ordinal scale). Specifically, zero-injury observations may not arise from a uniform source, because the underlying accident generation mechanisms may vary. For example, a portion of the zero-injury observations may be associated with very minor accidents, whose accident-specific conditions and contributing factors are unlikely to lead to a more severe, injury-involved outcome. Acknowledging that in the majority of injury-severity analyses the property-damage-only and possible injury outcomes are aggregated within the no-injury outcome (Savolainen et al., 2011), the aforementioned group of accidents is more likely to result in property damage only. Some indicative accident types relating to the aforementioned group of zero-injury observations include (but are not limited to): parking-related accidents, low-speed accidents, accidents at bottlenecks, or accidents involving low impact collision. Similarly, the remaining group of zero-injury observations may be associated with accidents that under different traffic-, weather-, roadway-, driver-, or vehicle-specific circumstances could naturally result in a more severe, injury-involved outcome. In the context of single-vehicle accidents, possible accident types associated with the latter group of zero-injury observations involve run-off-road accidents, collision with roadway structures, animal-involved accidents, and so on. Despite their observed no-injury outcome, the underlying mechanism corresponding to this group of zero-injury observations may share considerable (observed or unobserved) similarities with the mechanism of the injury-involved accidents.

From a theoretical perspective, the possible presence of alternate injury-severity mechanisms can lead to the consideration of two distinct injury-severity states in the analysis of accident data: (i) the zero-injury state; and (ii) the ordered injury-severity state. In line with the previous distinction of the zero-injury observations, the group of zero-injury observations that do not have the potential to result in more severe outcomes, form the basis of the zero-injury state. On the contrary, the second group of no-injury accidents (i.e., those that can potentially result in more severe outcomes) as well as all the injury-involved accidents, form the basis of the ordered injury-severity state. Note that the consideration of the ordered injury-severity state has a two-fold function: (i) to illustrate the generation mechanism of the non-zero-injury state; and (ii) to account for the inherent generation differences among the injury-severity outcomes.

From a modeling perspective, several statistical and econometric approaches have been developed to accommodate the possibility of excessive amount of zero observations in accident datasets. Such approaches include the zero-inflated count data models, such as the zero-inflated Poisson (Shankar et al., 1997; Boucher et al., 2009; Aguero-Valverde, 2013; Dong et al., 2014a,) and the zero-inflated negative binomial models (Jang et al., 2010; Usman et al., 2010; Dong et al., 2014b; Anastasopoulos, 2016; Cai et al., 2016; Liu et al., 2018) as well as the (univariate or multivariate) tobit models (Anastasopoulos et al., 2012a, 2012b; Anastasopoulos, 2016; Zeng et al., 2017). In accident research, these two streams of statistical techniques primarily account for the presence of zero-accident and non-zero-accident states in roadway segment based accident frequency and rate analysis, respectively (Anastasopoulos, 2016). However, the possibility of two distinct states at a more disaggregate level, and particularly at the level of accident observations, has been left under-explored, especially within the context of accident injury-severity research. Accounting for the presence of underlying states in the level of accident observations can result not only in more reliable parameter estimates, but also in the efficient use of after-crash information (i.e., accident-, vehicle-, driver-, time-varying weather-, and pavement-specific information) that cannot be used for the identification of underlying accident states at the roadway segment level.

In the context of injury-severity studies, Jiang et al. (2013) identified two possible underlying classes for the zero-injury accidents: (i) the injury-free accidents; and (ii) the injury-prone accidents. To statistically account for this possibility within an ordered probability setting, Jiang et al. (2013) employed a zero-inflated ordered probit approach; the latter can address the excessive amount of zero observations by identifying distinct regimes on the basis of binary probit and traditional ordered probit processes.

This study aims to extend the methodological potential of the zero-inflated ordered probit model, by additionally accounting for the common unobserved characteristics captured by the disturbance terms of the two distinct processes, and by simultaneously relaxing the fixed thresholds restriction of the traditional ordered probit formulation. To that end, a zero-inflated hierarchical ordered probit approach with correlated disturbances is employed for the first time – to the authors' knowledge – in accident research. The unrestricted formulation of the latter allows for capturing possible correlation between the disturbance terms corresponding to the binary and ordered probit components, while at the same time, decomposes the fixed thresholds as a function of threshold-specific explanatory variables. To examine the statistical merits of the proposed approach, various ordered probit counterparts (i.e., hierarchical ordered probit and zero-inflated ordered probit models) are estimated and compared *vis-à-vis* the former.

Methodology

The zero-inflated hierarchical ordered probit framework is formulated on the basis of two separate, but interrelated processes: (i) a binary probit process, which serves as an assignment function for the accidents between the zero-injury state and the ordered injury-severity state; and (ii) an ordered probit process, which can provide the determinants of the injury-severity outcomes, under the condition that the accident does not belong to the zero-injury state.

Following the formulation of Harris and Zhao (2007) and Greene (2016), the model structure of the zero-inflated ordered probit model consists of two latent variable equations corresponding to the aforementioned distinct processes. The splitting function between the zero-injury and ordered injury-severity state is expressed through a binary probit model (Harris and Zhao, 2007; Greene, 2016):

$$k_i{*} = \mathbf{dC}_i + w, \quad k_i = 1 \;\; (k_i{*} > 0) \tag{1}$$

where, k_i* denotes a latent variable being observed as a binary variable k_i, with the latter representing whether an accident belongs to the zero-injury-state ($k_\text{i} = 1$) or not ($k_i = 0$), **C** denotes a vector of explanatory variables that determine whether an accident belongs to the zero-injury state or not, **d** is a vector of estimable parameters, and w is the disturbance term, which is assumed to follow the standard normal distribution (with mean equal to zero and standard deviation equal to one).

In this context, the probability of an accident belonging in the zero-injury state is computed as (Washington et al., 2011):

$$P_i(k_i = 1) = \Phi(\mathbf{dC}_i) \tag{2}$$

where, Φ denotes the standardized cumulative normal distribution (with mean equal to zero and standard deviation equal to one), and all other terms are as previously defined.

Conditional on an accident belonging to the ordered injury-severity state, the specific accident injury-severity outcome is determined through the ordered probit model component. In this context, the latent variable z_i*, which constitutes the basis of the ordered probability formulation, is defined as (Harris and Zhao, 2007; Washington et al., 2011; Greene, 2016):

$$z_i^* = \boldsymbol{\beta}\mathbf{X}_i + \varepsilon, \tag{3}$$

and

$$z_i = \begin{cases} 0 \ \textit{if} \ z_i^* \leq 0 \ \textit{or} \ k_i = 1 \\ j \ \textit{if} \ \mu_{j-1} < z_i^* < \mu_j \ \textit{and} \ k_i = 0 \\ J \ \textit{if} \ z_i^* \geq \mu_{J-1} \ \textit{and} \ k_i = 0, \end{cases} \qquad \text{with } j=1,2\ldots J\text{-}1 \tag{4}$$

where, z_i is an integer corresponding to the observed injury-severity outcome of the accident i, **X** is a vector of explanatory variables, $\boldsymbol{\beta}$ is a vector of estimable parameters associated with **X**, j denotes the observed injury-severity level, J denotes the most severe injury outcome, μ denote the threshold parameters of the ordered probit model that distinguish the various injury-severity outcomes, and ε is a normally distributed disturbance term[1].

On the basis of Equation (4), the probabilities corresponding to various injury-severity outcomes are expressed as (Washington et al., 2011):

$$P_i(z = 0 \mid k = 0) = \Phi(-\boldsymbol{\beta}\mathbf{X}_i) \tag{5}$$

[1] It should be noted that the threshold μ_0 is assumed to be zero, without loss of generality (Washington et al., 2011). To that end, the number of estimable thresholds is equal to Γ-2, where Γ denotes the number of the ordered outcomes of the dependent variable. In this study, four injury-severity outcomes are considered, thus only two thresholds are estimated.

$$P_i(z = j \mid k = 0) = \Phi(\mu_j - \boldsymbol{\beta}\mathbf{X}_i) - \Phi(\mu_{j-1} - \boldsymbol{\beta}\mathbf{X}_i) \tag{6}$$

$$P_i(z = J \mid k = 0) = 1 - \Phi(\mu_{J-1} - \boldsymbol{\beta}\mathbf{X}_i)\,. \tag{7}$$

Note that the zero-inflated ordered probit model is formulated to statistically account for two different types of zeros for the no-injury outcomes, associated with: (i) accidents that belong to the zero-injury state ($k_i = 1$); and (ii) accidents that do not belong to the zero injury-severity state, but they result in no evident injury ($k_i = 0$ and $z_i = 0$). In this context, the unconditional probability of an accident to result in a no-evident injury, $P(z=0)$, is directly associated with the probabilities of the aforementioned conditions to occur, and is formulated as (Harris and Zhao, 2007; Jiang et al., 2013):

$$P(z = 0 \mid \mathbf{C}, \mathbf{X}) = P(k = 1 \mid \mathbf{C}) + P(k = 0 \mid \mathbf{C})\, P(z = 0 \mid \mathbf{X})\,. \tag{8}$$

In a similar fashion, the unconditional probability of an accident to result in an injury-severity outcome j, $P(z=j)$, is also dependent on the probability that the specific accident belongs to the ordered injury-severity state, and thus, is expressed as (Harris and Zhao, 2007; Jiang et al., 2013):

$$P(z = j \mid \mathbf{C}, \mathbf{X}) = P(k = 0 \mid \mathbf{C}) P(z = j \mid \mathbf{X})\,. \tag{9}$$

It should be noted that the binary probit and ordered probit model components may encounter similar unobserved characteristics, since the component-specific dependent variables stem from the injury-severity outcome of the same accident. In this context, there is strong possibility for the random disturbance terms of the two components to be correlated. Not accounting for the contemporaneous disturbance term correlation between the latent variables of the model formulation may result in inconsistent parameter estimates and inaccurate statistical inferences (Washington et al., 2011; Sarwar et al., 2017). To that end, the disturbance terms are specified to follow a standard bivariate normal distribution, which allows for possible correlation between the former through the estimation of a correlation coefficient.

To decompose the aforementioned unconditional probabilities, in terms of the explanatory variables involved in the binary probit and ordered probit model components, Equations (2) and (5) - (7) are incorporated in Equations (8) and (9), with the latter two being re-written, respectively, as:

$$P_i(z=0) = \Phi(d_i'C_i) + \Phi_2(-d_i'C_i, -\boldsymbol{\beta}\mathbf{X}_i, \lambda) \quad (10)$$

$$P_i(z=j) = \Phi_2(-d_i'C_i, \mu_j - \boldsymbol{\beta}\mathbf{X}_i, \lambda) - \Phi_2(-d_i'C_i, \mu_{j-1} - \boldsymbol{\beta}\mathbf{X}_i, \lambda) \quad (11)$$

where, Φ_2 denotes the cumulative function of the bivariate standard normal distribution, λ denotes the correlation coefficient between the disturbance terms of the binary and ordered probit components, and all the other terms are as previously defined.

To account for the heterogeneity arising from the fixed nature of the thresholds that determine the injury-severity outcomes, the hierarchical ordered probit formulation is incorporated into the zero-inflated model structure.[2] Specifically, the thresholds are allowed to vary across the accident observations, as a function of unique explanatory variables (Greene, 2016; Fountas and Anastasopoulos, 2017):

$$\mu_{i,j} = \exp(t_j + \boldsymbol{\gamma}\mathbf{U}_i) \quad (12)$$

[2] The generalized ordered logit/probit and the partial proportional odds models constitute alternate ordered probability approaches, in which the observation-specific thresholds are estimated as a function of threshold-specific explanatory variables (Eluru, 2008; Eluru and Yasmin, 2015).

where, t_j is a threshold-specific constant term, **U** denotes a vector of explanatory variables determining the thresholds, and **γ** is a vector of estimable parameters corresponding to **U**.

For the estimation of the model parameters, the maximum likelihood estimation technique is employed. To that end, the log-likelihood function is defined as (Harris and Zhao, 2007):

$$LL = \sum_{i=1}^{N}\sum_{j=0}^{J} v_{i,j} \ln[P(z=j \mid \mathbf{C}_i, \mathbf{X}_i, \mathbf{d}, \boldsymbol{\beta}, \mu_j, \lambda)] \qquad (13)$$

where, $v_{i,j}$ is a binary indicator indicating whether the specific injury-severity outcome is observed or not, and all the other terms are as previously defined.

To better interpret the effect of the explanatory variables on the probability of each injury-severity outcome, marginal effects are estimated (Harris and Zhao, 2007). As far as the binary probit model component is concerned, marginal effects provide the change in the probability of an accident to belong in the zero-injury state, due to a one unit change in the model component-specific explanatory variables, and are computed as:

$$ME_{\mathbf{C}} = \frac{\partial P(k=1)}{\partial \mathbf{C}} = \frac{\partial[\Phi(\mathbf{d}\mathbf{C}_i)]}{\partial \mathbf{C}}. \qquad (14)$$

In ordered probit component, marginal effects provide the change in the probability of a specific injury-severity outcome, due to a one unit change in the explanatory variables associated with the ordered injury-severity state, and are computed as:

$$ME_{\mathbf{X}} = \frac{\partial P(z=j)}{\partial \mathbf{X}} = \frac{\partial[\Phi_2(-d_i'C_i, \mu_j - \boldsymbol{\beta}\mathbf{X}_i, \lambda) - \Phi_2(-d_i'C_i, \mu_{j-1} - \boldsymbol{\beta}\mathbf{X}_i, \lambda)]}{\partial \mathbf{X}}. \qquad (15)$$

Note that in the case of indicator variables, marginal effects measure the difference in the predicted probability, due to a change from “0” to “1” in the value of the specific variable.

Data

To illustrate the potential of the proposed zero-inflated approach in identifying distinct states of the injury-severity mechanism, data from single-vehicle accidents – which occurred on urban and rural highways in the State of Washington – are analyzed. The dataset includes aggregate and highly disaggregate information from police-reported accidents that occurred throughout a three-year period, from 2011 to 2013.

The portion of the dataset with aggregate information includes accident-specific information (date, time and location of the accident; number of passengers; injury-severity outcome), as well as detailed collision-specific characteristics (type of collision; vehicle action during the accident; sequence of events after the collision; airbag deployment; environmental and lighting conditions). The aggregate accident-related information also consists of driver-specific characteristics (age; gender; driver's license status; level of drugs/alcohol consumption; level of driver's consciousness and inattention; seat belt use), and vehicle-specific characteristics (vehicle type, make and model; number of axles; vehicle condition before and after the accident; date of first vehicle registration). In addition, roadway-specific information (roadway geometrics; cross-section elements; horizontal and vertical curvature; functional class; access control), and traffic characteristics (average annual daily traffic, traffic counts for various vehicle types; posted speed limit; traffic control infrastructure and systems) are also included in the dataset. The portion of the dataset with aggregate information was acquired from the Highway Safety Information System (HSIS), and the SHRP2 Roadway Information Database (RID).

The portion of the dataset with disaggregate information includes a broad range of time-varying weather and pavement surface condition data. To account for their dynamic variations, the time-varying characteristics were clustered in 30-minute time intervals, corresponding to pre-crash and at-crash time points. In this context, time-varying information is available for the time of the accident (*t*) and for two time points preceding the time of the accident (*t*-30 and *t*-60, respectively). Note that the time-varying information was drawn from meteorological measurements incorporated in the Meteorological Assimilation Data Ingest System (MADIS). For a more detailed description of the time-varying characteristics, see previous studies that also used the specific dataset (Fountas et al., 2018a; Fountas et al., 2018b; Fountas et al. 2018c; Fountas, 2018).

The injury-severity outcomes of this study are expressed in a four-level ordinal scale: no-evident injury; non-incapacitating injury, incapacitating injury, and fatal injury (fatality). In consistency with previous studies (Fountas and Anastasopoulos, 2017; Fountas et al., 2018b), the reported injury-severity outcome is defined as the level of injury of the most severely injured person that was involved in the accident. In total 2,690 observations included complete information and were thus used for the empirical analysis. Figure 1 presents a histogram with the number of accidents per injury-severity outcome. The vast majority of the accidents considered in the analysis (approximately 72.8%) resulted in no-evident injury, as shown in Figure 1. Table 1 presents summary statistics for key variables – those that were found to be statistically significant determinants of the injury-severity outcomes.

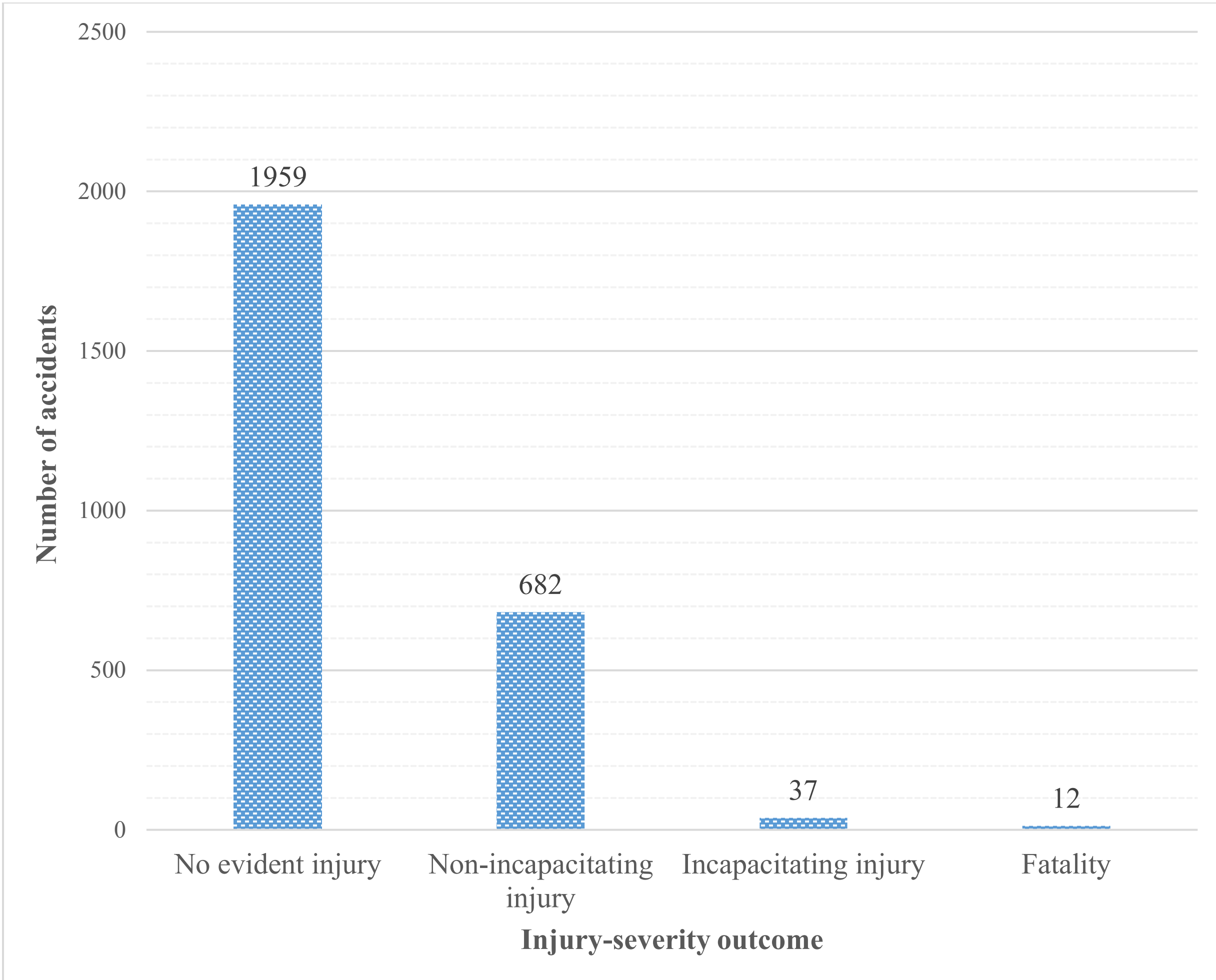

Figure 1. Number of accident observations per injury-severity outcome.

Table 1. Descriptive statistics of key variables.

	Mean or %	Std. Dev.	Min	Max
Roadway geometry characteristics				
Bridge indicator (1 if a bridge was present on the highway segment, 0 otherwise)	11.16%	–	0.000	1.00
Horizontal and vertical curvature indicator (1 if the accident occurred on a straight and level highway segment, 0 otherwise)	40.84%	–	0.000	1.00
Driver-specific characteristics				
Alcohol/Drugs indicator (1 if the driver was under the influence of alcohol or drugs, 0 otherwise)	10.49%	–	0.000	1.00
Driver's consciousness indicator (1 if the driver was apparently fatigued or ill, 0 otherwise)	6.37%	–	0.000	1.00
Restraints use indicator (1 if the driver did not use any restraint, 0 otherwise)	92.65%	–	0.000	1.00
Accident-specific characteristics				
Overturned vehicle indicator (1 if the vehicle overturned, 0 otherwise)	7.48%	–	0.000	1.00
Animal indicator (1 if an animal was involved in the accident, 0 otherwise)	3.51%	–	0.000	1.00
Pedestrian indicator (1 if a pedestrian was involved in the accident, 0 otherwise)	1.08%	–	0.000	1.00
Airbag Deployment Indicator (1 if airbag deployed, 0 otherwise)	23.60%	–	0.000	1.00
Towed vehicle indicator (1 if towed, 0 otherwise)	63.55%	–	0.000	1.00
Off-the-road vehicle indicator (1 if the vehicle ran-off-the-road during the accident, 0 otherwise)	27.81%	–	0.000	1.00
Time-varying weather characteristics				
Relative humidity indicator in t–30 minutes (1 if humidity was greater than 75%, 0 otherwise)	74.47%	–	0.000	1.00
Threshold parameters decomposition				
Gender indicator (1 if the driver is female, 0 otherwise)	40.46%	–	0.000	1.00
Vehicle direction indicator (1 if the vehicle was traveling on a straight ahead direction at the time of the accident, 0 otherwise)	86.11%	–	0.000	1.00
Lighting conditions indicator (1 if the accident occurred during the daylight, dawn or dusk time periods, 0 otherwise)	54.97%	–	0.000	1.00
Segment length indicator (1 if the segment length is greater than 0.55 miles, 0 otherwise)	20.65%	–	0.000	1.00
Zero-injury state				
Annual Average daily traffic (AADT) per lane Indicator (in thousands of vehicles per day)	18.75	8.44	0.000	45.82
Month indicator (1 if the accident occurred after December 31st and before April 1st, 0 otherwise)	27.42%	–	0.000	1.00

	Mean or %	Std. Dev.	Min	Max
County indicator (1 if the accident occurred in the King county, 0 otherwise)	62.35%	–	0.000	1.00
Truck involvement indicator (1 if a truck was involved in the accident, 0 otherwise)	35.17%	–	0.000	1.00
Traffic control indicator (1 if traffic was not controlled, 0 otherwise)	96.34%	–	0.000	1.00

Analysis and Results

To statistically assess the potential of the zero-inflated hierarchical ordered probit model with correlated disturbances, conventional hierarchical ordered probit and zero-inflated ordered probit models are also estimated.

Figure 1 shows that the accident injury-severity data exhibit significant clustering at zero observations (i.e., observations associated with a no-evident injury outcome). This supports the use of a modeling technique that can address the excessive amount of zero observations. In this context, the zero-inflated hierarchical ordered probit framework is a methodological candidate for identifying the determinants of the injury-severity outcomes.

Table 2 presents the model estimation results for the zero-inflated hierarchical ordered probit model with correlated disturbances, alongside its model counterparts (i.e., the zero inflated ordered probit, and the hierarchical ordered probit)[3]. Note that under the hierarchical ordered probit approach, all the zero observations (i.e., the observations corresponding to no evident injury outcome) are assumed to stem from a homogeneous underlying state, while the thresholds are allowed at the same time to vary as functions of explanatory variables (Eluru et al., 2008; Fountas and Anastasopoulos, 2017; Xin et al., 2017). On the contrary, the zero-inflated ordered probit approach allows for the presence of two underlying states, without accounting for threshold heterogeneity and for cross-equation disturbance correlation. Table 3 shows the marginal effects of the zero-inflated hierarchical ordered probit model with correlated disturbances. Specifically, two sets of marginal effects are estimated: (i) marginal effects of the explanatory variables associated with the probability of an accident belonging in the zero-injury state; and (ii) marginal effects of the explanatory variables associated with the overall probability of an accident resulting

[3] It should be mentioned that all competitive models are estimated on the basis of the same explanatory variables.

in a specific injury-severity outcome j. As it can be inferred from Equations (14) and (15), the first set of marginal effects is computed only for the group of explanatory variables affecting the zero-injury state probability, whereas the second set is computed for all the explanatory variables, regardless of the injury-severity state they are associated with.

Table 2. Estimation results for the zero-inflated hierarchical ordered probit model with correlated disturbances and its model counterparts.

	Hierarchical ordered probit model		**Zero-inflated ordered probit model**		**Zero-inflated hierarchical ordered probit model with correlated disturbances**	
	Coeff.	*t*-stat	Coeff.	*t*-stat	Coeff.	*t*-stat
Roadway geometry characteristics						
Bridge indicator (1 if a bridge was present on the highway segment, 0 otherwise)	0.132	1.53	0.196	1.77	0.219	2.13
Horizontal and vertical curvature indicator (1 if the accident occurred on a straight and level highway segment, 0 otherwise)	-0.148	-2.69	-0.166	-2.59	-0.170	-2.76
Driver-specific characteristics						
Alcohol/Drugs indicator (1 if the driver was under the influence of alcohol or drugs, 0 otherwise)	0.457	5.49	0.556	5.47	0.51	5.06
Driver's consciousness indicator (1 if the driver was apparently fatigued or ill, 0 otherwise)	0.347	3.21	0.344	2.61	0.312	2.44
Restraints use indicator (1 if the driver did not use any restraint, 0 otherwise)	-0.760	-12.87	-0.676	-9.33	-0.498	-4.97
Accident-specific characteristics						
Overturned vehicle indicator (1 if the vehicle overturned, 0 otherwise)	0.783	6.65	0.849	4.84	0.786	4.71
Animal indicator (1 if an animal was involved in the accident, 0 otherwise)	-0.660	-3.21	-0.654	-2.98	-0.562	-2.63
Pedestrian indicator (1 if a pedestrian was involved in the accident, 0 otherwise)	3.146	9.39	3.231	9.97	3.030	7.80
Airbag deployment indicator (1 if airbag deployed, 0 otherwise)	0.696	11.67	0.799	9.34	0.768	8.86
Towed vehicle indicator (1 if towed, 0 otherwise)	0.097	1.78	0.193	2.91	0.200	3.10
Off-the-road vehicle indicator (1 if the vehicle ran-off-the-road during the accident, 0 otherwise)	0.121	1.95	0.190	2.55	0.204	2.88
Time-variant characteristics						
Relative humidity indicator in *t*–30 minutes (1 if humidity was greater than 75%, 0 otherwise)	-0.316	-5.84	-0.276	-4.38	-0.237	-3.49
Threshold parameters						

	Hierarchical ordered probit model		Zero-inflated ordered probit model		Zero-inflated hierarchical ordered probit model with correlated disturbances	
	Coeff.	*t*-stat	Coeff.	*t*-stat	Coeff.	*t*-stat
Intercept for μ_1	0.252	2.38	–	–	0.272	2.08
Intercept for μ_2	0.625	5.96	–	–	0.617	4.82
μ_1	–	–	1.994	21.82	–	–
μ_2	–	–	2.710	20.84	–	–
Threshold parameters decomposition						
Gender indicator (1 if the driver is female, 0 otherwise)	0.236	2.540	–	–	0.231	2.65
Vehicle direction indicator (1 if the vehicle was traveling on a straight ahead direction at the time of the accident, 0 otherwise)	0.266	2.630	–	–	0.271	2.85
Lighting conditions indicator (1 if the accident occurred during the daylight, dawn or dusk time periods, 0 otherwise)	0.175	2.110	–	–	0.164	2.14
Segment length indicator (1 if the segment length is greater than 0.55 miles, 0 otherwise)	-0.183	-2.200	–	–	-0.183	-2.35
No-Injury State						
Annual Average daily traffic (AADT) per lane Indicator (in thousands of vehicles per day)	–	–	-0.021	2.63	-0.017	3.13
Month indicator (1 if the accident occurred after December 31st and before April 1st, 0 otherwise)	–	–	0.342	2.11	0.251	2.15
King county indicator (1 if the accident occurred in the King county, 0 otherwise)	–	–	-0.235	-1.54	-0.226	-2.22
Truck involvement indicator (1 if a truck was involved in the accident, 0 otherwise)	–	–	-0.331	-1.94	-0.264	-2.34
Traffic control indicator (1 if traffic was not controlled, 0 otherwise)	–	–	-0.288	-1.58	-0.369	-2.41
Correlation of disturbances						
Correlation coefficient (λ)	–	–	–	–	0.511	2.68
Number of observations	2690		2690		2690	
Log-Likelihood at zero, *LL*(0)	-1780.64		-1780.64		-1780.64	
Log-Likelihood at convergence, *LL*(**β**)	-1575.56		-1575.32		-1558.38	

Table 3. Marginal effects for the zero-inflated hierarchical ordered probit model with correlated disturbances.

	Zero-injury state *P*(*k*=0)	Overall marginal effects			
		No-evident injury P(*j*=0)	*Non-incapacitating injury P*(*j*=1)	*Incapacitating injury P*(*j*=2)	*Fatal injury P*(*j*=3)
Roadway geometry characteristics					
Bridge indicator (1 if a bridge was present on the highway segment, 0 otherwise)	–	-0.0863	0.0755	0.0093	0.0016
Horizontal and vertical curvature indicator (1 if the accident occurred on a straight and level highway segment, 0 otherwise)	–	0.0661	-0.0593	-0.0059	-0.0009
Driver-specific characteristics					
Alcohol/Drugs indicator (1 if the driver was under the influence of alcohol or drugs, 0 otherwise)	–	-0.2014	0.1684	0.0275	0.0055
Driver's consciousness indicator (1 if the driver was apparently fatigued or ill, 0 otherwise)	–	-0.1235	0.1062	0.0146	0.0027
Restraints use indicator (1 if the driver did not use any restraint, 0 otherwise)	–	0.1970	-0.1644	-0.0272	-0.0054
Accident-specific characteristics					
Overturned vehicle indicator (1 if the vehicle overturned, 0 otherwise)	–	-0.3029	0.2350	0.0547	0.0132
Animal indicator (1 if an animal was involved in the accident, 0 otherwise)	–	0.1983	-0.1846	-0.0121	-0.0016
Pedestrian indicator (1 if a pedestrian was involved in the accident, 0 otherwise)	–	-0.5894	0.2058	0.2604	0.1233
Airbag Deployment Indicator (1 if airbag deployed, 0 otherwise)	–	-0.2991	0.2476	0.0425	0.0090
Towed vehicle indicator (1 if towed, 0 otherwise)	–	-0.0776	0.0697	0.0068	0.0011
Off-the-road vehicle indicator (1 if the vehicle ran-off-the-road during the accident, 0 otherwise)	–	-0.0800	0.0707	0.0080	0.0013
Time-variant characteristics					
Relative humidity indicator in t–30 minutes (1 if humidity was greater than 75%, 0 otherwise)	–	0.0932	-0.0820	-0.0096	-0.0016

	Zero-injury state *P*(*k*=0)	Overall marginal effects			
		No-evident injury P(*j*=0)	*Non-incapacitating injury P*(*j*=1)	*Incapacitating injury P*(*j*=2)	*Fatal injury P*(*j*=3)
Threshold parameters decomposition					
Gender indicator (1 if the driver is female, 0 otherwise)	–	0.0000	0.0404	-0.0121	-0.0283
Vehicle direction indicator (1 if the vehicle was traveling on a straight ahead direction at the time of the accident, 0 otherwise)	–	0.0000	0.0522	-0.0145	-0.0377
Lighting conditions indicator (1 if the accident occurred during the daylight, dawn or dusk time periods, 0 otherwise)	–	0.0000	0.0294	-0.0087	-0.0207
Segment length indicator (1 if the segment length is greater than 0.55 miles, 0 otherwise)	–	0.0000	-0.0340	0.0097	0.0243
Zero-injury state variables					
Annual Average daily traffic (AADT) per lane Indicator (in thousands of vehicles per day)	-0.0050	-0.003	0.0024	0.0002	0.0004
Month indicator (1 if the accident occurred after December 31st and before May 1st, 0 otherwise)	0.0580	0.0314	-0.0282	-0.0023	-0.0009
King county indicator (1 if the accident occurred in the King county, 0 otherwise)	-0.0809	-0.0446	0.0400	0.0033	0.0013
Truck involvement indicator (1 if a truck was involved in the accident, 0 otherwise)	-0.0876	-0.0492	0.0440	0.0037	0.0015
Traffic control indicator (1 if traffic was not controlled, 0 otherwise)	-0.0158	-0.0090	0.0080	0.0007	0.0003

Table 2 shows that 5 explanatory variables are found to significantly affect the probability of an accident to be involved in the zero-injury state, under the zero-inflated hierarchical ordered probit approach. These variables include traffic characteristics (annual average daily traffic, and traffic control indicator), accident-specific characteristics (winter month indicator, and truck involvement indicator) and a location-specific attribute (County indicator). In contrast with the past zero-inflated ordered probit approaches, herein, a positive sign of a parameter associated with the zero-injury state implies that the probability of an accident belonging in the zero-injury state increases. Under this consideration, accidents occurred during the winter period are more likely to belong in the zero-injury state, with the corresponding probability increasing by 0.058, as shown in Table 3. This finding is intuitive and can be attributed to the possible risk-compensating behavior of drivers under inclement weather conditions (for a further discussion, see Fountas and Anastasopoulos, 2017; Fountas et al., 2018a; Fountas, 2018). In line with previous studies (Eluru et al., 2008; Xiong et al., 2014; Yasmin et al., 2016; Fountas and Anastasopoulos, 2017; Sarwar and Anastasopoulos, 2017; Fountas et al., 2018b), the annual average daily traffic, the traffic control indicator, the County indicator, and the truck involvement indicator are found to decrease the zero-injury state probability (by -0.005, -0.016, -0.081, and -0.088, respectively, as shown in Table 3). In line with Jiang et al. (2013), accidents occurred in highways with high annual traffic volumes have the potential to result in more severe injury outcomes, possibly due to significant speed fluctuations between peak and non-peak hours, or due to risk-taking driving patterns, which are more evident during congested traffic conditions (i.e., unsafe lane changes, tailgating, stop sign, or traffic signal violations). Similar effect is also observed for truck-involving accidents, since they are more likely to belong in the ordered injury-severity state. This type of single-vehicle accidents is associated with release of greater amount of energy, compared to accidents involving passenger cars, possibly resulting in more severe injury outcomes (Chen et al., 2015).

Moving on to the ordered injury-severity state results, Table 2 demonstrates that 12 explanatory variables generate statistically significant parameters (at a 0.95 level of confidence) under the zero-inflated hierarchical ordered probit approach with correlated disturbances. Specifically, one roadway characteristic (bridge presence indicator), two driver-specific characteristics (alcohol/drug consumption indicator, and driver's consciousness indicator) and five accident-specific characteristics (overturned vehicle indicator, pedestrian involvement indicator, airbag deployment indicator, towed vehicle indicator, and off-the-road vehicle indicator) are found to increase the likelihood of more severe injury outcomes (non-incapacitating injury, incapacitating injury, and fatality). Alcohol or drug-impaired drivers and drivers under the effect of fatigue or physical illness are associated with an intuitive propensity towards critical driving errors that may lead to injury outcomes of higher severity (Behnood et al., 2014). The accident-specific determinants may be capturing either unobserved features of the single-vehicle accident mechanism relating to the post-crash vehicle condition (Fountas and Anastasopoulos, 2017), or unobserved contributing factors relating to the actions of driver or other entities (e.g., pedestrians, passengers) that are involved in the accident (Mannering et al., 2016; Behnood and Mannering, 2016).

In contrast, one roadway characteristic (horizontal and vertical curvature indicator), one driver-specific characteristic (restraints use indicator), one accident-specific characteristic (animal involvement indicator), and one time-varying weather characteristic (relative humidity indicator) are found to increase the likelihood of a no evident injury, conditional on the accident belonging to the ordered injury-severity state. The effect of such determinants of injury-severity outcomes is intuitive and likely reflects the risk-compensating behavior of drivers under the corresponding driving circumstances. Similar findings – in terms of sign and magnitude – were also obtained by a number of studies, as for example in Abdel-Aty (2003), Quddus et al. (2009), Russo et al. (2014), Wu et al. (2014), and Seraneeprakarn et al. (2017), to name a few.

Turning to the results relating to the threshold parameters decomposition, positive sign of a threshold-specific parameter implies an increase in the values of both intermediate thresholds. Such increase is equivalent to a shift of the threshold values towards the right tail of the ordered probability distribution and, in turn, to a decrease of the likelihood corresponding to higher injury severity outcomes (for further details, see Washington et al, 2011). The opposite effect is observed when the sign of a threshold-specific parameter is negative. In this context, the variables indicating female drivers, straight-ahead direction of the vehicle at the time of the accident, and non-dark conditions at the time of the accident, are all associated with greater threshold values, and subsequently with lower likelihood of more severe injury outcomes (incapacitating injury and fatal injury). Given that the threshold between no evident injury and non-incapacitating injury is pre-specified as zero without loss of generality, the increase of the first threshold (μ_1) for female drivers implies that the accidents involving female drivers are more likely to result in incapacitating injury. Such effect is also shown by the marginal effects (Table 3) and is in line with Ulfarsson and Mannering (2004). Similar inferences can be drawn for the effect of vehicle direction indicator and lighting condition indicator on the likelihood of the non-incapacitating injury.

In contrast, longer highway segments (with length greater than 0.55 miles) are found to decrease the threshold values, and subsequently, to increase the likelihood of more severe injury outcomes (incapacitating injury and fatal injury). This finding is line with previous research

(Fountas et al., 2018a) and possibly captures the risk-taking behavior of drivers when driving on highway segments with consistent design elements throughout their lengths.

The correlation coefficient (λ) is statistically significant and strong in magnitude showing that the disturbance terms of the binary and ordered probit model components capture commonly shared unobserved characteristics. The latter is anticipated, since both zero-injury and ordered injury-severity states are associated with the same accident. However, the positive sign of the correlation coefficient may highlight the structural differences relating to the underlying mechanisms of the aforementioned states. In words, the unobserved characteristics that play an important role in the determination of the injury-severity outcomes may have opposite effects within each underlying state. For example, the unobserved characteristics that may be associated with an increase in the zero-injury probability have – under certain circumstances[4] – the potential to favor injury outcomes of higher severity. In fact, this finding can be attributed to the driver-specific unobserved heterogeneity that stems from the variations in the response of drivers against various internal or external stimuli (Kweon and Kockelman, 2003; Bunn et al., 2005; Awadzi et al., 2008; Mannering and Bhat, 2014; Mannering, 2018).

To determine which of the conventional ordered probit or the zero-inflated hierarchical ordered probit approach is more appropriate for statistically analyzing injury-severity data, the Vuong test statistic is employed and computed. Given that the two aforementioned approaches are non-nested, the Vuong test (Vuong, 1989) is based on the calculation of the statistic m_i for each accident observation, as follows (Vuong, 1989; Washington et al., 2011):

$$m_i = LN[f_1(q_i \mid \mathbf{X}_i) / f_2(q_i \mid \mathbf{X}_i)] \tag{16}$$

[4] Such circumstances may include heterogeneous drivers' reactions against variations in weather conditions, traffic conditions, or roadway characteristics (such as, roadway type, geometric design elements, pavement surface, etc.). For example, rapid changes of weather conditions may increase driving awareness for some drivers, or result in risk-taking driving behavior for other drivers.

where, $f_1(q_1|\mathbf{X}_1)$ and $f_2(q_1|\mathbf{X}_1)$ denote the probability density functions of the conventional ordered probit (traditional ordered probit or hierarchical ordered probit) and the zero-inflated hierarchical ordered probit with correlated disturbances, respectively. To test whether there is statistically significant difference in the correctly predicted outcomes provided by the competitive models, the Vuong statistic is calculated as (Shankar et al., 1997; Harris and Zhao, 2007; Washington et al., 2011):

$$V = \frac{\overline{m}\sqrt{N}}{\sigma_m} \tag{17}$$

where, $\overline{m}$ and σ_m denote the mean and the standard deviation of the observation-specific m_i, respectively, whereas N is the number of accident observations used for model estimation. For a 0.95 level of confidence, the critical value of the Vuong test statistic is equal to 1.96. Thus, large positive values of the Vuong statistic – greater than the critical value (i.e., if $V > 1.96$) – support the conventional ordered probit model, whereas large negative values of the Vuong statistic – lower that the negative equivalent of the critical value (i.e., if $V < -1.96$) – favor the zero-inflated model. Note that if the absolute value of the Vuong test statistic is lower that the critical value (i.e., if $|V| < 1.96$), the test statistic cannot provide conclusive results. The results of the Vuong tests conducted among the zero-inflated hierarchical ordered probit model with correlated disturbances, and the traditional and hierarchical ordered probit models are presented in Table 4, and support the use of the zero-inflated model.

Table 4. Statistical tests and goodness-of-fit measures.

	Zero-inflated hierarchical ordered probit model with correlated distrurbances vs ordered probit model	***Zero-inflated hierarchical ordered probit model with correlated distrurbances vs hierarchical ordered probit model***	***Zero-inflated hierarchical ordered probit model with correlated distrurbances vs zero-inflated ordered probit model***
Degrees of freedom	–	–	5
Level of confidence	–	–	0.95
Resulting χ^2	–	–	33.875
Critical χ^2	–	–	11.071
Vuong statistic	-5.557	-3.736	–
Statistically Superior Model	Zero-inflated hierarchical ordered probit model with correlated disturbances	Zero-inflated hierarchical ordered probit model with correlated disturbances	Zero-inflated hierarchical ordered probit model with correlated disturbances
	Hierarchical ordered probit model	**Zero-inflated ordered probit model**	**Zero-inflated hierarchical ordered probit model with correlated disturbances**
AIC[a]	3187.100	3188.600	3164.800
AIC_C[b]	3187.356	3189.050	3165.250
ρ^2	0.115	0.115	0.125
Corrected ρ^2	0.105	0.105	0.111

[a] The Akaike Information Criterion (AIC) is calculated as: $AIC = 2[K - LL(\boldsymbol{\beta})]$, where, K is the number of model parameters.

[b] The corrected Akaike Information Criterion (AIC_C) is computed as: $AIC_C = AIC + 2K(K+1)/(N-K-1)$, where N indicates the number of accident observations used for model estimation and all other terms as previously defined.

To compare the statistical performance of the competitive zero-inflated models (i.e., the zero-inflated probit model and the zero-inflated hierarchical ordered probit with correlated distrurbances), a likelihood ratio test is conducted. The likelihood ratio test statistic is defined as (Washington et al., 2011):

$$\chi^2 = -2\left[LL(\boldsymbol{\beta}_{z1}) - LL(\boldsymbol{\beta}_{z2})\right] \tag{18}$$

where, $LL(\boldsymbol{\beta}_{Z1})$ and $LL(\boldsymbol{\beta}_{Z2})$ denote the log-likelihood at convergence for the zero-inflated ordered probit and the zero-inflated hierarchical ordered probit with correlated disturbances, respectively. The likelihood ratio test statistic is chi-squared distributed, with the degrees of freedom being equal to the number of additional estimable parameters included in the zero-inflated hierarchical ordered probit model with correlated disturbances. Table 4 presents the results of the likelihood ratio test, which support the statistical superiority of the zero-inflated hierarchical ordered probit approach over its zero-inflated counterpart, with greater than 95% level of confidence.

To further compare the relative statistical fit of nested and non-nested models, information-based goodness-of-fit measures are estimated (Harris and Zhao, 2007). In line with previous work (Anastasopoulos, 2016; Sarwar et al., 2017; Fountas and Anastasopoulos, 2017; Guo et al., 2018), four different measures are examined (the corresponding mathematical formulations are presented in Table 4): (i) the ρ^2; (ii) the corrected ρ^2; (iii) the Akaike Information Criterion; and (iv) the Akaike Information Criterion with correction for the sample size. The resulting values of the goodness-of-fit measures are in line with the Vuong and likelihood ratio tests, and indicate the statistical superiority of the zero-inflated hierarchical ordered probit approach with correlated disturbances across all nested and non-nested comparisons.

In terms of explanatory power, Table 2 shows that accounting simultaneously for underlying injury-severity states, threshold heterogeneity, and disturbance term correlation can lead to the identification of statistically significant effects that cannot be identified through the lower-order model formulations. Such trend can be observed either for variables affecting the zero-injury probability, or for variables affecting the ordered injury-severity state probabilities. As far as the first set of variables is concerned, the bridge indicator (presence of a bridge on the highway segment where the accident occurred) is statistically insignificant in the hierarchical ordered probit model, and becomes statistically significant under the proposed approach. For the second set of variables, the effect of King County and traffic control indicators on the zero-injury probability is statistically insignificant under the zero-inflated ordered probit approach, and becomes significant in the zero-inflated hierarchical ordered probit approach with correlated disturbances.

Furthermore, the identification of different sets of explanatory variables for the zero-injury and ordered injury-severity states can shed more light on the decomposition of the overall probability for the no-injury outcome. Such decomposition can be demonstrated through the computation of the zero-injury state probability, as well as of the ordered no-injury probability, for the set of accidents with observed no-injury outcomes. Interestingly, the average portion of the no-injury probability attributed to the zero-injury state is equal to 26%, whereas the average portion of the no-injury probability attributed to the ordered injury-severity state is equal to 74%. Figure 2 illustrates these two portions of the overall no-injury probability across the no-injury accident observations. Specifically, the accumulation of no-injury observations in the upper left section of the Figure provides additional evidence with regard to the greater influence of the ordered-injury severity state mechanism on the overall no-injury probability.

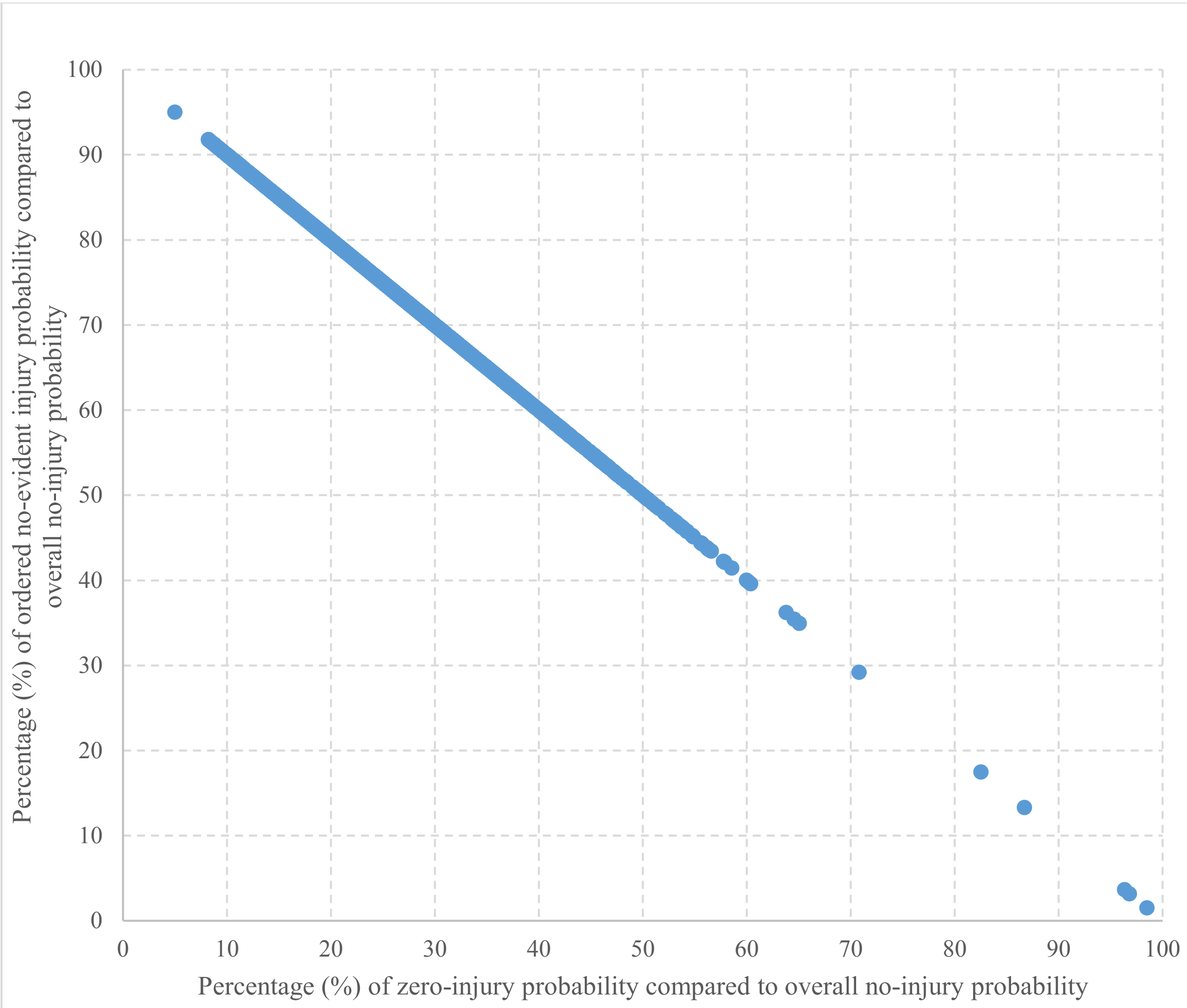

Figure 2. Portion of zero-injury probability versus the portion of the ordered no-evident injury probability relative to the overall no-injury probability.

To further assess the competitive modeling approaches, their forecasting accuracy is also investigated. To that end, two groups of forecasting accuracy measures are computed: (i) probability-based measures; and (ii) error-based and relative prediction performance measures. The first group of measures includes the percentage of correctly predicted outcomes and the average predicted probability for the observed outcomes (Yasmin et al., 2014; Fountas and Anastasopoulos, 2017; Fountas et al., 2018c). For each accident observation, the injury-severity outcome that corresponds to the highest model-predicted probability is determined as the predicted outcome. A prediction is considered as correct when the model-predicted outcome coincides with the observed outcome. For each injury outcome, this metric is calculated as the ratio of the number of observations with correctly predicted outcomes over the total number of observations associated with the specific injury outcome. Similarly, for the computation of the average predicted probability of observed outcomes, the model-exported probability corresponding to the observed injury-severity outcome of the specific observation is considered; thus, among the observations that result in a specific injury outcome, the average value of the latter probabilities is calculated. Table 5 summarizes the results of the two metrics for all competitive models, and shows that the zero-inflated hierarchical ordered probit model with correlated disturbances statistically outperforms its zero-inflated and hierarchical probit counterparts. Specifically, the comparison between the proposed approach and the hierarchical probit model shows that the addition of the zero-inflated structure in the hierarchical ordered framework results in forecasting accuracy improvements, especially for the lower injury-severity outcomes (i.e., no evident injury and non-incapacitating injury). Similarly, the simultaneous consideration of threshold heterogeneity (through the hierarchical structure) and disturbance term correlation improves the forecasting accuracy of the zero-inflated ordered approach, for the majority of injury-severity outcomes (no evident injury, incapacitating injury, and fatal injury). Overall, the proposed approach yields more robust forecasts for all injury-severity outcomes.

Table 5. Probability-based forecasting accuracy measures for the competitive ordered models.

	Hierarchical ordered probit model		***Zero-inflated ordered probit model***		***Zero-inflated hierarchical ordered probit model with correlated disturbances***	
	Percentage of correctly predicted outcomes	**Average predicted probability of observed outcomes**	**Percentage of correctly predicted outcomes**	**Average predicted probability of observed outcomes**	**Percentage of correctly predicted outcomes**	**Average predicted probability of observed outcomes**
No evident injury	94.45%	0.748	94.80%	0.756	95.81%	0.760
Non-incapacitating injury	17.53%	0.333	20.39%	0.340	20.39%	0.340
Incapacitating injury	2.90%	0.091	0.00%	0.074	2.90%	0.083
Fatal injury	27.27%	0.151	0.00%	0.101	27.27%	0.150

Turning to the second group of forecasting accuracy measures (i.e., the error-based measures), it should be noted that the prediction error is derived from the difference between the observed outcome and the model-predicted probability for each observed outcome.[5] In this context, 7 error-based accuracy measures are computed: mean absolute deviation (MAD); sum of squared error (SSE); mean squared error (MSE); root mean squared errors (RMSE); standard deviation of error (SDE); symmetric mean absolute percentage error (SMAPE); and mean squared log of the accuracy ratio (MSLAR). Note that the last measure is relatively new in the econometric research, and its robust formulation allows for a better evaluation of the relative prediction accuracy across various competitive models (Tofallis, 2015). Specifically, for each accident observation, the accuracy ratio is defined as the ratio of the model-predicted probability over the

[5] It should be noted that for the calculation of the prediction error, the observed outcome is represented by 1; thus, for each observation, the prediction error is computed as the difference between 1 and the model-predicted probability corresponding to the observed outcome.

observed outcome.[6] Note that for all the measures of the second group, lower values indicate lower prediction error and, in turn, better prediction performance. Table 6 presents the relevant results, which offer supplemental evidence with regard to the statistical superiority of the zero-inflated hierarchical ordered probit approach with correlated disturbances, in terms of forecasting accuracy.

More specifically, the proposed approach provides the lowest prediction error across all competitive models. Another interesting finding arises from the values of the Mean Squared Log of the Accuracy Ratio, for all competitive models. In contrast to the other error-based measures, the former shows that the hierarchical ordered probit produces lower prediction error compared to the zero-inflated ordered probit model. This finding is in line with the results of the non-nested goodness-of-fit measures and may highlight the potential of the specific metric for unbiased model selection, using the relative accuracy as primary criterion of selection (Tofallis, 2015).

[6] For the computation of the accuracy ratio, the observed outcome is represented by 1. To that end, the log of the accuracy ratio is equal to zero when the model-predicted probability of the observed outcome is equal to 1. In this context, lower values of the mean squared log of the accuracy ratio reflect higher forecasting accuracy.

Table 6. Error-based and relative forecasting accuracy measures.

	Hierarchical ordered probit model	***Zero-inflated ordered probit model***	***Zero-inflated hierarchial ordered probit model with correlated disturbances***
Mean absolute deviation (MAD)[b] $MAD = \frac{\sum_{i=1}^{n} \lvert \varepsilon_i \rvert}{n}$	0.371	0.363	0.361
Sum of squared error (SSE)[b] $SSE = \sum_{i=1}^{n} \varepsilon_i^2$	431.275	417.618	414.840
Mean squared error (MSE)[b] $MSE = \frac{\sum_{i=1}^{n} \varepsilon_i^2}{n}$	0.196	0.190	0.188
Root mean squared error (RMSE)[b] $RMSE = \sqrt{\frac{\sum_{i=1}^{n} \varepsilon_i^2}{n}}$	0.442	0.435	0.434
Standard deviation of errors (SDE)[b] $SDE = \sqrt{\frac{\sum_{i=1}^{n} \varepsilon_i^2}{n-1}}$	0.443	0.435	0.434
Symmetric Mean Absolute Percentage Error (SMAPE)[a] $SMAPE = \frac{100\%}{N} \sum_{i=1}^{N} \frac{\lvert p_i - o_i \rvert}{(\lvert p_i \rvert + \lvert o_i \rvert)/2}$	0.498	0.490	0.487
Mean Squared Log of the Accuracy Ratio (MSLAR)[a] $MSLAR = \frac{\sum_{i=1}^{N} (\ln \frac{p_i}{o_i})^2}{N}$	0.842	0.851	0.807

[a] p_i=model-predicted probability o_i=observed outcome
[b] $\varepsilon = o_i - p_i$

Summary and Conclusion

In injury-severity analysis, an inherent limitation of the traditional ordered probit model arises from the assumption that all the zero-injury accidents are governed by a homogeneous underlying (non-)accident-generation mechanism. Due to the heterogeneous conditions that generate the zero-injury accidents, it is likely that the generation mechanisms of such accidents is subject to the effect of two underlying regimes. To statistically account for the possible presence of such regimes, which likely reflect the mechanisms of a zero-injury and an ordered injury severity state, the zero-inflated ordered probit framework was employed. The inflation of the zero-injury observations was accommodated through a joint estimation of a binary probit model and an ordered probit model. To account for the cross-equation disturbance term correlation, as well as for the heterogeneity stemming from the fixed nature of the ordered thresholds (Eluru et al., 2008; Fountas and Anastasopoulos, 2017), a zero-inflated hierarchical ordered probit model with correlated disturbances was estimated for the first time, to the authors' knowledge.

The model structure of the zero-inflated hierarchical ordered probit model with correlated disturbances allowed for the identification of three sets of determinants: (i) determinants of the probability that an accident belongs to the zero-injury state; (ii) determinants of the probability that an accident results in a specific injury-severity outcome, given that the specific accident belongs to the ordered injury-severity state; and (iii) determinants of the thresholds that in turn determine the various injury-severity outcomes. Model estimation results showed that the aforementioned sets of determinants include various roadway- and traffic-specific characteristics, pre-crash time-varying weather conditions, as well as after-crash accident-, driver-, and vehicle-specific attributes. However, each set of determinants was found to be associated with unique explanatory variables illustrating, in such way, the potential of the proposed approach to explicitly identify the influential factors of the underlying states. In addition, the statistical significance of the coefficient reflecting the correlation between the disturbance terms of the model components further supports the use of the proposed approach, especially in terms of capturing commonly shared unobserved characteristics.

The comparative assessment of the zero-inflated hierarchical ordered probit model with correlated disturbances, and of its non-zero-inflated and zero-inflated counterparts (i.e., the hierarchical ordered probit, and zero-inflated ordered probit models) highlighted the statistical superiority of the former. Specifically, the simultaneous consideration of the zero-inflated and hierarchical modeling schemes allowed the identification of statistically significant effects that likely remain masked under the more parsimonious model structures. The results of nested and non-nested statistical tests and goodness-of-fit measures demonstrated the capability of the zero-inflated ordered probit modeling framework to address the preponderance of the zero observations, as well as the statistical merits of the proposed methodological extensions. Through the use of a broad range of absolute and relative forecasting accuracy measures, it was shown that the zero-inflated hierarchical ordered probit approach with correlated disturbances offers prediction improvements, especially for the zero-injury observations.

Despite the statistical and explanatory benefits gained by the proposed approach, the introduction of additional estimation layers on the zero-inflated ordered probit framework may induce computational complexities or data-specific effects. The latter is particularly important when the developed modeling context is implemented on datasets that may exhibit clustering at zero observations, but such clustering cannot be theoretically attributed to the presence of underlying and distinct zero and non-zero states. In cases of datasets with the specific characteristics, the appropriateness of the proposed zero-inflated approach should be thoroughly investigated; specifically, the identification of two underlying sources for the zero-injury observations constitutes critical criterion that should not be overlooked. Otherwise, incorrect implementation of such approach may lead not only to heavily data-specific inferences, but also to biased parameter estimates and inaccurate predictors.

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