Revisiting the particle size effects in centrifuge modelling

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ABSTRACT: Geotechnical centrifuge modelling provides an opportunity to examine novel and complex events in a well-controlled and repeatable environment. While grain interaction and contact dynamics are considered in centrifuge modelling, the soil is treated as a continuum, consistent with standard geotechnical analysis. In the last four decades, particle size effects have been normally approached by the ratio of median particle diameter to critical dimension of modelled structure. The current study considers the response of a granular medium in a centrifuge model by coupling physical tests and equivalent discrete element simulations. The response of a strip footing on uniformly graded glass ballotini is investigated. This is chosen as the sample characteristics can be accurately replicated in a discrete element simulation. Particle size distribution, gravity and footing width are scaled in the context of model-the-model technique and the sensitivity of the bulk response to rapid increase in stress level is explored. This will help establishing the link between the micro phenomena and the macro response and contribute towards improving geotechnical design. The paper describes the work conducted to overcome challenges related to physical modelling including particle mixing, sample preparation, image analysis, and loading apparatus.

1 INRODUCTION

It has been understood that the soil behaviour is highly nonlinear in varying stress-level conditions. Thus representing full scale stresses and stress distribution is important. Centrifuge modelling consists of subjecting a small scale model to an inertial radial acceleration field of 'N' times greater than earth's gravity (Taylor, 1995). This causes a rapid increase in the stress level from surface to bottom of the model which can be determined by soil density and 'N'. If the vertical stress at depth h_m in a centrifuge model is:

$$\sigma_{v,m} = \rho g N h_m \qquad (1)$$

where ρ is density and g is gravity. Considering the prototype vertical stress ($\sigma_{v,prototype}$) at depth h_p is equal to model vertical stress ($\sigma_{v,m}$) at h_m:

$$\sigma_{v,m} = \sigma_{v,prototype} \qquad (2)$$

$$\rho g N h_m = \rho g h_p \qquad (3)$$

Therefore:

$$\frac{h_m}{h_p} = \frac{1}{N} \qquad (4)$$

In other words, it is assumed that the prototype model is exactly 'N' times the size of the centrifuge model. Schofield (1980) describes the relationships between centrifuge and prototype model as scaling laws. The length should be scaled according to these scaling laws which basically come from Equation 4. But, the question is should the particle size in the model also be reduced to correctly represent the prototype particle size? This may lead to the use of soil in the model with different properties.

A number of researchers have investigated particle size effects and suggested using the ratio between median particle size (d_{50}) and the dimension of modelled structure for reducing this effect. This information was collected and summarised by Garnier *et al.* (2007). Tatsuoka *et al.* (1991) recommended using the ratio of particle size to the thickness of the shear band at failure. Taylor (1995) has advised that particle size effects may be important in some circumstances and the model test series should contain satisfactory relevant investigation.

Regarding soil-footing interaction, Ovesen (1979) showed that the effect of reducing the footing size while retaining particle size is negligible when B/d_{50} is more than 30 (where B is the footing width). Lau & Bolton (2011) observed that there is no particle size effects for B/d_{50} of varying from 165 to 8333 when investigating the bearing capacity of a circular

footing. These studies raise the question of the reliabilities of tests where there is no consideration of soil grading, particle shape and particle strength. For example, can the study of Lau & Bolton (2011) on fine silica sand and silt justify that scaling effects are negligible for an investigation on coarse grained Devonian Limestone (Halai *et al.* 2012) or a case of coarse carbonate sand under different loading conditions?

In recent years, soil-footing interaction under combined loading has received increasing attention, in particular for the design of offshore structures. For instance, in the latest international conference on the jack-up platforms (McKinley, 2015), six papers focused on centrifuge modelling of footing-soil interaction. Govoni *et al.* (2011) studied a shallow footing under combined loading and also noted that a comprehensive investigation of the particle size effects is required.

The aim of this study is to revisit the particle size effects from particle scale behaviour. In order to reduce the number of variables and couple physical model with an equivalent discrete element simulation, smooth spherical particles (glass beads) were used for the granular medium. A strip footing in the context of model-the-model technique was used and particle size distribution, gravity and footing width were scaled. Model-the-model technique refers to two different physical dimensions tested, which represent the same prototype dimensions at different gravities. The work conducted to overcome some challenges related to physical modelling and verification tests are presented here.

2 CENTRIFUGE MODELLING

2.1 *Centrifuge Facility*

The London geotechnical centrifuge centre at City University London utilises an Acutronic 661 centrifuge. The centrifuge has a working radius of 1.6 m and a maximum acceleration of 200g. It is classed as a 40 g-tonne machine; it can carry a maximum of 400kg at 100g, and 200kg at 200g. Schofield & Taylor (1988) describe the specifications of this machine in details. McNamara *et al.* (2012) describe the recent upgrade works.

2.2 *Model Assembly*

A schematic of the plane strain test of a strip footing is shown in Figure 1. Two stiff aluminium plates with 10mm thickness were machined to represent 20mm and 40mm wide footing with a length of 200mm. This will enable plane strain conditions which correspond to a 2m wide strip footing at 100g and 50g, respectively. Glass beads were glued to the bottom of footing to create an interaction surface.

The loading apparatus developed by Taylor *et al.* (2013) has been modified for the purpose of this study. This includes a 10kN screw jack load actuator, servo motor and gearbox. The plate was driven into the soil at a constant rate of penetration of 1 mm per minute until achieving a settlement of at least 0.1B. Two LVDTs were used to measure the displacement of the footing and quantify if there is any tilting in the major dimension. The force sandwich plates are shown in Figure 2. They consist of three load cells located between two aluminium plates with 6 mm thickness. The load is applied at the centre between the three load cells. The reading of load
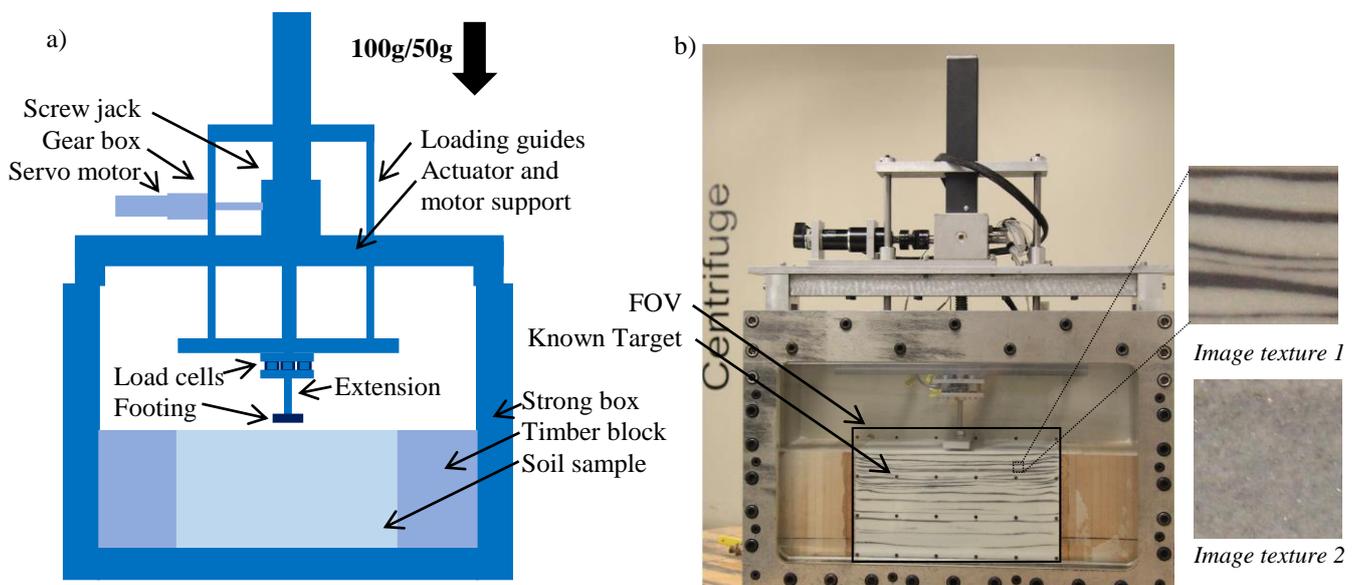

Figure 1. a) Schematic of the actuator and strongbox assembly, b) a general view of set-up and image analysis parameters.

cells will demonstrate the un-level seating of the footing on soil.

The internal dimensions of the strong box were reduced using timber blocks to make a soil bed with dimension of 300 × 200 × 160 mm. The timber bolcks facilitated the process of creating top surface of sample which is critical in this experiment, while reducing the number of particles for DEM simulations.

FE simulations were carried out using PLAXIS 2D incorporating Mohr-coulomb model, with c'=0.15kPa and φ=26°, to investigate the maximum zone of deformation (Figure 3). These simulations confirm that there is no boundary effect in the reduced dimensions of sample.
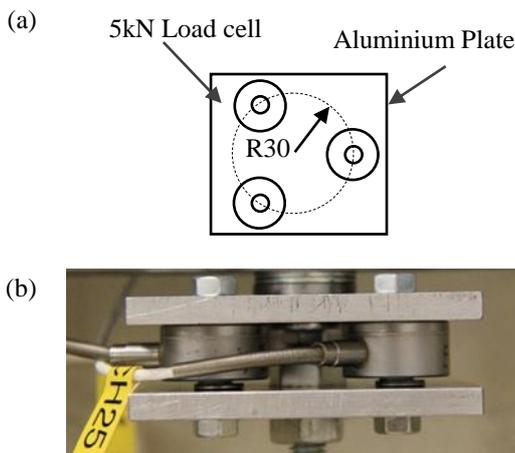

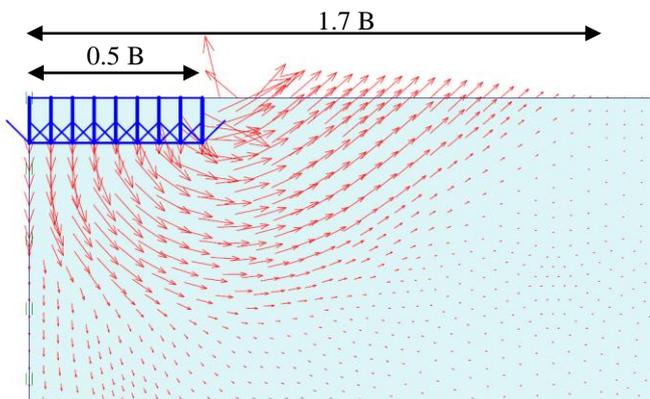Figure 2. Force plate for measuring axial reaction load, a) plan and b) general view.

Figure 3. PLAXIS results showing deformation zone for strip footing problem.

2.3 Sample Preparation

Six particle sizes of glass ballotini with specific gravity of 2.57 and 95% roundness were used (Table 1). They are commercially supplied by Sigmund Lindner as type S beads. The maximum and minimum void ratios of 0.63 and 0.54 have been reported by Barreto (2009).

Table 1. Glass ballotini sizes and percentage (by mass) to prepare two uniform graded mixes.

	% of mix	Particle size (mm)
Mix1	20	0.5-0.75
	50	1.0-1.3
	30	1.7-2.1
Mix2	20	1.25-1.65
	50	2.0-2.4
	30	3.4-4.0

It was decided to mix three sizes of glass ballotini to produce two uniformly graded samples shown in Figure 4. This was based on the following considerations: 1) to limit the size of particles for DEM simulation, 2) different ranges of B/d_{50} ratio for model-the-model technique and 3) the need to consider the size of beads due to experimental conditions. There are some challenges associated with the preparation of uniform graded sample of glass ballotini in the strong box. These include: 1) granular mixing (also known as solid-solid mixing), 2) pouring/packing, and 3) trimming the top surface. The issues are described in the following section.
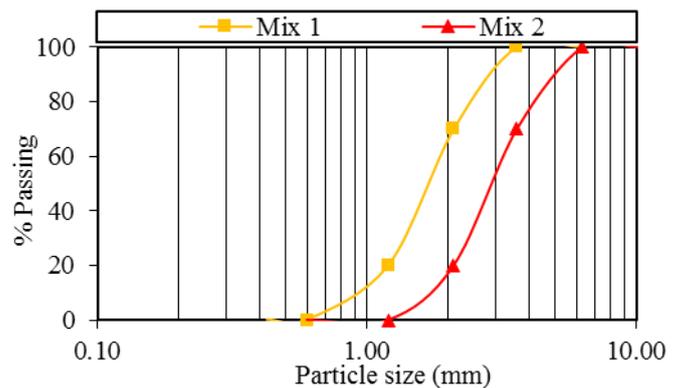

Figure 4. Particle size distribution of two glass ballotini mixes.

2.3.1 Granular mixing

In order to produce near homogenous and repeatable samples, a mixing device was developed. Mixing of particulates is an important process in many industries and is an active area of research (e.g. Muzzio et al. 2003). Mixing devices can be classified as gravity controlled, bladed and high shear (or high energy) mixers. Measurement of mixing state is non-trivial in experiments. Cleary & Sinnott (2008) carried out several DEM simulations to assess the characteristics of granular mixing devices for the various methods and to quantify the degree of mixing. Two schemes have been used by the authors to measure the degree of mixing. In the first scheme, an assembly of particles is divided into groups and the centre of mass of each group is calculated (before mixing). The current distance of the centroids for each group over the global centre of mass presents the degree of mixing. In the second scheme, a 3D grid is con-

structed over the assembly. The local averages of the selected properties such as mass, density and diameter characterises the degree of mixing. More details can be found in Cleary (1998).

The comparison of different mixing devices demonstrates that the final mixing state of high shear mixer with a blade impeller is much higher than the other methods. For example the model of disc impeller was able to produce a mix with only 70% homogeneity when compared to the 95% for the blade impeller. The blade impeller was chosen here as it produces the best state of mixing. A schematic of the mixer with a blade impeller is shown in Figure 5, following Cleary & Sinnott (2008).

Figure 6 shows a general view of the mixing device developed here. For this mixer, the mixing state as a function of time is presented in Figure 7 obtained from DEM simulations. The probability of close to homogeneous mixing is very strong with this type of mixer.
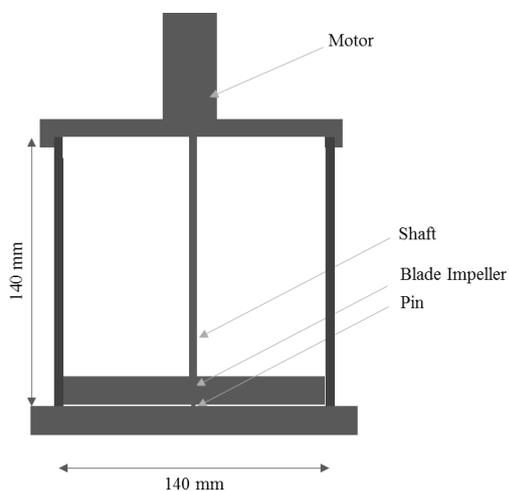

Figure 5. Schematic of the mixer with a blade impeller following Cleary & Sinnott (2008).

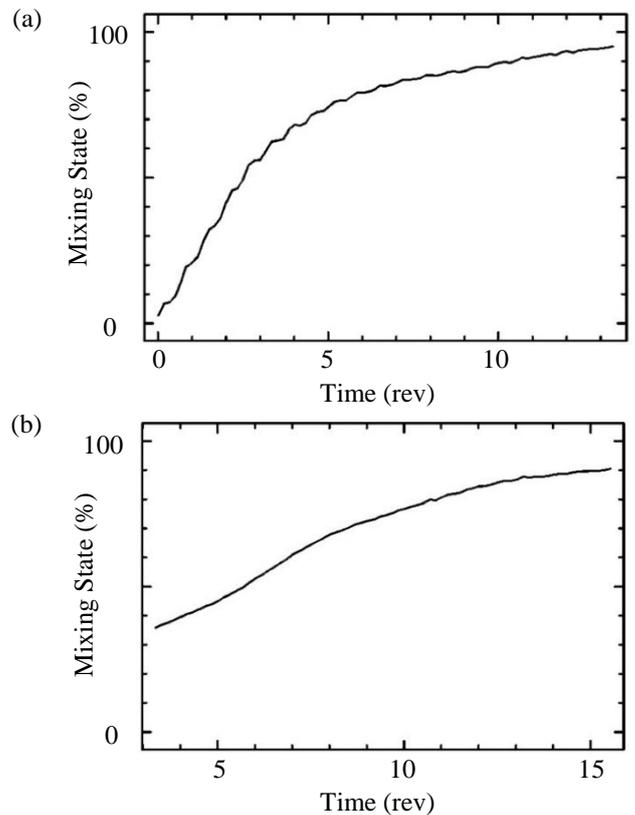Figure 6. A general view of mixer developed for this study.

Figure 7. The mixing state for a high shear mixer with a blade impeller as a function of time for: (a) azimuthal and (b) radial mixing from DEM simulations by Cleary and Sinnott (2008).

Figure 8 shows an experiment for mixing dyed and non-dyed glass ballotini. The initial state of particles is shown in Figure 8.a. The mixing has been carried out for 60sec with a speed of 100rpm. The final mixing state is presented in Figure 8.b. As shown in the figure a good mixing state was achieved. A quantitative description of the mixing using image processing is under progress.
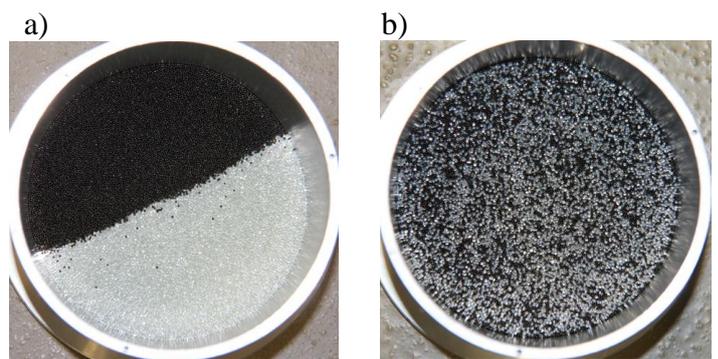

Figure 8. Mixing of dyed and non-dyed glass ballotini, a) initial state, b) mixing state after 60 sec.

2.3.2 *Pouring/Packing*

Cavarretta (2009) proposed a simple method for air pluviation of glass ballotini using a funnel. In this method, the funnel throat is kept in contact with the soil surface (no drop height) and is raised without excessive agitation, to achieve a void ratio of 0.60 ± 0.02 for triaxial specimens. This method was adopt-

ed in this project as there is concern that the drop height may induce particle segregation. As the dimensions of the sample in this study are large, the direction of pouring is changed from layer to layer: one layer parallel and the next layer normal to the strong box window. At the end, the top surface was trimmed to the same level as the timber blocks.

2.4 *Deformation monitoring*

The pre-failure deformation behaviour of the footing and foundation soil was monitored using particle image velocimetry (PIV) technique. This technique allows more flexible image analysis for the wide range of physical models in comparison with tracking target markers proposed by Taylor *et al.* (1998). In order to make high quality image texture for PIV analysis, firstly some glass ballotini has been dyed (Figure 9). In order to investigate the effect of dying on frictional behaviour of glass ballotini, a series of direct shear tests were conducted.
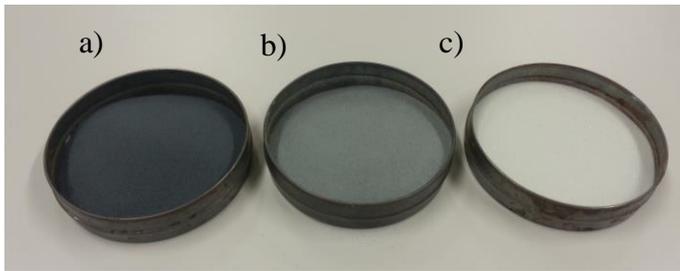
Figure 9. a) Black dyed, b) grey dyed and c) non-dyed fine glass ballotini.

The direct shear tests showed a slight increase in friction angle due to dying (Figure 10.a). Using a microscope, it has been observed that dying of glass ballotini causes particle clumping and changes in surface roughness (Figure 10.b). It is important to note that gentle sieving carried out before the experiments could not remove all particle clumping. Therefore, a thin layer coated glass bead was ordered from the same supplier mentioned earlier. The coating surface includes a 2µm silver base and 1-3µm colouring of a Sol-Gel.

Reference again to Figure 1, black beads can be placed in the form of strip lines or mixed to make visualisation and monitoring of deformation possible. Here, Image texture 2 (shown in Figure 1) which is mix of beads has been used. Image texture plays an important role in the precision of image analysis using PIV. More detail can be found in Nadimi *et al.* (2016).
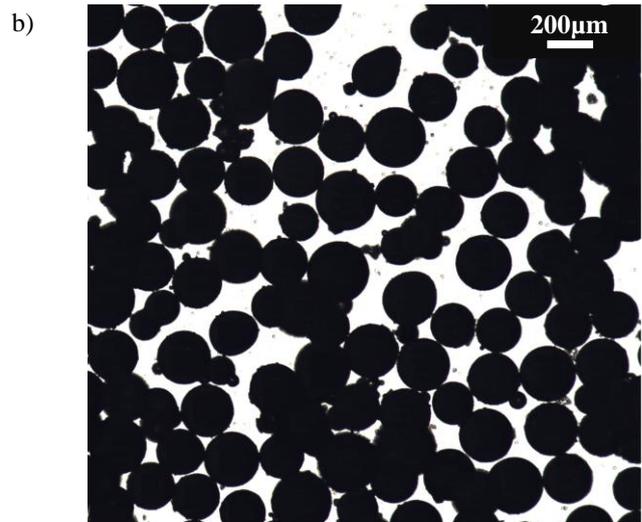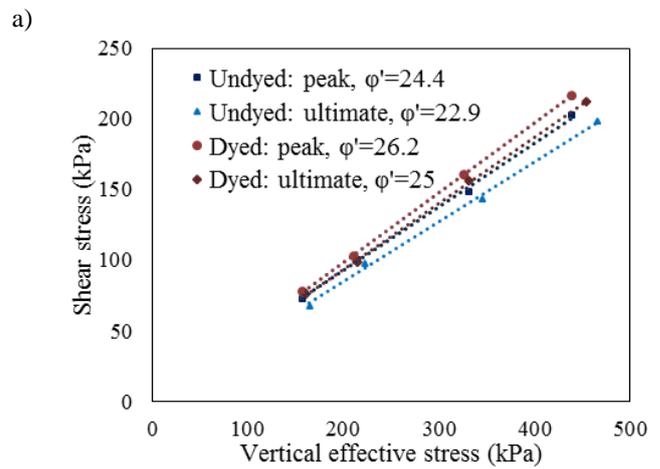
a)

b)

Figure 10. a) Direct shear results for dyed and non-dyed glass ballotini, b) dyed glass Ballotini viewed under a microscope.

2.5 *Preliminary Tests*

Six preliminary tests have been conducted with the aim of checking the loading apparatus, image analysis, model design, instrumentation and software. Testing consisted of accelerating the model using the centrifuge to 50g or 100g, while the footing was hold above the sample surface. At the target gravity the footing was loaded at a rate of 1mm/min to touch the surface and then depress into the ballotini up to 0.2B. Figure 11 shows a typical stress vs. settlement curve obtained from LVDTs and load cell readings for a 40 mm footing at 1g. When the footing is not in contact with soil at target gravity, the curve shows zero stress, in this case up to 5mm settlement. The friction test has been carried out to find the optimum gap between the footing and the strong box, which consists of loading a footing without soil. Load cell readings show the amount of frictional force. In order to avoid filling of the gap with glass beads, the smallest size of particles (0.5mm) was selected to be larger than the gap <0.4mm). The loading guides shown in Figure 1

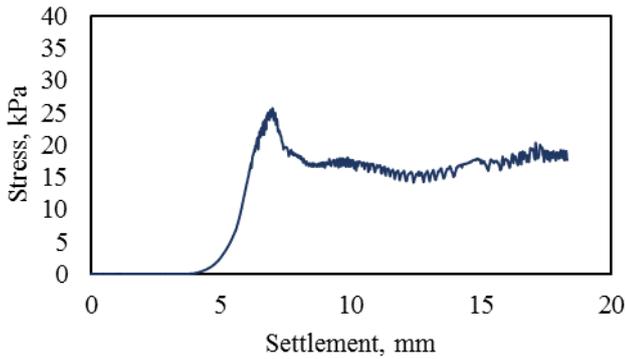have been used to control the backlash in the screw jack and create a purely vertical loading.

Figure 11. Typical Stress vs. settlement curve obtained from a centrifuge experiment.

2.6 *Verification Tests*

The purpose of these tests is to verify the consistency of the apparatus and testing regime. Two separate tests with exactly the same conditions were conducted. Figure 12 and 13 show the results of two centrifuge tests in terms of stress vs. time and stress vs. settlement for a 20 mm strip footing at 100g. The data show good agreement between two tests presenting the reproducibility of the methodology.
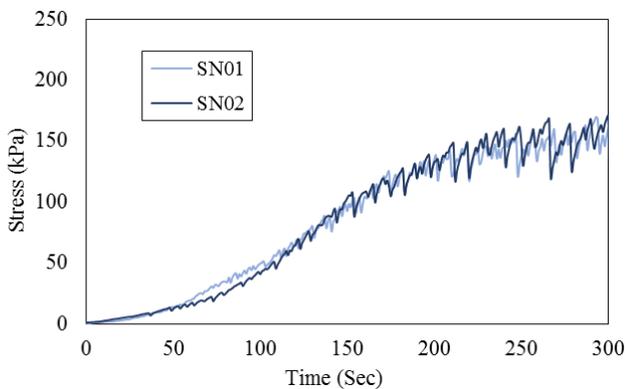

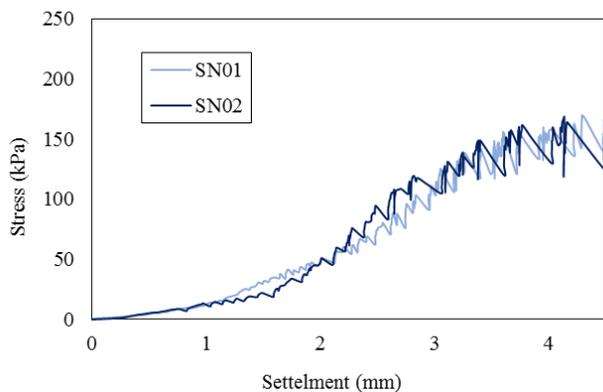Figure 12. Stress vs. time curves for centrifuge verification tests.

Figure 13. Stress vs. settlement curves for centrifuge verification tests.

3 DEM MODELLING

In contrast to continuum (FE) modelling, the Discrete Element Method (DEM) proposed by Cundall & Strack (1979) accounts for the particulate nature of soil. A DEM simulation involves cyclic calculations. At any time t, inter-particle forces are calculated from the relative velocities of particles in contact. From particle forces, new particle accelerations are obtained using Newton's second law. Numerical integration of the accelerations, using a time-centred explicit finite difference scheme, provides new particle velocities which give new displacements from which new particle positions are obtained. A typical DEM simulation repeats the cycle of updating contact forces and particle locations for a pre-defined number of (small) time-steps.

The main advantage of using DEM simulations is the possibility of monitoring parameters that cannot be easily measured from experimental tests and from which micro-mechanical analyses can be performed to gain further insight into the particle-scale interactions that underlie the observed macro-scale behaviour of soils. A parallel set of DEM simulations which replicates each of the experimental (centrifuge) tests is currently being carried out. These simulations will enable detailed analysis on the development and thickness of shear bands to verify the findings by Tatsuoka *et al.* (1991), amongst others, and to further understand the nature and importance of particle size effects in centrifuge modelling (Taylor, 1995). Furthermore, O'Sullivan (2011) highlights that DEM has been mostly used for the simulation of element tests. The modelling of boundary value problems using DEM is more limited and it is significantly constrained by their associated computational cost (proportional to the number of particles used). Taking advantage of the plane strain (2D) nature of the centrifuge tests, 2D as well as 3D simulations with differing model thicknesses (in the out-of-plane direction − hence with significantly different number of particles), will be compared to 1) establish better DEM simulation requirements, 2) explicitly assess the need to perform 3D simulations for plane strain (2D) problems and 3) to expand the existing knowledge on the influence of differing gravitational levels on granular materials' behaviour at a particle scale.

4 CLOSING REMARKS

This study aims to revisit particle size effects in centrifuge modelling. The work carried out to overcome challenges regarding physical modelling has been described here. Model design and sample preparation are two key steps for carrying well-controlled and repeatable experiments. As the sample represents uniform grading of glass ballotini, a

mixing apparatus has been developed based on DEM simulation results to make a near homogenous sample. The macro response of the centrifuge model and deformation pattern will be compared with DEM results. The DEM model can provide micro parameters such as contact forces to investigate the effect of varying gravity on contact force distribution. It is suggested that coupling established physical modelling with advances in discrete computational methods can improve understanding for geotechnical design.

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