

Closed-Form Pathloss Formulation for IRS-Assisted 6G Wireless Networks

Petros Karadimas

School of Computing, Engineering and the Built Environment
Edinburgh Napier University
Edinburgh, UK
p.karadimas@napier.ac.uk

Abstract—Intelligent reflecting surfaces (IRSs) are smart devices that can manipulate electromagnetic wave propagation to optimize performance of future 6G wireless networks. In IRS-assisted wireless communications, one of the most important performance metrics to optimize is the received signal-to-noise ratio (SNR). As the SNR depends on the received signal power, it is thus dependent on the underlying path loss (PL) of the wireless channel. There exist dozens of PL formulations for wireless links facilitated through an IRS. However, IRS-assisted wireless communications comprise of a transmitter (Tx)-Receiver (Rx) link together with the one through the IRS, that is, the Tx-IRS-Rx link. The Tx-Rx and Tx-IRS-Rx links are subject to different PL formulations. The PL of the complete wireless channel comprised of both links has never been formulated. The latter constitutes the subject of this paper, where we present a closed-form PL solution for IRS-assisted wireless communications comprised of both Tx-Rx and Tx-IRS-Rx links. The presented solution is subject to the PLs of the individual links and demonstrates the great benefit that IRSs can have on overall PL reduction.

Keywords—Intelligent Reflecting Surfaces (IRS), Path Loss (PL), Wireless Channel Modelling.

I. INTRODUCTION

In future 6G wireless networks, the requirements for high data rates can be met by maximizing the received signal-to-noise ratio (SNR). The received SNR is directly related to the path loss (PL) which in turn depends on the physical propagation mechanisms of the wireless communication channel [1]. Intelligent reflecting surfaces (IRSs) have emerged as smart devices aiming to control electromagnetic wave propagation resulting in maximum received SNR [2], [3], [4]. Thus, devising accurate PL models for IRS-assisted wireless communications is of great importance to showcase potential benefits of IRS deployment for future 6G wireless networks.

IRS-assisted wireless communications comprise of a transmitter (Tx)-Receiver (Rx) link together with the link through the IRS, that is, the Tx-IRS-Rx link, or possibly multiple IRS-enabled links [3], [4], [5], [6], [7], [8], [9]. There exist several recently published PL formulations for the Tx-IRS-Rx link arisen from physical electromagnetic wave propagation modelling [10], [11], [12], [13]. All such formulations have resulted in a similar PL dependence with respect to the squared Tx-IRS and IRS-Rx distances. The PL of the Tx-Rx link has been extensively studied in the open literature with hundreds of

contributions published since many decades ago. A distance dependent PL formulation is the most commonly used for Tx-Rx links [14]. Hence, the individual Tx-Rx and Tx-IRS-Rx links are rationally subject to different PL formulations as was also reported in [6], [7].

A complete PL formulation for IRS-assisted wireless communications comprised of both Tx-Rx and Tx-IRS-Rx links is an open problem which is addressed in this work. Departing from a channel model consistent with the main IRS operation of maximizing the received SNR [2], [3], [4], a simple yet accurate closed-form PL solution is derived. The methodology is generic, hence, it applies on every IRS-assisted wireless scenario such as indoors, cellular outdoors, and high mobility vehicular communications [2], [4], [5], [9]. The presented solution is a function of the PLs of the individual Tx-Rx and Tx-IRS-Rx links. It is revealed that including a Tx-IRS-Rx link, together with the Tx-Rx one, results in significant PL reduction even if the PLs of the individual links are quite large. Such finding can constitute a major advance towards the adoption of IRS technology in future 6G wireless networks [15].

The remainder of the paper is organized as follows. Section II presents the theoretical channel model drawn from [2], [4] and derives the mean received power as required for PL analysis. In Section III, the closed-form PL solution for IRS-assisted wireless communications is derived followed by illustrative examples highlighting the great benefit that IRSs can have on overall PL reduction. Finally, Section IV draws the conclusion.

II. CHANNEL MODELING AND RECEIVED POWER

Consider an IRS-assisted wireless communication scenario comprised of a Tx-Rx and a Tx-IRS-Rx link as in Fig. 1. The IRS contains N reflecting elements. Using standard wireless channel modelling theory, e.g., [1], [2], the Tx- n th IRS element narrowband channel response $h_{1,n}$ will be

$$h_{1,n} = \sum_{l=1}^L a_l \exp(j\varphi_l) = |h_{1,n}| \exp(j\Phi_n) \quad (1)$$

where a_l is the real amplitude and φ_l the phase of the l th multipath component incident on the n th reflecting element of the IRS. Similarly, we can write the n th IRS element-Rx and Tx-Rx channel responses $h_{2,n}$ and h_0 , respectively, as

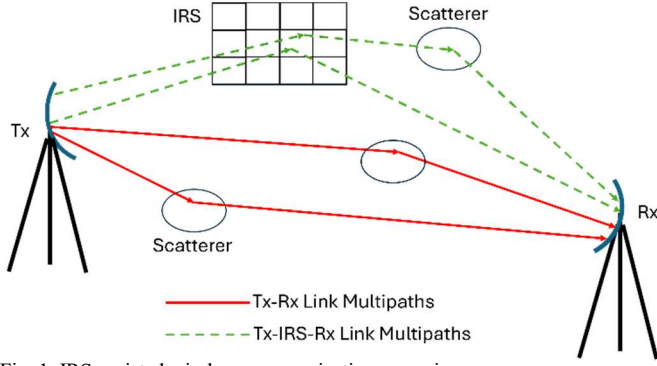


Fig. 1. IRS-assisted wireless communication scenario.

$$h_{2,n} = |h_{2,n}| \exp(j\theta_n) \quad (2)$$

$$h_0 = |h_0| \exp(j\psi_d). \quad (3)$$

The role of the IRS is to induce phase shifts in order the signals received through the Tx-Rx and Tx-IRS-Rx links to be added coherently, that is, to be aligned in phase [2], [4]. Such operation maximizes the received SNR, and the total channel response will be [2], [4]

$$h_{total} = |h_0| \exp(j\psi_d) + \sum_{n=1}^N |h_{1,n}| \exp(j\phi_n) |h_{2,n}| \exp(j\theta_n) C_n \exp(j\psi_n) \quad (4)$$

where $C_n \in [0,1]$ and $\psi_n \in [0,2\pi]$ are the amplitude attenuation and the adjustable phase induced by the n th reflecting element of the IRS. We should have $C_n = 1$ and $\psi_n = \text{mod}[\psi_d - (\phi_n + \theta_n), 2\pi]$ for maximum received SNR [2], [4]. If there is no Tx-Rx channel, e.g., if it is severely obstructed, and communication just takes place through the IRS, the solution for ψ_n will be $\psi_n = \text{mod}[-(\phi_n + \theta_n), 2\pi]$. With such definitions for C_n and ψ_n , the received power P becomes maximum as [2], [4]

$$P = \left[|h_0| + \sum_{n=1}^N |h_{1,n}| |h_{2,n}| \right]^2 = V^2. \quad (5)$$

A. Single Tx-IRS-Rx link

To obtain better intuition, we first study the Tx-IRS-Rx link. We assume that there is no contribution from the Tx-Rx link and the received power is exclusively contributed by the Tx-IRS-Rx link as follows

$$P_1 = \left[\sum_{n=1}^N |h_{1,n}| |h_{2,n}| \right]^2 = V_1^2. \quad (6)$$

Considering the trivial assumptions of a large number of IRS elements, that is, $N \gg 1$, statistically independent $|h_{1,n}|$, $|h_{2,n}|$ for all $n = 1, 2, \dots, N$ [2], [4], and employing the central

limit theorem (CLT) in its most general form [16, p. 278], the mean value and variance of $V_1 = \sum_{i=1}^N |h_{1,n}| |h_{2,n}|$ will be

$$E[V_1] = N \cdot E[|h_1| |h_2|] = N \cdot E[|h_1|] \cdot E[|h_2|] \quad (7)$$

$$\begin{aligned} \text{Var}[V_1] &= N \cdot \text{Var}[|h_1| |h_2|] = \\ &= N \cdot E[|h_1|^2] \cdot E[|h_2|^2] - N \cdot E[|h_1|]^2 E[|h_2|]^2 \end{aligned} \quad (8)$$

where $E[\cdot]$ and $\text{Var}[\cdot]$ are the expectation and variance operators, respectively. We derived (7) and (8) by also assuming $|h_{1,n}|$ for all $n = 1, 2, \dots, N$ to be identically distributed, likewise $|h_{2,n}|$ for all $n = 1, 2, \dots, N$, see [4].

For PL analysis, we need to derive the mean received power as follows

$$P_{r1} = E[P_1] = E[V_1^2] = E[V_1]^2 + \text{Var}[V_1]. \quad (9)$$

Using (7) and (8) in (9), we obtain after some manipulations

$$\begin{aligned} P_{r1} &= N^2 \cdot E[|h_1|]^2 E[|h_2|]^2 + \\ &= N \cdot E[|h_1|^2] \cdot E[|h_2|^2] - N \cdot E[|h_1|]^2 E[|h_2|]^2. \end{aligned} \quad (10)$$

Employing the standard assumption of a large number of IRS elements, that is, $N \gg 1$ [2], [4], the mean received power in (10) is approximated as

$$P_{r1} \approx N^2 \cdot E[|h_1|]^2 E[|h_2|]^2 = E[V_1]^2. \quad (11)$$

By a simple inspection of (9) and (11), we can primarily notice the great impact of IRSs on wireless communications, that is, they can significantly mitigate the stochastic random nature of wireless channels. Such theoretical observation can be also deduced from recently published measurement campaigns [17].

B. Both Tx-Rx and Tx-IRS-Rx links

We expand the analysis conducted for a single Tx-IRS-Rx link by including the contribution of the Tx-Rx link according to (5). Considering statistically independent $|h_0|$, $|h_{1,n}|$, $|h_{2,n}|$ for all $n = 1, 2, \dots, N$ and adopting all other assumptions reported just below (6) and (8), we can derive the mean value and variance of $V = |h_0| + \sum_{n=1}^N |h_{1,n}| |h_{2,n}| = |h_0| + V_1$ by employing the CLT in its most general form [16, p. 278]. Thus,

$$E[V] = E[|h_0|] + E[V_1] \quad (12)$$

$$\text{Var}[V] = \text{Var}[|h_0|] + \text{Var}[V_1]. \quad (13)$$

Using (7) in (12) and (8) in (13), we obtain

$$E[V] = E[|h_0|] + N \cdot E[|h_1|] \cdot E[|h_2|] \quad (14)$$

$$\begin{aligned} \text{Var}[V] &= \text{Var}[|h_0|] + \\ &= N \cdot E[|h_1|^2] \cdot E[|h_2|^2] - N \cdot E[|h_1|]^2 E[|h_2|]^2. \end{aligned} \quad (15)$$

Similarly to (9), the mean received power will be

$$P_r = E[P] = E[V^2] = E[V]^2 + \text{Var}[V]. \quad (16)$$

Using (14) and (15) in (16), we obtain after some manipulations

$$\begin{aligned} P_r = & E[|h_0|^2] + N^2 \cdot E[|h_1|^2]E[|h_2|^2] + \\ & 2N \cdot E[|h_1|] \cdot E[|h_2|] \cdot E[|h_0|] + \text{Var}[|h_0|] + \\ & N \cdot E[|h_1|^2] \cdot E[|h_2|^2] - N \cdot E[|h_1|]^2 E[|h_2|]^2. \end{aligned} \quad (17)$$

Employing again the standard assumption of a large number of IRS elements, that is, $N \gg 1$ [2], [4], we can neglect the last two terms of (17) as compared with the term $N^2 \cdot E[|h_1|^2]E[|h_2|^2]$ similarly to the case of a single Tx-IRS-Rx link. We will not however neglect the term $2N \cdot E[|h_1|] \cdot E[|h_2|] \cdot E[|h_0|]$ as we want to account for the actual scenario of having a functional Tx-Rx link, that is, a Tx-Rx link with non-zero mean power. Thus, using $E[|h_0|^2] + \text{Var}[|h_0|] = E[|h_0|^2]$, the mean received power in (17) is approximated as

$$P_r \approx N^2 \cdot E[|h_1|^2]E[|h_2|^2] + E[|h_0|^2] + 2N \cdot E[|h_1|] \cdot E[|h_2|] \cdot E[|h_0|]. \quad (18)$$

It is observed in (18) that the first term represents the mean power contributed by the Tx-IRS-Rx link, that is, $P_{r1} \approx N^2 \cdot E[|h_1|^2]E[|h_2|^2]$, see also (11). The second term is the mean power contributed by the Tx-Rx link, that is, $P_{r0} = E[|h_0|^2]$. Eventually, the third term can be written as $2N \cdot E[|h_1|] \cdot E[|h_2|] \cdot E[|h_0|] \approx 2\sqrt{P_{r1}} \cdot E[|h_0|]$. Thus, (18) becomes

$$P_r \approx P_{r0} + P_{r1} + 2\sqrt{P_{r1}} \cdot E[|h_0|]. \quad (19)$$

We just need to associate the term $E[|h_0|]$ with P_{r0} . To do so, we consider a Nakagami- m (m, Ω) statistical model for $|h_0|$ due to its wide flexibility, that is, accounting for less fading severity than the Rayleigh model but including the Rayleigh as special case ($m = 1$), ability to approximate the Rician model, and mathematically tractable formulas for its moments such as the mean value, mean squared value (power), and variance [18]. Moreover, the Nakagami- m model can closely approximate the random electromagnetic wave propagation physical problem [18]. Hence, $E[|h_0|]$ is determined as [18]

$$E[|h_0|] = \frac{\Gamma(m + \frac{1}{2})}{\Gamma(m)\sqrt{m}} \sqrt{\Omega} = \frac{\Gamma(m + \frac{1}{2})}{\Gamma(m)\sqrt{m}} \sqrt{P_{r0}} \quad (20)$$

where $\Gamma(\cdot)$ is the Gamma function, m the parameter accounting for the fading severity, and Ω the mean squared value, that is, the mean power of the Tx-Rx link. Thus, $\Omega = E[|h_0|^2] = E[P_0] = P_{r0}$. Using (20) in (19), we obtain the desired formula associating the total mean received power with the contributions from the Tx-Rx and Tx-IRS-Rx links as follows

$$P_r \approx P_{r0} + P_{r1} + 2 \frac{\Gamma(m + \frac{1}{2})}{\Gamma(m)\sqrt{m}} \sqrt{P_{r0}P_{r1}}. \quad (21)$$

For a Rayleigh ($m = 1$) Tx-Rx link, (21) simplifies to

$$P_r \approx P_{r0} + P_{r1} + \sqrt{\pi P_{r0}P_{r1}}. \quad (22)$$

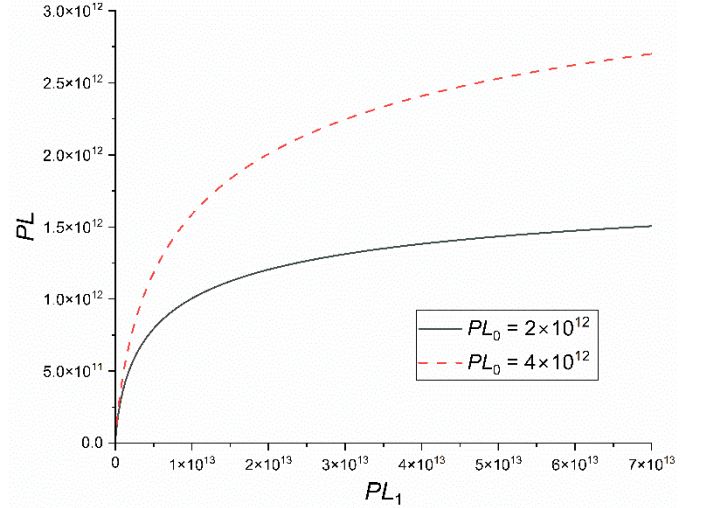


Fig. 2. Path loss illustration for IRS-assisted wireless communications.

Formulas (21) and (22) associating the received power with the contributions from the Tx-Rx and Tx-IRS-Rx links are novel findings. These formulas will be leveraged in the next Section to derive the closed-form PL solution for IRS-assisted wireless communications.

III. PATH LOSS FORMULATION AND ILLUSTRATIVE EXAMPLES

A. Path Loss Formulation

Consider P_t to be the transmitted power, PL_0 and PL_1 the PLs of the Tx-Rx and Tx-IRS-Rx links, respectively. We then define $P_{r0} = P_t/PL_0$ and $P_{r1} = P_t/PL_1$. Using such definitions in (21), we have

$$P_r \approx P_t \left[\frac{1}{PL_0} + \frac{1}{PL_1} + 2 \frac{\Gamma(m + \frac{1}{2})}{\Gamma(m)\sqrt{m}} \sqrt{\frac{1}{PL_0 PL_1}} \right]. \quad (23)$$

From (23), we derive the PL of IRS-assisted wireless communications by using the definition $P_r = P_t/PL$. Thus, we have after some manipulations

$$PL = \frac{P_t}{P_r} \approx \frac{PL_0 PL_1}{PL_0 + PL_1 + 2 \frac{\Gamma(m + \frac{1}{2})}{\Gamma(m)\sqrt{m}} \sqrt{PL_0 PL_1}}. \quad (24)$$

For a Rayleigh ($m = 1$) Tx-Rx link, (24) simplifies to

$$PL \approx \frac{PL_0 PL_1}{PL_0 + PL_1 + \sqrt{\pi PL_0 PL_1}}. \quad (25)$$

The closed-form solutions (24) and (25) are novel findings associating the PL of IRS-assisted wireless communications with the PLs of the individual Tx-Rx and Tx-IRS-Rx links. Note from (24) the following properties hold after some elementary algebra, that is, $\frac{dPL}{dPL_1} > 0$ and $\lim_{PL_1 \rightarrow \infty} PL = PL_0$. Thus, PL is a monotonically increasing function with respect to PL_1 and the maximum PL equals to PL_0 when $PL_1 \rightarrow \infty$. In other words, the

worst case scenario is $PL = PL_0$ when $PL_1 \rightarrow \infty$. In all other cases, $PL < PL_0$ and in fact $PL \ll PL_0$ for practical IRS-assisted wireless communication scenarios. This will be further demonstrated next by studying some illustrative examples.

B. Illustrative Examples

As a first example, consider a Rayleigh Tx-Rx link and a relatively “good” Tx-IRS-Rx link with its PL being almost three times less than the PL of the Tx-Rx link, that is, $PL_1 = PL_0/\pi$. After some elementary algebra, we find from (25) $PL = PL_0/(2\pi + 1)$. In other words, use of an IRS results in an almost seven times overall PL reduction, that is, around 85% reduction compared to the Tx-Rx PL_0 . An even better Tx-IRS-Rx link with less PL_1 will result in better overall PL reduction.

As another example, consider again a Rayleigh Tx-Rx link and a relatively “bad” Tx-IRS-Rx link with its PL being almost three times higher than the PL of the Tx-Rx link, that is, $PL_1 = \pi PL_0$. After some elementary algebra, we find from (25) $PL = (\pi PL_0)/(2\pi + 1)$. In other words, use of an IRS results in an almost 60% overall PL reduction compared to the Tx-Rx PL_0 . For even “worse” Tx-IRS-Rx links with higher PL_1 , the overall PL reduction will be further compromised. However, keep in mind that it is not actually a realistic scenario to deploy IRSs such that the Tx-IRS-Rx link to be “worse” or “much worse” than the Tx-Rx link.

The above mentioned findings are graphically demonstrated in Fig. 2 for the case of a Rayleigh Tx-Rx link, where we plot the PL of (25) against PL_1 . For the sake of illustration, we consider $PL_0 = 2 \cdot 10^{12}$ and $4 \cdot 10^{12}$ for the Tx-Rx link. The monotonically increasing behaviour of PL with respect to PL_1 is clearly evident and PL will approach PL_0 for relatively large values of PL_1 as was discussed previously. Significant overall PL reduction compared to PL_0 is revealed for rational values of PL_1 even for “bad” Tx-IRS-Rx links with relatively high PL_1 . Indicatively, for $PL_0 = 2 \cdot 10^{12}$, a relatively “very bad” Tx-IRS-Rx link with $PL_1 = 10^{13}$ results in $PL \approx 10^{12}$, almost half of the Tx-Rx link PL_0 . In the scenario that the 6G network engineer wisely installed the IRS to get a relatively “very good” Tx-IRS-Rx link with $PL_1 = 4 \cdot 10^{11}$, this will result in $PL \approx 2 \cdot 10^{11}$, almost ten times less than the Tx-Rx link PL_0 .

Similar observations can be reported should we consider a Nakagami-m Tx-Rx link and employ the more generic PL formula (24). Such findings clearly demonstrate the great benefit that IRSs can have on overall PL reduction making them a quite promising technology for future 6G wireless network deployments [15].

IV. CONCLUSION

A closed-form PL solution for IRS-assisted wireless communications comprised of both Tx-Rx and Tx-IRS-Rx links was presented. Such solution relies solely on the PLs of the individual Tx-Rx and Tx-IRS-Rx links. Through further mathematical processing, illustrative examples and graphical presentations, use of IRSs was proven extremely beneficial in terms of overall PL reduction. Even employing a relatively “bad” Tx-IRS-Rx link, with higher PL compared to that of the Tx-Rx link, results in significant overall PL reduction. Such

findings are quite promising towards adopting IRS technology in future 6G wireless network planning and optimization.

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