Harmonic errors associated with the use of choppers in optical experiments

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Abstract

Rotating chopper wheels are used to modulate optical radiation in many experimental systems, typically for measuring the frequency response of an optical system. The assumption is often made that the chopped radiation varies sinusoidally. In practice, the radiation may have a square wave profile. This introduces high frequency harmonics that can distort the results of frequency measurements, particularly at low frequencies. Furthermore, the use of chopper wheels with different numbers of slots in order to cover different frequency ranges can introduce further effects. A simple change to the experimental set-up can produce a signal that has an approximately trapezoidal profile. Although not an ideal sine wave, we show that the trapezoidal modulation produces a much smaller error than for square wave modulation. In our case, the measurements are applied to the frequency response of pyroelectric infrared detectors, though the results are applicable to more general measurements on optical systems.

1. Introduction

It is very common in optical experiments to modulate the radiation source that is being used. There are a number of reasons for doing this. Firstly, we may be interested in the frequency response of the system. Secondly, noise may be a problem, and the use of modulated radiation, together with lock- in/synchronous amplification can improve the signal-to-noise ratio. Thirdly, the sensing element may respond to the rate of change of radiation, rather than to the absolute level of radiation. In our experiments, we are trying to develop pyroelectric detectors for mid-wavelength infrared radiation. These detectors produce an output that varies with the rate of change of the incident radiation, so it is essential that the radiation is modulated.

1.1 Modulation techniques

Optical sources can be modulated in a number of ways. Some sources are suitable for direct modulation. This is usually controlled electronically, for instance by modulating a control voltage or current. However, many sources cannot be modulated directly, and have to be modulated by interposing a modulator between the source and the detector. The exact type of modulator and its position within the experimental system can vary considerably.

As an example, consider the sort of experiment where the frequency response of an infrared detector is being measured. This type of experiment has been reported widely in the literature (e.g. Schopf et al. [1], Simonne et al.[2].) Although solid-state modulators, such as LCD shutters, are available, they generally do not work in the mid to far infrared. Modulation therefore is commonly done using a physical chopper. Two main types are used: rotating disk, and tuning fork type choppers. Tuning fork choppers have a long life, but can only be used at a fixed resonant frequency. For many experiments, the rotating chopper type is used, as it has advantages in terms of the physical area that is chopped, and in the range of frequencies that can be produced. Mounted on a small, and well-controlled electrical motor, they can cover a wide range of frequencies. This is partly achieved by controlling the rate of rotation of the motor, but also by using chopper wheels with different numbers of slots. Obviously, for a given rate of rotation, the number of slots in the wheel will determine the frequency at which the radiation is modulated.

1.2 Chopper wheel profiles

A variety of chopper wheel profiles are available. In the simplest, slots are cut in the wheel with edges along the radii of the rotating wheel. This is then mounted on an accurately controlled motor (figure 1). These are often the simplest to use. Different profiles are available: these can affect the time profile of the radiation that is passed by the slots in the wheel.

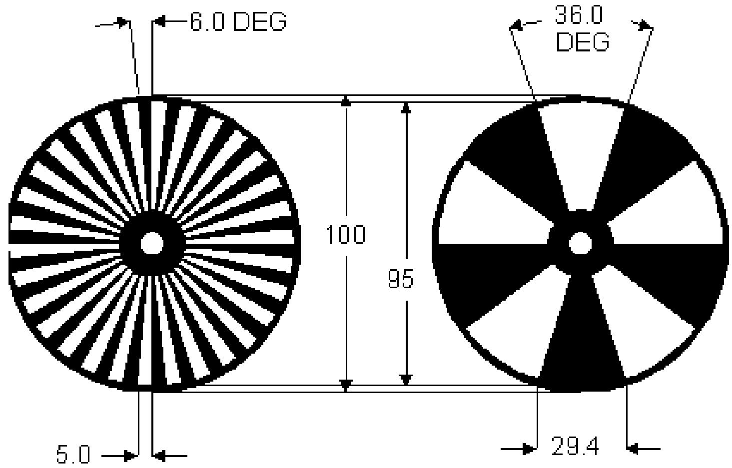

Fig. 1. Profile of 30- and 5-slot chopper wheels, with dimensions in mm (taken from the manual for a typical rotating chopper system [3])

1.3 Experimental set-up

Typically the radiation is modulated in such a way that all the optical radiation is passed when a slot happens to line up with the optical beam, and is obscured entirely when an opaque section of the disk coincides with the beam. However, the transition between these two cases can affect the time profile of the radiation that is passed.

The main consideration when positioning a chopper wheel is whether to put it at a focus in the optical path (figure 2). If a rotating chopper wheel is positioned so that the optical beam is tightly focussed at the plane of the wheel (position A in figure 2), then the profile of the radiation that is passed by the chopper will be approximately a square wave. This will always be an approximation, in that even at a tight focus, the beam has a finite width. However, in many cases this will be much smaller than the aperture in the chopper wheel, and so can be safely ignored.

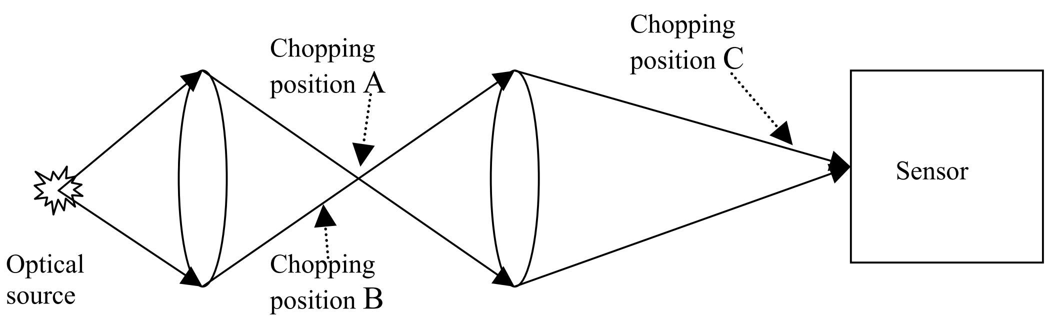

Fig. 2. A typical optical set-up showing a number of possible positions for placing an optical chopper in the optical path.

If the chopper is positioned where the optical beam has a significant diameter, then the chopped radiation will not have a square profile (positions B and C in figure 2). From a position where the beam is entirely obscured, the wheel will rotate so that an increasing amount of light is transmitted as the edge of the aperture moves through the finite-width beam. Eventually, the entire beam will be transmitted as it lines up with an aperture. The amount of light will then remain constant until the beam starts to be obscured by the next edge of the chopper wheel. In most circumstances, it is desirable to ensure that the beam is fully obscured, or fully transmitted, for at least part of the time. This can be achieved by ensuring that the beam is smaller than the width of the smallest aperture that is used.

The result of this is that the optical radiation after the chopper has a profile that is approximately trapezoidal in shape (figure 3). The ratio of the width between the flat and sloping sections of the trapezoid depend on the ratio of the width of the optical beam and the width of the chopper wheel apertures. This has an unfortunate consequence (that we believe has been overlooked by many authors) that changing the chopper wheel, as is often done to achieve different frequency ranges, has a significant effect, since the width of the slots changes with the number of slots.

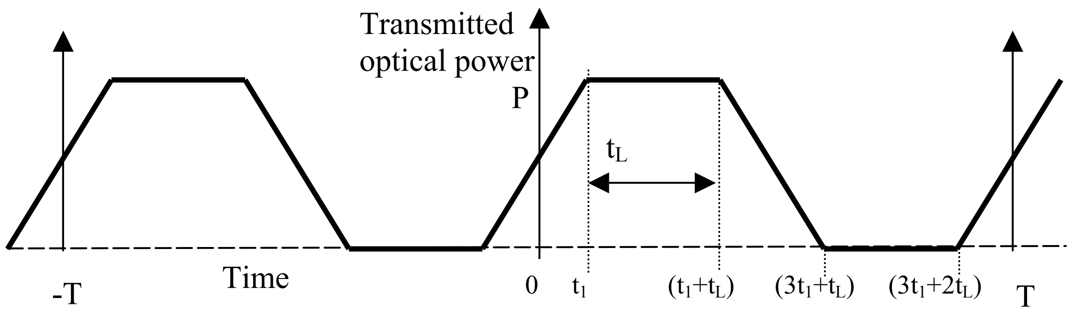

Fig. 3. Trapezoidal modulation. The period is T, and the proportion of the time when the beam is partially obscured depends on t_l.

2 Square Wave Modulation

2.1 Frequency response

A typical experiment will measure the output of a sensor as the frequency is varied. As an example of this, the response of pyroelectric detectors [4] is given by the responsivity equation:

$$V = \eta P_0 p \omega A \frac{R_t}{\sqrt{1+\omega^2\tau_t^2}} \frac{R_e}{\sqrt{1+\omega^2\tau_e^2}} \tag{1}$$

Where η is the emissivity of the surface of the sensor, P_0 is the peak value of the modulation of the applied infrared power, p is the pyroelectric coefficient, A is the sensor area, R_t and R_e are the thermal and electrical resistances, τ_t and τ_e are the thermal and electrical time constants and ω is the angular frequency of the radiation modulation. However, this assumes that the radiation is a sinusoidally varying signal.

If the chopping arrangement used is that of position A in figure 2, or the optical beam has a very small diameter compared with the size of the slots in the chopper wheel, then the modulation, to a good approximation, will be of a square wave form. Logan and McLean [5] have discussed the features of square wave modulation, but with reference to thermal spread. Here, we wish to examine the effect of harmonics on the responsivity.

In general, the square wave can be represented by a Fourier series:

$$f(t) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{2n\pi}{T} t + b_n \sin \frac{2n\pi}{T} t \right) \qquad (2)$$

In the case of the square wave, with a time origin on a rising edge, the cosine and even sine terms are zero, and the series reduces to:

$$f(t) = \frac{P}{2} + \frac{2P}{\pi} \left(\sin \frac{2\pi}{T} t + \frac{1}{3} \sin \frac{3.2\pi}{T} t + \frac{1}{5} \sin \frac{5.2\pi}{T} t + \frac{1}{7} \sin \frac{7.2\pi}{T} t + \ldots \right) \qquad (3)$$

This series has a constant term, P/2, added to the various components. For our experiment, the constant term can be ignored, as the detector will only respond to changing components. The first harmonic term, i.e. the fundamental frequency term, is $2P/\pi$. If the modulation had been sinusoidal, there would have been only this term, but with a coefficient of P/2. Thus, even at the fundamental frequency, the effect of using square wave modulation is different from sinusoidal modulation (by 27%). As higher order terms are added, further differences from the sinusoidal case arise.

2.2 Error calculation

Terms from the series derived in equation 3 can be used in equation 1 to see the effect that they have on the responsivity of the detector. Of course, the series is infinite, so practically the error introduced by truncating the series has to be investigated. The largest contributions are from the first few terms. Taking the fundamental alone leaves an error of approximately 19%. Adding the third harmonic leaves an error of slightly less than 10%, and so on. However, the series is not geometric, and the terms reduce only slowly. This can be seen in the top trace of figure 4.

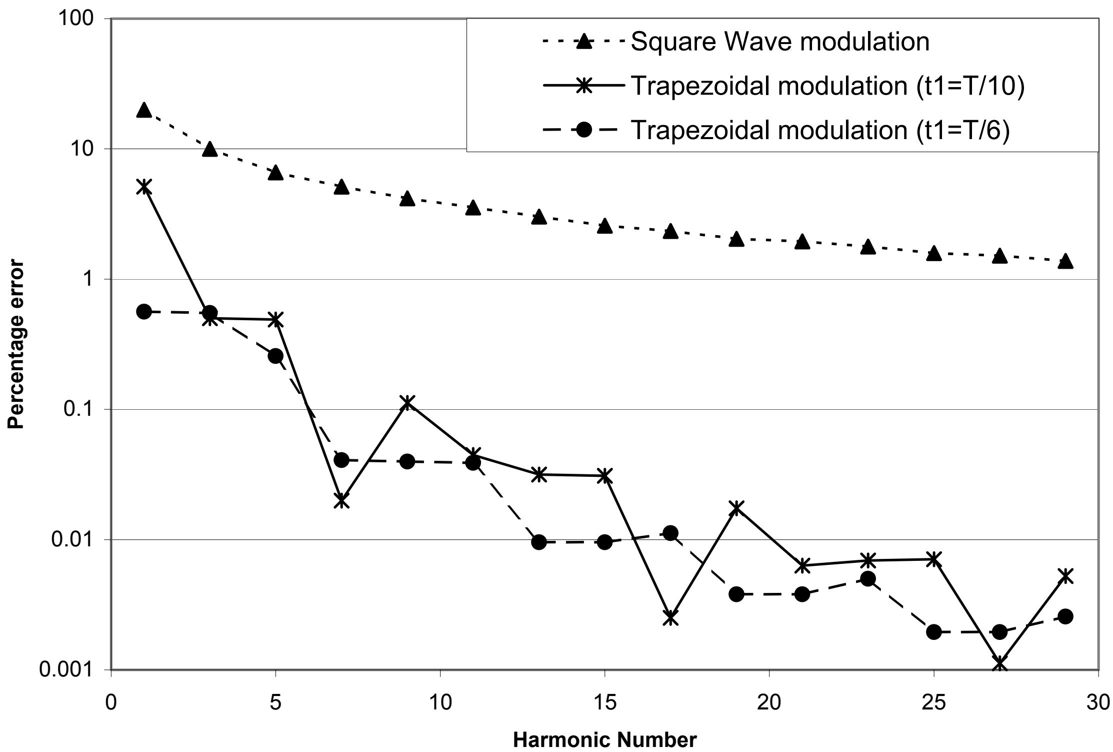

Fig. 4. Percentage error caused by truncating the Fourier expansion at different harmonic numbers.

2.3 Generation of harmonic voltages

The harmonics that are present in a square wave signal have a significant effect on the signal produced by a sensor. In the case of the pyroelectric detector, whose voltage responsivity is given by equation 1, it is possible to calculate the expected contributions to the output for the terms in the Fourier expansion. The results of this calculation can be seen in figure 5, where the calculation has been repeated for different numbers of harmonics.

It can be seen that the biggest deviation from that of the 'ideal' sinusoid occurs at low frequencies. This can be explained as follows. At very low frequencies, equation 1 reduces to a simple form where the output voltage is proportional to ω (and at high frequencies the voltage is proportional to 1/ω). In the low frequency case, when the fundamental frequency is well below the peak in the response curve, the higher harmonics will still have a significant effect since the responsivity increases with frequency. Conversely, at high frequencies, the harmonics have less effect because the responsivity is decreasing with frequency.

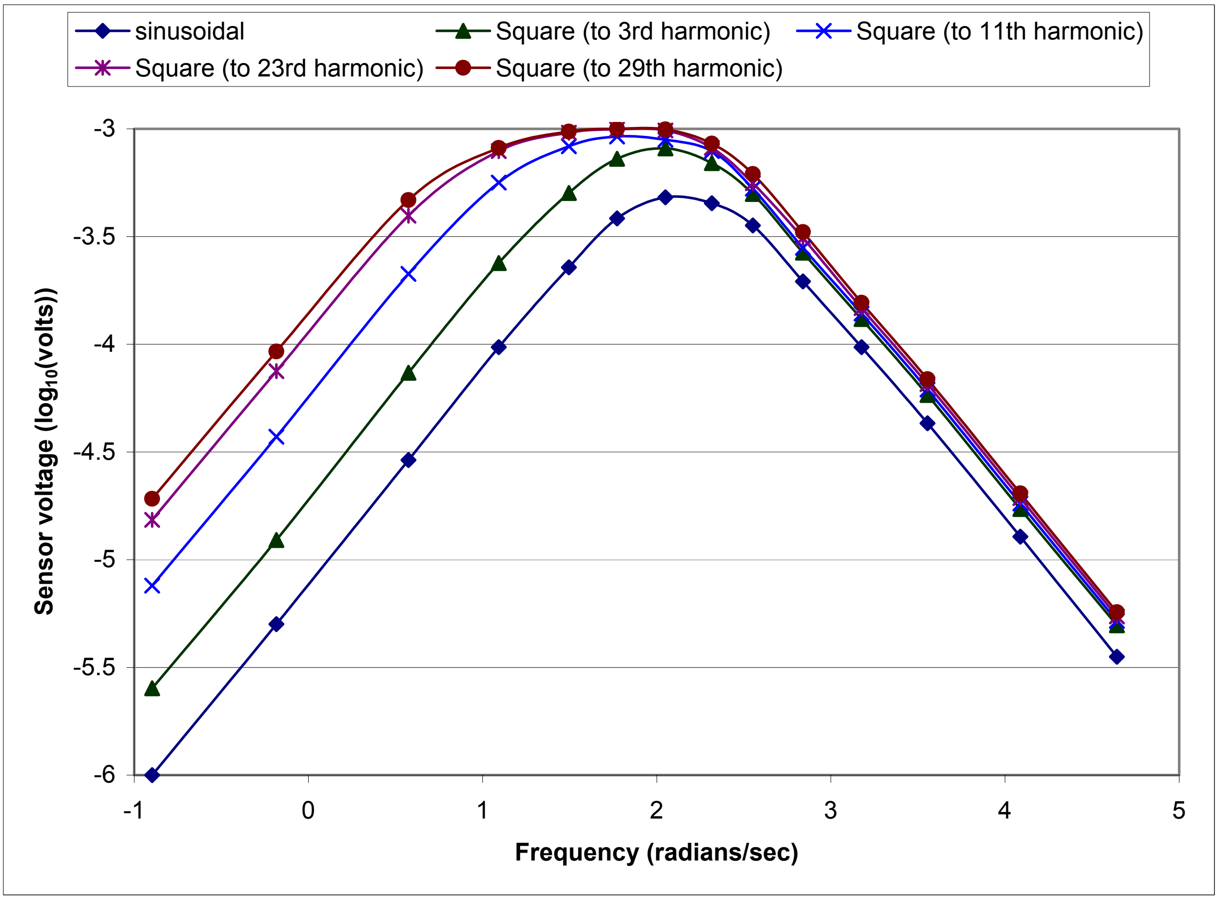

Fig. 5. Output voltage generated by Fourier representations of square wave modulation.

3 Trapezoidal modulation

If the optical beam has a finite size, then the modulation will not be a true sine wave. In practice, the profile will depend on the shape and profile of the beam, and where on the chopper slots it is positioned. A trapezoidal shape is a reasonable approximation to this. Figure 6 compares a simple trapezoidal profile with a profile generated by a chopper blade intersecting a circular beam where the optical beam has a diameter of one third of the slot size. The differences are small (less than 5% maximum) and the trapezoidal profile can be taken as a reasonable approximation to the actual profile. This makes subsequent calculations more tractable.

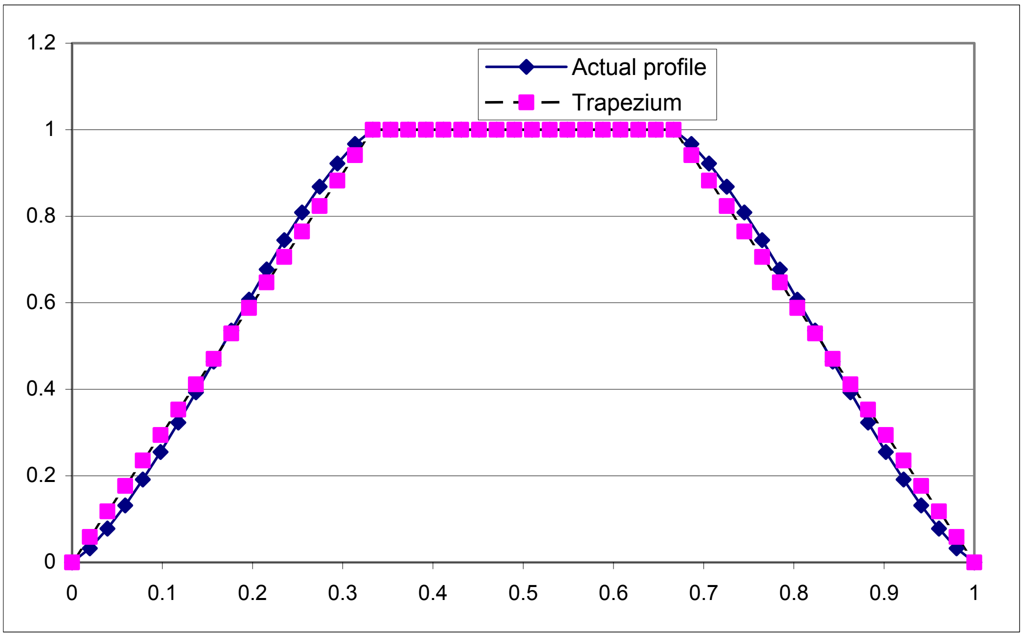

Fig. 6. Comparison of Trapezoidal approximation with actual profile

Taking an origin as in Figure 3, the expressions for the radiation as a function of time are:

$$f(t)=\begin{cases}\dfrac{P}{2}+\dfrac{P}{2t_1}t & 0\le t\le t_1\\ P & t_1\le t\le t_1+t_L\\ P-\dfrac{P}{2t_1}\left[t-(t_1+t_L)\right] & t_1+t_L\le t\le 3t_1+t_L\\ 0 & 3t_1+t_L\le t\le 3t_1+2t_L\\ \dfrac{P}{2t_1}\left[t-(3t_1+2t_L)\right] & 3t_1+2t_L\le t\le 4t_1-2t_L(=T)\end{cases} \quad \text{for} \tag{4}$$

Taking the Fourier expansion of this, the trapezoidal waveform can be represented as:

$$f(t)=\frac{P}{2}+\frac{PT}{t_1\pi^2}\left(\sin\frac{2\pi t_1}{T}\sin\frac{2\pi t}{T}+\frac{1}{3^2}\sin\frac{2.3\pi t_1}{T}\sin\frac{2.3\pi t}{T}+\frac{1}{5^2}\sin\frac{2.5\pi t_1}{T}\sin\frac{2.5\pi t}{T}+\cdots\right) \tag{5}$$

In order to be able to calculate actual values, specific values for t_1 and T must be chosen. In our standard experimental configuration, the chopper is paced at position C (figure 2). With a standard ten-slot wheel, this happens to be at a ratio of t_1/T of 0.1. For this particular trapezoidal waveform equation 5 becomes:

$$f(t)=\frac{P}{2}+\frac{10P}{\pi^2}\left(\sin\frac{\pi}{5}\sin\frac{2\pi t}{T}+\frac{1}{3^2}\sin\frac{3\pi}{5}\sin\frac{2.3\pi t}{T}+0+\frac{1}{7^2}\sin\frac{7\pi}{5}\sin\frac{2.5\pi t}{T}+\cdots\right) \tag{6}$$

The first thing to note is that the fundamental has a coefficient of $10/\pi^2 \sin(\pi/5)$. This is closer to the pure sinusoidal value of 0.5 than is obtained with the square wave modulation. Comparing equation 6 with equation 3, it can be seen that the trapezoidal series is likely to decline more rapidly at higher terms, due to the $1/n^2$ terms in equation 6, compared with the $1/n$ terms in equation 3. This can be seen in figure 4, where the error is shown as a function of the number of harmonics considered. Figure 4 also includes a plot for a trapezoidal profile where the ratio of t_1/T is 1/6. The decrease is not monotonic for several reasons. Every fifth harmonic contributes zero (since $\sin(m\pi)$ is zero). Also, for some terms, $\sin(n\pi/5)$ is negative. When the effect is calculated on the responsivity, figure 7 can be obtained. It is found that almost all the information is in the lower harmonics (figure 4). Little extra accuracy is gained by considering terms above the 11th harmonic; in fact for many purposes, the fundamental can be considered in isolation will little loss of accuracy.

The third harmonic term is usually the most important of the harmonic terms. In this case it turns out that, with the ratio $t_1/T = 1/6$, the third harmonic term is zero, as are the ninth, fifteenth, eighteenth, and so on. This gives a performance very close to the performance provided by sinusoidal modulation.

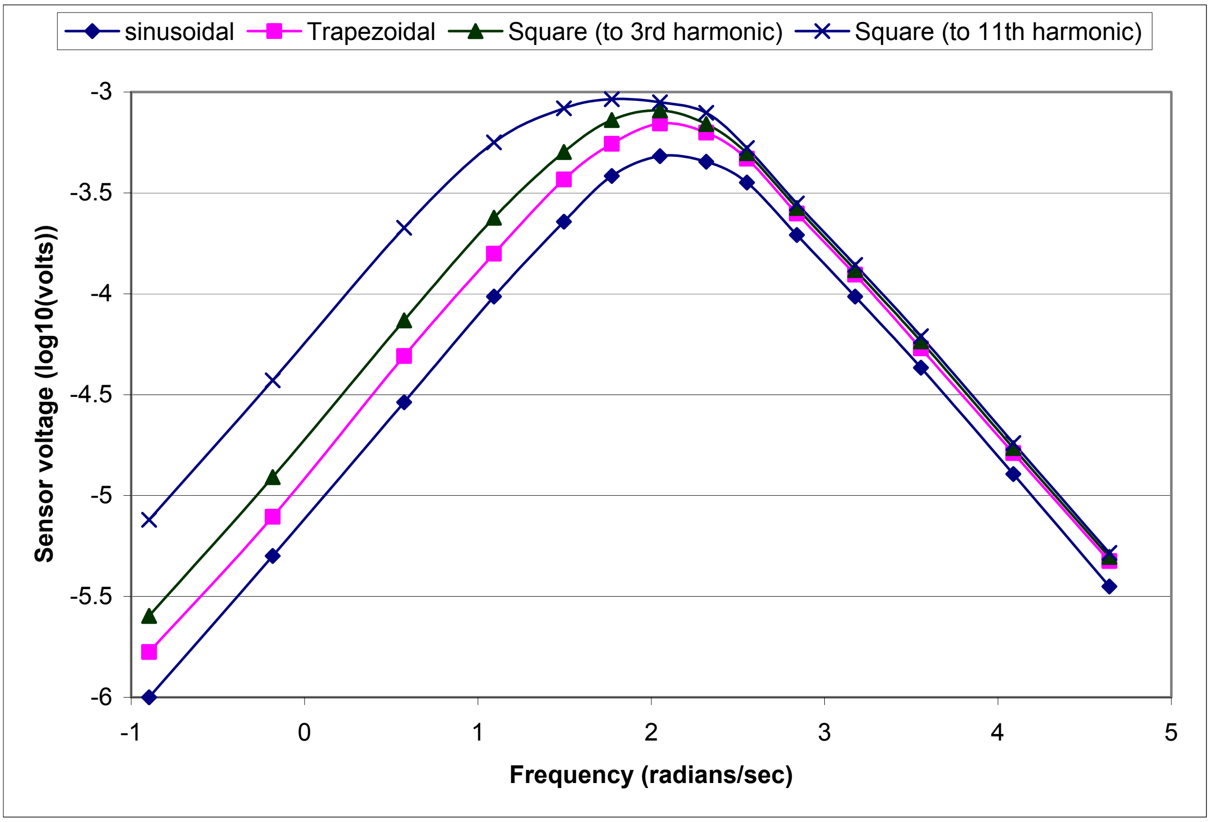

Fig. 7. Output voltage generated by a Fourier representation of trapezoidal wave modulation, compared with square waves.

4. Discussion

The main conclusion that can be drawn from the examination of the effect of the harmonics is that the trapezoidal waveform is much closer than the square waveform to the ideal sinusoidal waveform. There is still some error compared with the sinusoidal case, but it is significantly smaller. In particular, the bandwidth is much closer to that of the sinusoidal case. Several groups have published responsivity curves that use sinusoidally modulated radiation [2][6], and we believe that their results will be accurate. Some results were achieved using modulation of an unspecified form (e.g. [7]). However some groups [8] have used square wave modulation, stating that only the fundamental frequency was taken into consideration. The presence of high frequency harmonics is likely to lead to overestimates of both the peak responsivity and the bandwidth of such a system.

5. Conclusion

Care must be taken when making frequency measurements using optical chopper systems. In some circumstances, a sinusoidal waveform can be generated, and this is likely to give the most accurate results, as the sine wave only contains one frequency component. Using other types of modulation can introduce high frequency components that can distort results. This is particularly the case with square wave modulation, where the low frequency measurements are likely to be affected by the presence of the high frequency harmonics. If it is not possible to use sinusoidal modulation, then trapezoidal modulation will produce a much lower distortion than a square wave modulation. This can be achieved by ensuring that the chopper intersects the optical beam at a point where the beam size is a sizable fraction of the slot size.

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