Using topology optimization technique to determine the optimized layout of steel reinforcing bars in concrete structures

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1. Abstract
This study presents an optimization procedure based on the modified Bi-directional evolutionary structural optimization (BESO) approach to optimize both location and orientation of discrete reinforcing steel bars within concrete structures, while satisfying the prescribed realistic volumetric ratio of steel amount into the continuum concrete. Opposed to the strut-and-tie model (STM) mechanism, both tension and compression are taken into account in reinforcing bars. The optimization variables are only applicable to steel reinforcements that are modelled as discrete truss bars embedded into a concrete domain. The flexible orientation of each reinforcing bar is achieved by employing a heuristic orientation finding scheme according to principal strain direction into a two-dimensional (2D) BESO algorithm. Also, apart from ranking the sensitivity number of each reinforcing bar, an update scheme for design variables is developed due to the asymmetric property of concrete in tension and compression. The capability of the proposed optimization method is shown through two cases. It can be concluded that the proposed method obtains a rational reinforcement layout under a volume constraint on steel used in reinforced concrete (RC) structures. Reinforcing bars playing high contribution to the structural behavior are remained within a constant concrete domain, which provides a valuable suggestion for the distribution of steel reinforcements.

2. Keywords: Reinforced concrete structures, BESO topology optimization, discrete reinforcing bar, heuristic orientation scheme, update scheme

3. Introduction
In structural concrete design, disturbed regions, so called as ‘D-regions’, has been a challenge for decades. Opposed to ‘B-regions’ (Bernoulli or Beam regions) where design procedure is maturely established according to beam theory and cross-sectional analysis, D-region is defined by a structural part with nonlinear strain distribution, for which traditional approaches for slender beams are not appropriate for design. Current practice towards design and analysis of such regions of the structure is using strut-and-tie model (STM) [1] which is well known as a generalization of truss analogy model [2, 3]. The concrete struts represent elements in compression while the tensile ties are carried by steel reinforcements. However, the selection of STM is usually uncertain, especially for an irregular RC structural member under complicated loading and boundary conditions, because it is mainly based on stress trajectories, load path methods or empirical observations. The optimization technique has been regarded by researchers as an efficient tool to distribute reinforcements within the concrete structure.
Initially, the discrete topology optimization based on the truss ground structure approach, that allows the truss topology design problem to be viewed as a generalized sizing problem, has been used to search for the optimal STMs in reinforced concrete structures [4, 5]. The continuous reinforced concrete domain is discretized by a predefined layout where the fixed truss ties correspond to the actual reinforcements and the ties with cross-sectional areas equal to zero or nearly zero are removed through the topology optimization process. In both works of Biondini [6] and Ali and White [7], an automatic search technique for truss models consistent with the elastic stress trajectories in reinforced concrete members were proposed based on ground structure approach and linear mathematical programming technique. Also, genetic algorithms have been applied to truss topology optimization to seek the best layout of the location of reinforcing ties and compressive struts within the reinforced concrete beam [8]. More recently, Amir and Sigmund [9] presenting a truss topology optimization by embedding a truss ground structure into a concrete continuum damage model, so that the distribution of embedded steel reinforcement is optimized. However, the predefined ground structure have dominant influence on the resulting topology, which is chosen mainly relying on the intuition and experience of designers.
As opposed to truss topology optimization that require designers to define node locations and element connections a priori, using continuum topology optimization to achieve a novel layout design of reinforced concrete structures has attracted a large amount of researchers in recent years [10-16]. However, these studies all proposed to use a truss-like structure obtained from single-material topology optimization so as to predict a strut-and-tie model. As the name suggests, the reinforced concrete structure is composed of two materials: concrete and steel. Hence, incorporating different mechanical properties of concrete and steel into topology optimization has emerged to gain a more effective reinforced concrete structure [17-19]. Luo and Kang [20] developed a two-material topology optimization, in which the resulting topology is much like a steel-concrete composite structure, with volume constraint on steel and strength constraint on concrete. Rather than impose stress constraints, the complete non-linear elasto-plastic response for both concrete and steel were modelled in Bogomolny and Amir et al. [21]. Also, Luo et al. [22] proposed an effective continuum topology optimization method, aiming at minimizing the costs of steel reinforcements subjected to a shrinkage volume constraint and yield constraints on concrete phases. From the review of these works, RC structure is commonly treated as composite material structure, however, in real application, the volumetric ratio of steel used into the continuum concrete is rarely over 1% [9], which cannot be achieved properly by modelling steel reinforcements as continuum elements in topology optimization problem. Also, from a construction perspective, the required postprocessing of continuum members in tension regions to discrete bars is less practical.
In order to benefit from both continuum and truss topology optimization, the truss ground structure and continuum finite elements are combined onto a mesh of shared nodes where tension members are presented by truss elements resulting in reinforcing steel design while continuum elements are implemented as concrete to carry compression. This idea was initially proposed in Moen and Guest et al. [23] and then topology optimization using a hybrid truss-continuum model was further developed in Gaynor et al. [24] that using

the bilinear truss-continuum topology optimization approach to prevent the strut-only solution from missing transverse tensile stresses caused by load spreading. Also, following this approach implemented in 2D structural domains, it has been extended to 3D design models by Yang, Moen and Guest [25] and developed to generate a structural system performing practical in construction as well by considering a tradeoff between material and construction cost [26]. However, the stiffness of truss steel element modeled in the hybrid approach is high in tension but negligible in compression, just as in most of the review studies with the goal of achieving a strut-and-tie model. It is important to point out that although it opposed to the STM mechanism, both tension and compression are acceptable in steel in the real application.

As a result of this, the current work develop a truss-continuum embedded model that reinforcing bars are modeled as truss elements embedded onto the continuum concrete. Furthermore, it focus on obtaining an optimized steel reinforcement layout accepting both tension and compression within a constant concrete domain. The optimization variables are only applicable to steel reinforcements. The idea of using embedded formulation for discrete components was previously explored in optimization of reinforced concrete structure [9, 27, 28]. The key difference in this paper is that a heuristic orientation finding scheme is proposed to be employed into a 2D BESO algorithm. This enable the placed reinforcing bar can adjust its efficient orientation following the principal strain direction in each iteration, which get rid of the drawback of conventional truss topology optimization based on ground structure approach that the layout for reinforcing bars is dominantly influenced by the predefined truss layout.

BESO method is an improvement of evolutionary structural optimization (ESO), which removes the unnecessary material from a structure while the efficient ones to be added. Apart from the gradient based method, e.g. the Solid Isotropic Material with Penalization method (SIMP), the heuristic BESO method is another promising choice. More recently, the solution to problems of checkerboard pattern, mesh-dependence and non-convergence, and also a material interpolation scheme with penalization was employed in the BESO technique [29]. Its simple concept and easy implementation has gained widespread popularity among researchers and designers and has been used for a wide range of applications in engineering field [30]. For these reasons the author decides to achieve the goal by using the evolutionary technique (BESO) in this paper. Moreover, due to the property of concrete that strong in compression but weak in tension, a design variable updating scheme is developed based on the allowable strain for concrete in tension and compression separately, which prevents unnecessary reinforcement being distributed to regions where concrete can suffer from itself.

4. Modeling of reinforcing bars

In order to achieve a more visualized discrete reinforcement layouts approaching to real design, the reinforcing steel bar is modeled as a one-dimensional (1D) truss element embedded in a 2D concrete element. The discrete modeling of steel reinforcements used was proposed by Kwak and Filippou et al. [31]. Perfect bond between the two candidate material of steel and concrete is assumed so that the displacements of truss (steel) elements are consistent with those of the surrounding continuum (concrete) elements. Also, the location and orientation of the steel bar superimposed on a concrete element is arbitrary, not necessarily sharing the same nodes, hence the orientation of the truss element is flexible through the optimization process. The elemental stiffness of each individual steel bar can be added to nodes of the hosing concrete elements, in other words, the strain energy of the embedded steel bar can be evaluated through the nodal displacements and its elemental stiffness at a global coordinate system. Based on the assumption that the reinforcing bar only has stiffness along the longitudinal axial direction with a constant cross-sectional area, the elemental stiffness matrix for a 1D steel bar is given by,

$$\boldsymbol{K_{sl}} = \frac{A_s E_s}{L_e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \tag{1}$$

Where A_s, E_s and L_e represent the cross-sectional area, the elastic Young's modulus and the elemental length of an individual steel bar respectively. Then the local elemental stiffness matrix $\boldsymbol{K_{sl}}$ can be transformed to $\boldsymbol{K_{sg}}$ in a global coordinates through a transformation matrix $\boldsymbol{N_1}$

$$\boldsymbol{K_{sg}} = \boldsymbol{N_1^T K_{sl} N_1} \tag{2}$$

$$\boldsymbol{N_1} = \begin{bmatrix} cos\theta & sin\theta & 0 & 0 \\ 0 & 0 & cos\theta & sin\theta \end{bmatrix} \tag{3}$$

Only 2D cases are considered in this paper, and θ expresses the angle between the axis of the reinforcing bar and the x-axial direction of the structure. Hence, the transformation matrix $\boldsymbol{N_1}$ relates the local truss elements to a global coordinate system by applying a rotationθ. As the location of the truss element embedded in the continuum element is arbitrary that the truss nodes may not coincide with the continuum nodes, another transformation matrix $\boldsymbol{N_2}$ is needed to map $\boldsymbol{K_{sg}}$ to a stiffness matrix $\boldsymbol{K_s}$ in terms of the degree of freedoms (DOFs) of continuum nodes. Based on the principal of energy conservation, $\boldsymbol{N_2}$ can be derived by the following steps.

$$\boldsymbol{U_{sg}^T K_{sg} U_{sg}} = \boldsymbol{U_e^T K_s U_e} \tag{4}$$

Where $\boldsymbol{U_e}$ represent a nodal displacements matrix associated to continuum nodes. Due to the assumption that the displacement of the embedded reinforcing bar is compatible with those of the continuum concrete elements, the global displacement matrix $\boldsymbol{U_{sg}}$ at the end nodes for a truss element can be expressed in terms of $\boldsymbol{U_e}$ through shape functions $\boldsymbol{N}$.

$$\boldsymbol{U_{sg}} = \boldsymbol{N U_e} \tag{5}$$

By substituting Eq. (5) into Eq. (4), the desired matrix $\boldsymbol{K_s}$ takes the form

$$\boldsymbol{N^T K_{sg} N} = \boldsymbol{K_s} \tag{6}$$

When the 4-node isoparametric element type is applied to a two-dimensional mesh domain, the transformation matrix $\boldsymbol{N_2}$ is kind of a simplified form of the shape function $\boldsymbol{N}$ based on the side of a continuum element where the end points of a truss element cross. A fully description of stiffness matrix $\boldsymbol{K_s}$ can be summarized by the following relation

$$\boldsymbol{K_s} = \boldsymbol{N_2^T N_1^T K_{sl} N_1 N_2} \tag{7}$$

Therefore, the assembled stiffness $\boldsymbol{K_e}$ can be obtained by adding $\boldsymbol{K_s}$ to the elemental concrete stiffness matrix $\boldsymbol{K_c}$ to model the reinforced concrete element in a finite element method. The final expression of the elemental stiffness matrix including both the truss and the continuum is given by

$$\boldsymbol{K_e} = \boldsymbol{K_s} + \boldsymbol{K_c} \tag{8}$$

In this study, matrix $\boldsymbol{N_2}$ varies subject to four types of rotation of reinforcing bar that being allowed from axis x within the continuum element, as shown in Figure 1 in which the grey area denotes the continuum concrete element while reinforcing bar is represented by black bold line.

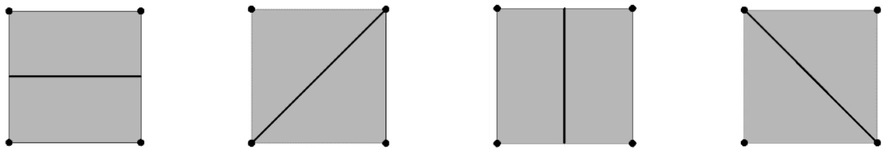


Figure 1. Four different types of reinforcing bar embedded in a continuum element

5. Topology optimization algorithm

The optimization algorithm applied in this work is explored dominantly based on the BESO optimization approach [30]. Although the contribution of the linear elastic hosting concrete discretized by finite elements has been taken into account through finite element analysis (FEA), the optimization design domain is only applicable to layout of steel reinforcements. Currently, topology optimization is widely used to design the reinforced concrete structures that result in a truss-like layout. However, the achieved topology cannot be applied straight forward to the real design problems due to the requirement for structural integrity and the minimum reinforcing ratio in some specified regions. Therefore, this drives to remain the concrete domain constant and optimize the steel reinforcement layout only at the initial research stage.

5.1 Problem statement and design parametrization

The objective of designing a steel reinforcement layout having maximal stiffness, which is equivalent to minimize the mean compliance, with a volume constraint on steel in RC structure are as expressed in Eq.(9).

$$\begin{aligned} \min \quad & C = \frac{1}{2}\boldsymbol{U}^T\boldsymbol{K}\boldsymbol{U} \\ s.t. \quad & V_s^* - \sum v_e x_e = 0 \\ & x_e = x_{min} \; or \; 1 \end{aligned} \tag{9}$$

Where C = mean compliance; $\boldsymbol{U}$ = vectors of nodal displacement obtained from FEA of the reinforced concrete model; $\boldsymbol{K}$ = global stiffness matrix of the structure; V_s^* = the prescribed target volume fraction of the design domain which corresponding to the realistic reinforcement volume ratio into the concrete domain; v_e = volume for each truss element; x_e = truss elemental design variable that are restricted to be either lower-bound x_{min} of 0.001 or 1 throughout this paper.

As elements are defined as either absent or existence without intermediate density, the resulting topology corresponds to a reinforced concrete structure with only one type of steel reinforcement. The evolutionary optimization starts from the full design domain that means the continuum concrete is initially over-reinforced by steel reinforcements, and its amount decreases gradually by applying an evolutionary ratio (ER) till the target volume V_s^* is achieved. In order to avoid the topology may be influenced by this parameter, a constant value of 2% that has been widely tested in previous studies is used throughout all the examples herein. Furthermore, a suitable convergence criterion is required to terminate the optimization algorithm. As suggested in BESO optimization method [29], the optimization stops when a change of 0.01% in the mean compliance over the last 10 iterations is achieved in this research.

5.2 Reinforcing bar orientation

Amir and Sigmund et al. [9] states the outcome of optimization is obviously influenced by the selection of the ground structure. Therefore, it is vital to consider the efficient orientation of steel without establishing a very dense ground structure. In Setoodeh et al [32], fiber orientation and the corresponding topology are combined by cellular automata (CA) framework. Also, for the orthotropic fiber reinforced materials, a heuristic optimization technique for achieving convergence of both orientation and distribution is proposed in a 2D BESO algorithm. In literature, very little efforts has been made to combine orientation and topology into optimization for steel reinforced concrete structure. As a heuristic approach, BESO provide researchers a promising basis of extending to various design problems and implementing ideas into its algorithm. This research focus on steel reinforced concrete structure, allowing the reinforcing bars modeled as discrete truss elements accepting both tension and compression, to optimize orientation and topology.

At the initial design domain, two reinforcing bars are embedded into each concrete continuum element in the forms of along the longitudinal axis x and y respectively (Figure 4a). After the first iteration, with the goal of considering both contribution of steel reinforcement playing in tension and compression, their orientation will be adjusted corresponding to the maximum and minimum principal strain direction respectively. Although the angle of rotation from x axis can also be obtained by the principal stresses, achieving the principal stress is not as easy as strain based on this finite element embedded model that consist of both discrete truss and the continuum. Four different 2D principal strain states which are tension dominated or compression dominated are shown in Figure 2.

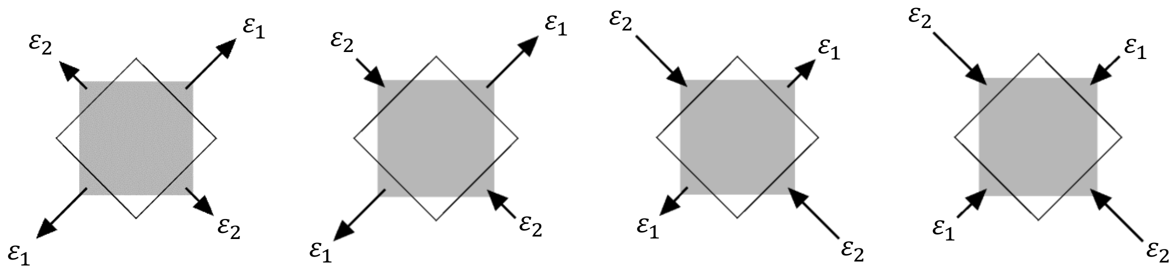


Figure 2. 2D principal strain states

As the purpose of the initial research is to verify the proposed heuristic orientation finding system rather than generate a very complicated reinforcement layout, only two types of embedded model are considered to simplify the problem in which reinforcing bars are located horizontally and vertically or in a diagonal form. The maximum principal strain orientation is defined roughly in four groups: 0° (180°), $45°, 90°$ *or* $135°$, in terms of rotating anticlockwise from x axis (Figure 3). Hence, Figure 4 shows the reinforcing bar in black can be distributed in those four orientations while the bar in blue corresponding to minimum principal strain direction is perpendicular to it.

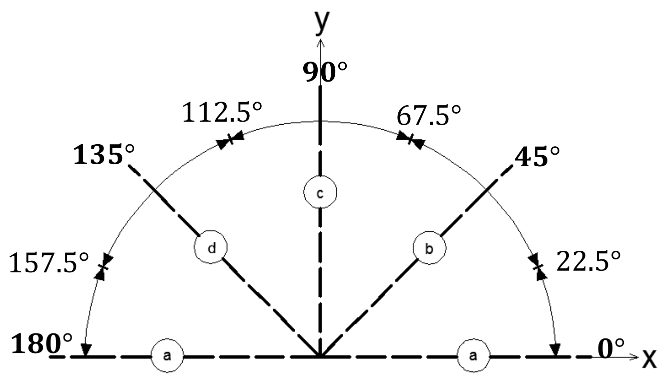
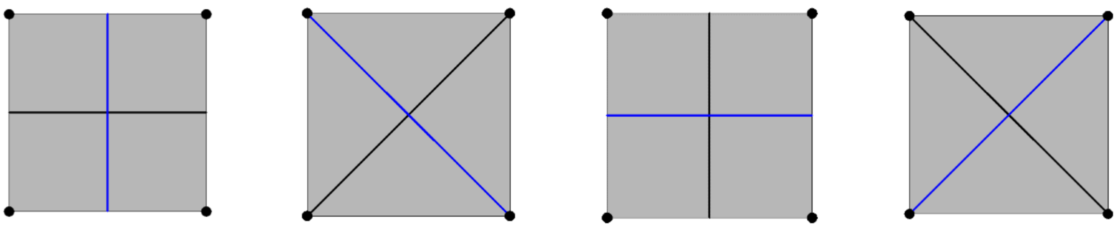


Figure 3. Four simplified maximum principal strain orientation rotating anticlockwise from x axis



(a) (b) (c) (d)

Figure 4. Presentation of embedded model

A Pseudocode for the bar orientation finding scheme is depicted in Figure 5. Since numerical instabilities such as checkerboard or mesh-dependencies usually arise in topology optimization. In order to achieve a more reasonable and stable reinforcement layout, a filtering scheme with same concept adopted in Huang and Xie et al. [32] is applied to modify the obtained strain that being smoothed by the strain in neighboring elements within the filtering circle through the relative weight factors.

For each truss-continuum element **do**
  calculate the centroid strain components ε_x, ε_y, γ_{xy}
  apply filtering to ε_x, ε_y, γ_{xy}
  calculate principal strain ε_1, ε_2
  apply filtering to ε_1, ε_2
  calculate angle θ between maximum principal strain direction and x axis in terms of anticlockwise rotation
  If $\theta \leq 22.5°$ or $157.5° \leq \theta \leq 180°$ **then**
    $\theta = 0°$
  Else if $22.5° < \theta \leq 67.5°$ **then**
    $\theta = 45°$

Else if $67.5° < \theta \le 112.5°$ **then**
$\theta = 90°$
Else if $112.5° < \theta < 157.5°$ **then**
$\theta = 135°$
End if
rotate reinforcing bar in black by θ
locate reinforcing bar in blue perpendicular to the one in black
End for

Figure 5. Pseudocode of the heuristic orientation finding scheme for the distribution of reinforcing bars

However, an oscillation in mean compliance without convergence is observed after satisfying the volume constraint in optimization, which means the rotation angle θ is oscillating. This leads to a non-convergence results and non-optimal bar orientation. At this point, a possible solution to this problem is proposed to stabilize the orientation. The rotation angle is defined by weighting the current orientation θ and the orientation obtained in last iteration θ_{old} to a threshold θ_{th} that may be $22.5°$, $67.5°$, $112.5°$ or $157.5°$ as introduced above (Figure 5). See Figure 6 for the pseudocode of describing one situation which may occur. Although, it cannot guarantee an optimal solution but play a positive role in converging material orientation for all examples in this paper.

Do while $\theta \le \theta_{th}$ and $\theta_{old} > \theta_{th}$
If $\frac{\theta_{th}-|\theta|}{\theta_{th}} \le \frac{|\theta_{old}|-\theta_{th}}{\theta_{th}}$ **then**
$\theta = \theta_{old}$
Else if
$\theta = \theta$
End if
End do

Figure 6. Pseudocode of solution to orientation oscillation for reinforcing bar

5.3 Update scheme

As the optimized topology is achieved by the relative ranking of sensitivity numbers in BESO method, no matter an individual truss element acting in tension or compression along its longitudinal axial direction, its corresponding sensitivity is involved in the optimization algorithm. However, the resulting topology based on the contribution of steel reinforcement to the objective function may not reflect the real design problem. Some steel reinforcements may be remained in regions where concrete itself can withstand the compression since its higher sensitivity than that of those in tension areas. Opposed to steel having constant and high strength in all direction, concrete is strong in compression but very weak in tension due to its quasi-brittle nature. In this study, a proposed scheme updating the design variables while taking the host concrete into consideration is applied into the BESO algorithm. Homogenized average strain is derived from a truss-continuum combined element model. When the maximum principal strain or the minimum principal strain exceeds the threshold value of the allowable tensile or compressive strain that measured by uniaxial tension and compression tests, the sensitivity of the corresponding truss element is modified to be ranked over the ones with strain less than the allowable value.
The threshold values for concrete in tension and compression are set to be 1×10^{-4} and -3×10^{-3} respectively throughout the paper. The sensitivity modification formulation is constructed as

$$s_i = s_h - 0.001 \times (s_h - s_i) \quad (i = 1,2) \tag{10}$$

Where s_i is the sensitivity number with respect to the reinforcing bar being placed based on the maximum $(i = 1)$ or minimum $(i = 2)$ principal strain direction. And s_h is the highest sensitivity value among those of all the candidate truss elements. Note that the sensitivities of elements having allowable strain do not need undergo artificial modification. The example given in the next section shows different topologies obtained from an optimization algorithm with and without adding this heuristic update scheme and proves its capacity in achieving a reasonable reinforcement layout.

6. Examples

Two 2D cases were tested in this section to present the implementation of the proposed optimization algorithm in obtaining steel reinforcement layout for concrete structure design. The Young's modulus for steel and concrete$E_s = 210\ Gpa$, $E_c = 25\ Gpa$ and the poisons ratio for both concrete and steel $v = 0.3$ are constant throughout all examples. Four-node quadrilateral plane-stress elements are used to model the 2D continuum concrete element. The filter radius applied in the filtering scheme is equal to 3 times of the element size. Since reinforcing bars are embedded into individual continuum element, using smaller element size leading to a denser reinforcement layout while reinforcements are placed with larger space based on a coarser FE mesh. In all simulations, the thickness of a 2D structure is 0.001m and the volumetric ratio of steel used in a continuum element is set to 2%. Although the cross-sectional area of the reinforcing bar cannot correspond to the practical bar types used in real design, there would not be a significant influence on determining the regions where to be reinforced by compressive or tensile bars. In the resulting topologies, black solid line represents reinforcing bar in tension while the one in compression is described by blue solid line.

6.1 A 2D cantilever beam

A cantilever short beam with length-to-height ratio equal to 8/4 is shown in Figure 7. It is fully clamped along the left edge and a downward concentrated load $F = 300N$ is applied at the centroid of the right edge. Using an element size of 20mm, the whole domain is discretized into 800 (40×20) continuum finite elements. It is aiming to achieve a practical volumetric ratio of 0.48%

from an initial over-reinforced reinforcing ratio (2%), in other words, only 24% amount of reinforcing bars remain in the continuum concrete through optimization.

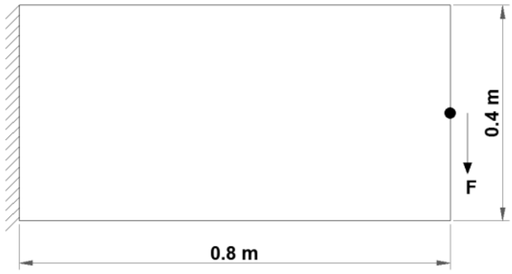
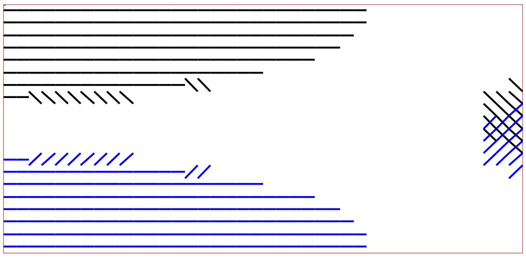
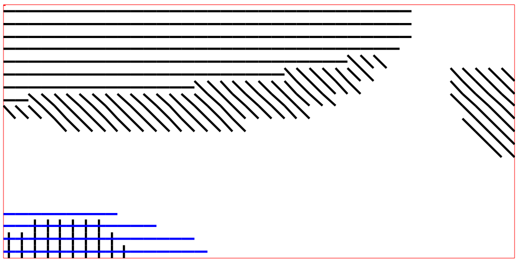


Figure 7. A 2D cantilever beam

Firstly, the influence of the implementation of the proposed update scheme in the optimization algorithm on the resulting topology is studied. As is shown in Figure 8a, a symmetric layout of reinforcing bars located in tension and compression regions is achieved for this particular case. Same amount of steel bars are distributed in the upper and lower parts of the cantilever beam when the asymmetric property of concrete is not considered. While Figure 8b represents the topology obtained by taking the allowable tensile and compressive strain for concrete into account in the update system of ranking elemental sensitivity numbers to update the corresponding design variables. It can be observed that more reinforcing bars are distributed in tension regions which is due to concrete has a strong nature in compression. Also, it should be pointed out that a small amount of bars perpendicular to the compressive reinforcing bars exist in the left lower area to reduce the concrete potential crack widths suffer from tensile stresses.
Furthermore, the cantilever beam with various length-to-height ratio of 9/4 and 10/4 are tested respectively, under the same loading and boundary condition, which is given in Figure 8c and d. Through comparison with the result described in Figure 8a, an increasing amount of vertical tensile steel bars are distributed to reinforce concrete in compression-dominant regions. Moreover, the length of the horizontal bar acting in both tension and compression are extended along the geometrical increase in length of the domain. And rows of compressive horizontal bar located at the bottom area increase from four (Figure 8b) to five (Figure 8d). However, the limitation of target volume of steel used and high demanding in other critical bar positions reduce the amount of diagonal reinforcements.



(a)



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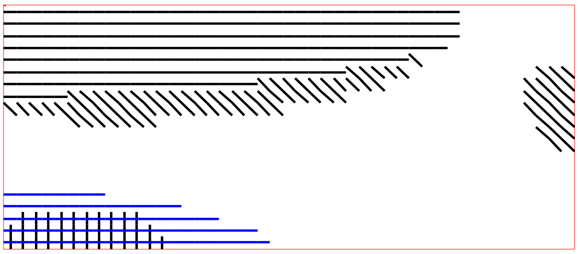

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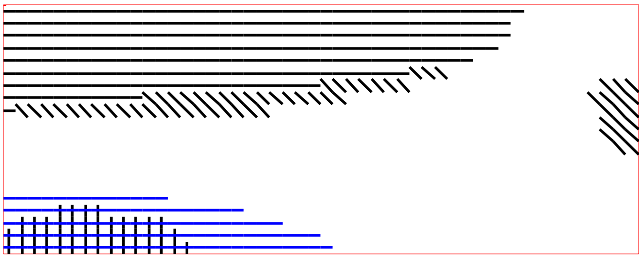


(d)

Figure 8. Comparison of resulting topologies. **a** Optimized layout without applying the novel update scheme into BESO algorithm. Optimized layout with applying the novel update scheme into BESO algorithm: **b** 2D structure with length-to-height ratio of 8/4. **c** 2D structure with length-to-height ratio of 9/4. **d** 2D structure with length-to-height ratio of 10/4.

6.2 A 2D deep beam

This example considers the layout design of steel reinforcing bars in a deep beam as is shown in Figure 9. It has fixed support on left and roller support that allow to move along x axial direction on right. And a downwards point load of 1500N is applied on top middle area. Also, it is aiming to investigate the effect of the mesh size of continuum elements on the optimized reinforcement layout. The target volumetric ratio of 0.4% for steel embedded into the continuum concrete are constant in all simulations.

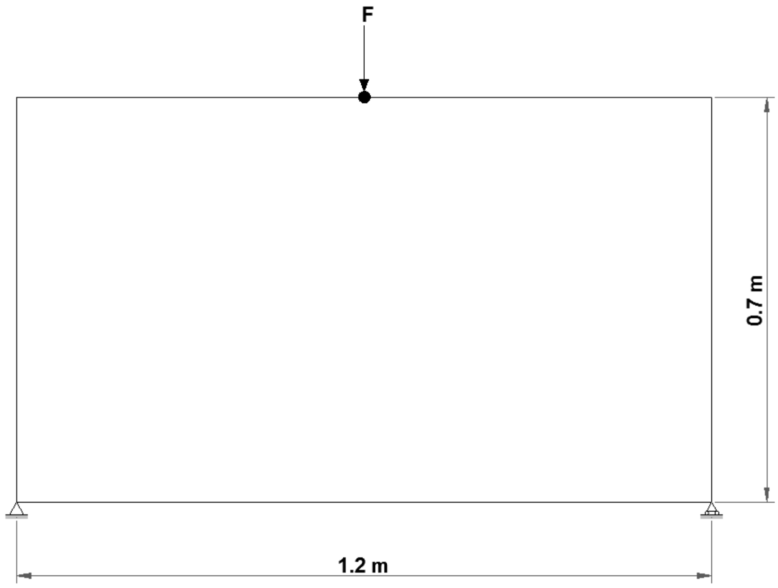
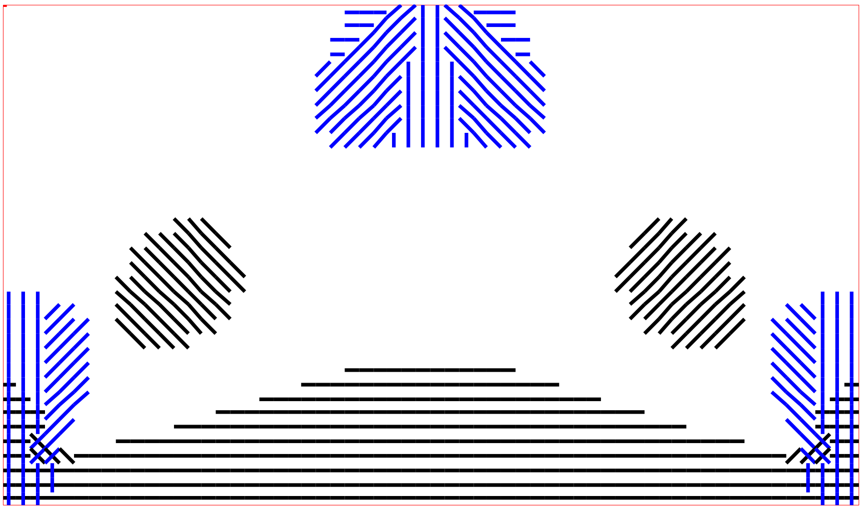


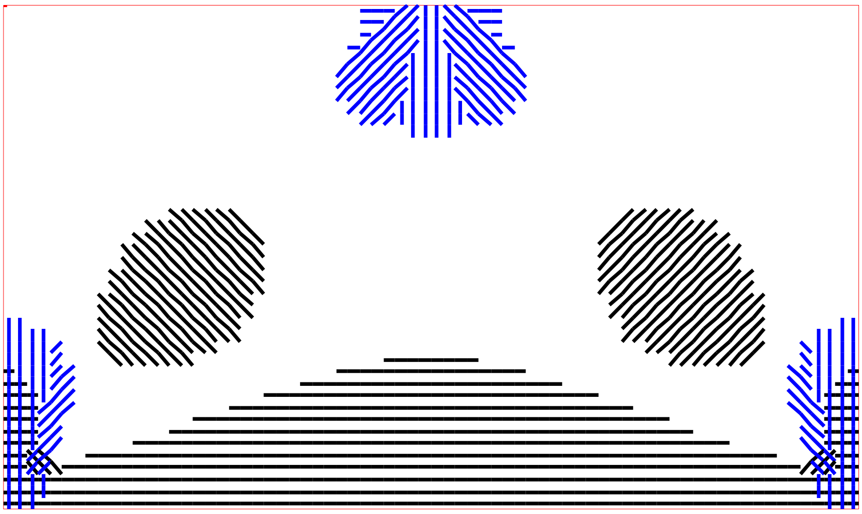
Figure 9. A 2D deep beam

Initially, the concrete domain is discretized by 2100 continuum elements by using the element size of 20mm. From the results presented in Figure 10a, it can be observed that large amount of tensile reinforcing bars are located at the bottom of the concrete domain where flexural failure easily exist. While the top middle zone in which the downwards pressure is applied, a group of steel bars are remained to strengthen concrete and act in compression. Also, double reinforcements are placed in the boundary support regions due to their high concentrated stress. As expected, there is an obvious distribution of diagonal reinforcing bars in tension appears to reinforce the shear part of the deep beam.

Since the reinforcing bars are embedded in continuum elements, a denser mesh for a continuum domain leads to a denser truss layout. The cross-sectional area of a bar decreases with the reduction of element size to ensure the reinforcing ratio in an individual concrete element is constant. It can be observed from Figure 10a-c that using a finer finite element mesh, more discrete tension-reinforced bars are distributed to prevent from missing any tensile strain developed in a compression-dominated phase that exceeds the allowable tensile strain for concrete. Simultaneously, some continuum elements having compressive strain are found to be able to withstand by concrete itself so that the volume of steel bar act in compression decreases. Particularly in Figure 10c, tensile-load carrying reinforcing bars exist around the top central region where the point load applied, which does not exist in other obtained topologies. As mentioned above, although the cross-sectional area of the reinforcing bar is unrealistic in this paper, it provides a rational reinforcement layout under a limited amount of steel used for designers.



(a)



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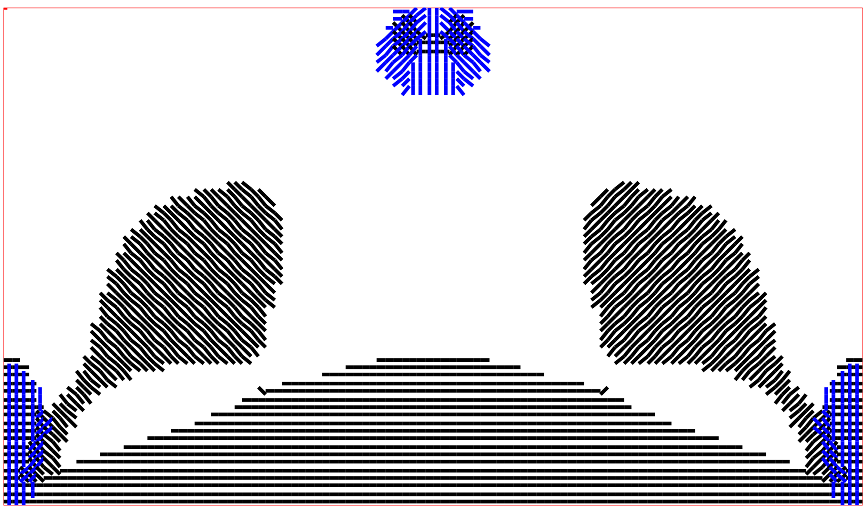

(c)

Figure 10. Reinforcement layout obtained with various mesh dense discretization for the continuum domain. **a** 2100 (60 × 35) finite elements. **b** 3024 (72 × 42) finite elements. **c** 6804 (108 × 63) finite elements

7. Conclusions and discussions

This paper proposed an optimization procedure on the basis of BESO approach to design reinforcement layout within a constant concrete domain. Two reinforcing bars modeled as 1D truss elements are embedded in each continuum element along maximum principal strain and minimum principal strain direction respectively. A heuristic orientation scheme is designed for rotating the bar into its most efficient orientation so that the resulting reinforcement topology is not influenced by the initial truss layout. To just corroborate the implementation of the proposed orientation finding system, only two types of embedded model in which reinforcing bars are located horizontally and vertically or in a diagonal form are considered. Furthermore, a novel scheme of updating design variables besides the relative ranking of sensitivity numbers is applied to the BESO algorithms to avoid reinforcing regions where concrete can withstand itself, especially for regions suffering from compression.

The optimization algorithms that incorporates volume constraint on steel are implemented in two 2D examples. From the results, it can be observed that reinforcing bars in tension are mainly distributed due to high tensile strains, while in order to strength some critical area with high compressive strain, reinforcement bars in compression are also located. Also, through the optimization procedure start with a denser initial truss layout, the homogenized average strain is derived more accurately from a finer finite element mesh. Hence, more truss elements are detected to have the maximum tensile strain exceeding the allowable tensile strain for concrete, and discrete reinforcing bar appears in more compression-dominant elements acting in tension.

Although the total volumetric ratio of steel amount into the continuum concrete satisfies the realistic practical design requirement, the cross-sectional area of an individual bar is in a very small scale which is unrealistic. Also only one type of steel reinforcement is achieved as the truss design variables are restricted to either absence or existence. In the future study, this can be improved by embedding the reinforcement in a group of continuum elements with reasonable space between bars in a three-dimensional (3D) model to generate a practical reinforcement layout. And the relationship between cross-sectional area and axial stress of bar is going to be added into the optimization algorithms. Nevertheless, the current study succeeds in achieving both location and orientation of reinforcing bars playing roles in tension or compression, which provides a valuable suggestion for the regions in a concrete structure to be reinforced by steel.

8. Reference

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