Prediction of modal frequencies, modal shapes and static point load deflections of I-joist timber flooring systems using finite element method

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PREDICTION OF MODAL FREQUENCIES, MODAL SHAPES AND STATIC POINT LOAD DEFLECTIONS OF I-JOIST TIMBER FLOORING SYSTEMS USING FINITE ELEMENT METHOD

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ABSTRACT: The vibrational behaviour of structural timber flooring systems can result in serviceability problems due to discomfort experienced by the occupants. However, the dynamic response of such structural systems cannot be easily determined by hand calculations and as a result, finite element method (FEM) was used, which provides a suitable tool for numerical modelling for evaluation of dynamic parameters. In this study, timber flooring structures constructed with I-joists and particleboard decking were modelled for eigenproblem analyses to predict the most critical modal frequencies with corresponding modal shapes and also the static deflection under a point load. Joint elements were introduced at the interfaces between deck and joists to account for the spring stiffnesses due to the connection with metal fasteners. Spring stiffnesses were also assigned to the supports considering slip and withdrawal effects. Some sensitivity studies were then performed to identify the influence of introducing the spring stiffness in the individual translational directions and to determine the appropriate withdrawal stiffness. This paper presents the details of the modelling and also the correlation of predicted and measured results of six full-scale timber I-joist flooring systems. It shows the capability of the model in determining the most critical frequencies (e.g. of the first five principal first order modes), accurately in particular the fundamental frequencies and the modal shapes as well as good prediction of deflections under static point loads.

KEYWORDS: Dynamic Response, Finite Element Method, Eigenproblem Analysis, Flooring Systems, I-joists

1 INTRODUCTION

The use of finite element method (FEM) of analysis can permit accurate prediction of structural behaviour of complex systems so as to minimise the need for relatively expensive experimental investigations. The method is particularly suitable for structures, whose responses cannot be easily determined by hand calculations.

The method, in principal involves breaking down of complex structural system into a number of interdependent finite elements (mesh), to determine structural responses from external influences. Each single element holds nodes with a number of degrees of freedom. Coinciding nodes of different elements interact. A set of equations is composed of mathematical expressions formulated for the response of each element. The degrees of freedom are the unknowns. A matrix technique is used to solve the equations [1]. Detailed background information on the basic principles of the finite element analysis (FEA) is given by Henwood and Bonet [2].

This paper presents the development of a finite element model to predict natural frequencies with corresponding modal shapes of timber I-joist flooring systems, based on an eigenvalue analysis, using the commercial FEM software LUSAS. To support model verification, the deflection under static point load was also investigated. This study forms part of an extensive research project on the dynamic response of structural timber flooring systems [3].

Generally, a model is composed of geometric features to which attributes are assigned. The structural geometry is created by selecting coordinates to define the geometric points, which in turn define geometric lines. These are used to compose a surface, and a number of surfaces can be combined to form a volume [1]. The different geometric features can be merged with adjoining elements of the same type to gain full composite action.

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Mesh properties (mesh refinement, finite element types), material properties, geometric properties, boundary conditions, etc. are then assigned to the geometric features. The mesh refinement is dependent on the required accuracy of the results and an acceptable calculation time. If the mesh refinement is raised, the accuracy should be increased, and so is the required calculation time.

The element types used are selected by considering the needs and demands of the model and structural responses under investigation. Over 100 element types can be chosen from in LUSAS and are divided into groups such as Beams, Plates, Shells, Joints, etc. See [4] for a full list of element types available in LUSAS.

The objective was to produce convincing correlations of predicted and measured responses while preventing the model from becoming too complex but considering necessary details. It further aimed at demonstrating the influence of spring stiffness assigned at the supports and interface of deck and joists on dynamic floor responses.

2 LITERATURE REVIEW ON ANALYTICAL AND NUMERICAL METHODS TO PREDICT DYNAMIC TIMBER FLOOR RESPONSES

For the prediction of dynamic floor responses of a flooring system tested in laboratory, Ohlsson [5] employed a grillage model. Joists were represented by beam members and the decking by cross beams. Since no glue was used in reality and no ceiling attached to the floor, a torsion-weak model was assumed. It consisted of five main and five cross beams. Mode shapes and natural frequencies were obtained using the computer programme SFVIBAT-II for a dynamic analysis. While the mode shapes were observed to be very well predicted, the predicted natural frequency satisfactorily matched the measured one. It was believed that a better correlation of the latter may have been achieved if torsional stiffness and elasticity of the connections had been considered.

A mathematical model, which was based on the Rayleigh-Ritz method, was developed by Chui [6] to predict the dynamic response of timber floors with solid joists and continuous decking, assuming the joist ends to be simply supported and having the two edge joists simply supported or free along their length. The decking sheets were to be rigidly or semi-rigidly connected to the joists. Model validation was obtained by comparing predicted results with experimentally determined ones. The predicted modal shapes were observed to be identical to the ones obtained from measurements. There was a general variation of up to 5% for the fundamental frequencies with a maximum of 13%. Higher natural frequencies were generally under-estimated by usually below 20% with rising inaccuracy for successive modes. Negligence of transverse shear deformations in the model was believed to be the cause of this since such deformations would get more significant with increasing mode number. The model was also used to predict acceleration responses. It was concluded that the model would be acceptable for design purposes.

Hu [7] developed a numerical model based on the modal synthesis method to predict natural frequencies and acceleration responses of ribbed plates. Comparison of natural frequency predictions to results obtained from experimental work on timber I-joist floors showed errors under 10% for 29 floors and above 25% for three floors. The model was further validated against 17 other I-joist floors, yielding generally similar agreement levels as before for the higher natural frequencies and an error of 7.4% on average for the fundamental frequencies. Studying the modal shapes of three test floors, the shapes and the number of nodes and anti-nodes were found to be predicted well.

With respect to the examined I-joist floors, the model was identified to produce more accurate predictions of the natural frequencies compared to the models developed by Chui (see [6]) and Filiatrault [8]. However, the greater accuracy was not fully given when predicting natural frequencies of two floors with lumber joists, one tested by Chui (see [6]) and one by Ohlsson (see [5]), comparing Hu's model predictions to Chui's and Filiatrault's, and Chui's and Ohlsson's respectively. The model of Hu, which considers shear deformation effects and rotatory inertia in ribs, was concluded to be applicable for the prediction of natural frequencies and modal shapes of ribbed plates of various materials with a similar or improved accuracy in comparison to other models.

Studies by [9]-[11] describe the development of a FE model for complex wood-based flooring systems constructed with I-joists or truss joists to predict fundamental frequency and static point load deflection. Shell elements were used to model deck and ceiling, beam elements to model joists and transverse stiffening members. Connector elements were developed to model the fasteners (see [11] for details), adopting two-noded elements for semi-rigid connections of joists and transverse members and a four-noded interface element for the connection of deck or ceiling to the joists. The correlation coefficient for predicted and measured fundamental frequencies was $R^2 = 0.7327$. It was found that the fundamental frequencies of the floors were generally over-predicted. This was believed to be caused by modelling the supports as simply rigid, and this assumption was confirmed by re-examining one flooring system assigning flexible supports. The deflections were predicted with a correlation coefficient $R^2 = 0.8698$. Furthermore, several floors were also investigated with respect to relative variation in response due to the structural modifications of introducing transverse stiffeners. The prediction of relative changes was found to work rather well and the contribution from the transverse reinforcements to be slightly under-estimated, resulting in conservative predictions.

Numerical and analytical methods were used in the past to establish models for the prediction of the dynamic responses of timber floors. Difficulties lay especially in modelling of decking layers, end fixity and elasticity of connections. Even very complex models may not yield excellent predictions of floor’s vibrational behaviour. Establishing universally applicable flooring models is difficult due to various aspects of building practice and ranges of materials that can be used, such a solid timber joists, I-joists, metal-web joists, etc.

Especially since appropriate computer programmes became more powerful and the technology to effectively perform analyses affordable, relatively complex systems can be modelled. However, it is desirable to use models whose calculation times are economical with respect to standard computational equipment. The study presented in this paper focused on response prediction for I-joist flooring systems and considered discontinuities in the deck, its flexible connection to the joists and the degree of end fixity while using only one element type for all timber and timber-based materials.

3 MODELLING TIMBER I-JOIST FLOORS

Complexity when modelling timber flooring systems lies in the composite nature and anisotropy of the structure and sometimes materials. The flooring structure is basically assembled of joists, rim boards, decking sheets and fasteners. Nowadays engineered timber joists can be composed of various material types with different dimensions, such as timber I-joists with OSB web and solid timber or plywood type flanges. The decking is usually composed of several adjoining sheets, causing discontinuities in the deck.

3.1 FLOORING STRUCTURES SELECTED FOR MODELLING

The model was created with respect to floors of the JJI test series (see [3] for details), starting with Floor JJI 2 A. This floor had a size of 3.50 m x 2.44 m, was constructed with five composite timber I-joists of 220 mm depth and $b \times h$ = 45 mm × 45 mm flanges, which ends were connected to glulam rim boards by screws, and decked with 1 layer of 3 particleboard sheets, screw-fixed to the joists and rim boards. Each flange end was fixed to the supports by one screw on either side of the web. The supporting structure consisted of timber wall plates, which were assembled to a frame and fixed to the concrete floor of the laboratory. Two further wall plates were fixed on top of the frame, one at each of the opposite edges with shorter length so as to have the floor supported at two sides.

Two other floors (JJI 2 G, JJI 2 J) of the same dimensions were modelled, in which the central single floor joist was exchanged by double joists. For one of these two cases (Floor JJI 2 G), the flange widths of the double joists were raised to 97 mm each. Thereafter, three floors with the properties of the above types but with greater joist depth (h = 245 mm) were modelled and denoted "1" instead of "2". These were six of 67 floorings structures investigated experimentally in laboratory conditions. Stiffening dynamically sensitive locations was the reason for the use of double joists, which is discussed in [3],[12].

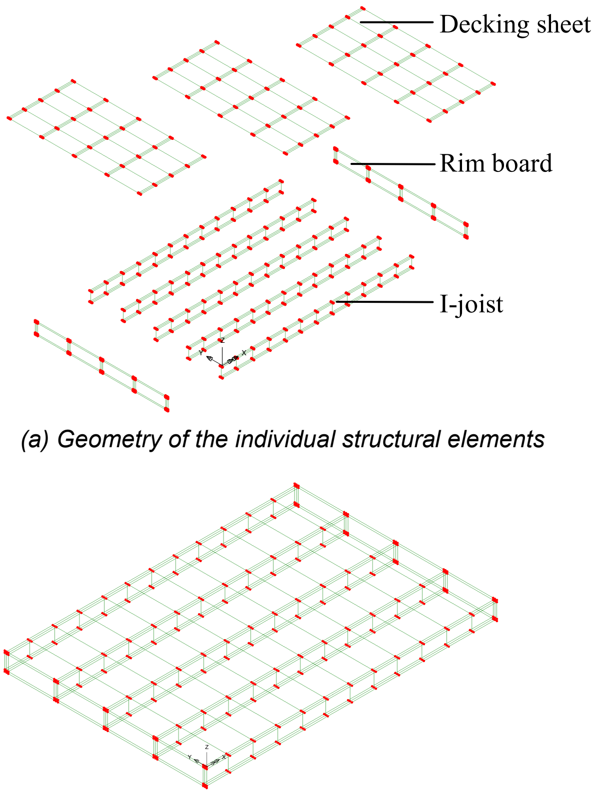
3.2 CREATING THE MODEL

Using shell elements was found to be suitable to model all timber(-based) materials with the same element type, allowing accurate reproduction of material properties and being appropriate for an eigenvalue analysis to identify the principal bending modes of the structure.

This means that geometric surfaces were defined for the flanges and web of each I-joist, for the rim boards and for the decking sheets. The geometry of the floor was subdivided at the locations of fasteners along the joists and with regard to further meshing purposes. Geometries were then merged where web and flanges of the I-joists meet and at locations where the surfaces were separated for meshing purposes only. Thus, the decking sheets could be placed next to each other without being (fully) connected.

Isotropic quadrilateral 8-noded (3D) thick shell elements (QTS8) with 6 degrees of freedom (3 translations and 3 rotations) were assigned to the surfaces. The material thickness and potential eccentricity of the nodal plane to the bending plane can be set through the geometric properties. Some eccentricity occurred from the locations of the deck and rim board geometry. The material attributes, which had to be assigned, were taken from literature [13]-[15].

Figure 3.1 illustrates the geometry of Floor JJI 2 A, showing the separated structural elements, in which the geometric features of each individual member (joist, decking sheet, rim board) are fully merged (Figure 3.1a), and the combined, but not fully merged, geometry (Figure 3.1b).



(a) Geometry of the individual structural elements

(b) Combined geometry

***Figure 3.1**: Model geometry of Floor JJI 2 A* [3]

The model was created gradually so as to examine the degree of possible model simplification. This involved principally the introduction of spring stiffness at the supports and as joint elements at the interface of different geometries. First, the model was established without these spring stiffnesses, assuming full composite action at coinciding geometric points of different geometries and pin-supports with fully restrained translations (free rotation), and an eigenvalue analysis performed.

Spring stiffness with respect to the translational directions was then assigned to each point of the bottom flange ends to modify the end fixity. The spring stiffness was first only assigned to the lateral y-direction, then only to the x-direction, then only to the (vertical) z-direction, then only to x/y-directions while restraining other translational movements, and finally to all three translational directions. Each time an eigenvalue analysis was carried out.

This procedure was followed by the implementation of 2-noded 3D joint elements, which connect two nodes with the same coordinates, at those geometric points, which represented the fasteners to connect decking sheets and joists/rim boards. The selected joint elements (JNT4) connect the nodes by springs in the translational directions and have no rotational stiffness. Joint stiffness was assigned so as to allow movement in the lateral directions only. The remaining coinciding geometric points of joists and deck were unmerged.

As before, the influence of the newly assigned spring stiffness was examined gradually, keeping one direction "fixed" (using very high stiffness) and conducting eigenvalue analyses. In practice each decking margin was fixed with screws. In the model, coinciding geometric points of adjoining sheets were merged. Therefore, the spring stiffness assigned to the points representing the screws was (manually) doubled at locations where decking sheets meet. To examine the influence on natural frequencies of adding joint elements only, the support conditions were set to be fully restrained.

Finally, spring stiffness was assigned to the translational directions at the supports and to the translational directions in plane of the joint elements for the connection of deck and joists. Deck to rim board connection was only considered at the location of I-joists. Connecting the joists' ends to the rim boards was simplified by merging coinciding geometric points of these elements but keeping the geometric lines unmerged.

As the stiffness values for movement in plane direction were assumed to be equal to the slip modulus k_{ser} [16], they were calculated from:

$$k_{\mathrm{ser}} = \rho_{\mathrm{m}}^{1.5} \frac{d}{23} \qquad (3.1)$$

where ρ_{m} is the mean density of the jointed members in [kg/m^3] and d is the fastener diameter in [mm].

The assigned withdrawal stiffness of the screws at the supports was initially based on minimum and maximum values reported by [9], who conducted experimental investigations on axial load-displacement moduli of fastener-to-wood connections.

The modelled bottom flange ends consisted of three geometric points to which the support conditions were assigned to. Since two screws were used per joist end to fix them to the supports, the spring stiffness determined for one screw was multiplied by a factor of 2 and then divided by 3 for distribution onto the geometric points. To establish the final model, the withdrawal stiffness was the property that was used for further adjustments until an excellent correlation of the predicted and measured fundamental vibration mode of Floor 2 A was obtained. The identified appropriate stiffness value was then kept for modelling the other five floors.

The most suitable mesh refinement was determined by running a number of analyses with the initial model using increasing numbers of finite elements until the variation in results from two successive analyses became marginal. Then, the mesh with lower refinement but similar accuracy was selected to save calculation time. Figure 3.2 shows the final mesh for the floor model where no spring stiffness was assigned.

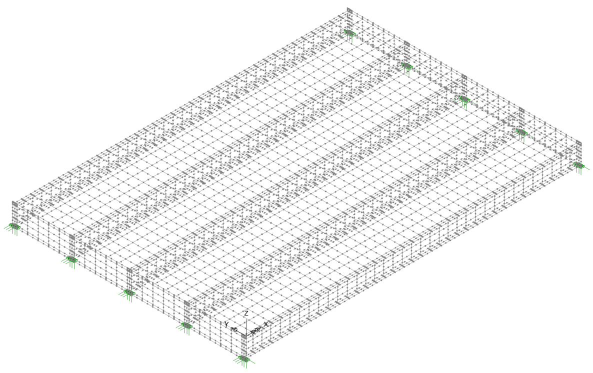


Figure 3.2: *Mesh refinement of model (without spring stiffness at supports and joint elements)* [3]

3.3 INTERMEDIATE RESULTS

The gradually assigned and varied spring stiffness values for Floor JJI 2 A are shown in Table 3.1, accordingly predicted natural frequencies *f* of the first five principal vibration modes are given in Table 3.2.

Assigning the spring stiffness to the supports had little effect on the modes by slightly reducing the natural frequencies, apart from the fourth principal bending mode with some increase in frequency, if it was applied to only the transverse direction (horizontal y-direction). It clearly decreased the natural frequencies if applied only to the longitudinal direction. Assigning spring stiffness to both horizontal directions at once therefore led to results close to the latter with slightly higher frequency of the fourth principal mode.

Lowering the restraints in vertical direction by using the minimum and maximum values for withdrawal stiffness as reported by [9] showed greatest impact on the frequencies corresponding to the first two principal modes, notable influence on the frequency of the third mode and little effect on higher ones.

Following these initial analyses, spring stiffness was assigned to each of the three translational directions, conducting two further runs under consideration of the different values in the vertical direction.

The introduction of joint elements at the interface of deck and joists lowered all examined natural frequencies with strongest impact on the frequency of the third principal mode if fully restraining movement in vertical direction, assigning very high stiffness to one of the horizontal directions and the calculated spring stiffness to the other one. While the frequencies of the first three principal bending modes were affected to a similar degree whether the calculated stiffness was assigned to the x- or y-direction, there was a noteworthy difference in the degree of the effect for the two highest examined natural frequencies.

The gradual investigation of the FE-model demonstrated that consideration of spring stiffness at the supports and for the connection of deck and joists is required for an eigenproblem analysis. The estimation of frequencies corresponding to the lower vibration modes was mostly affected by accounting for spring stiffness at the supports, and the estimation of higher natural frequencies was mainly influenced by the introduction of joint elements at the interface of deck and joists (see Table 3.1 and Table 3.2).

Table 3.1: *End fixity and elasticity of deck-to-joists connections for various models* [3]

Floor JJI 2 A	Support			Deck/Joists		
	x	**y**	**z**	**x**	**y**	**z**
	$k_{ser,x}$ [N/m]	$k_{ser,y}$ [N/m]	$k_{withdraw,z}$ [N/m]	$k_{ser,x}$ [N/m]	$k_{ser,y}$ [N/m]	$k_{withdraw,z}$ [N/m]
(A): complete floor without support springs	Fixed	Fixed	Fixed	Rigid	Rigid	Rigid
(B): (A) + support y-springs + x/z-fixed	Fixed	1.366×10^6	Fixed	Rigid	Rigid	Rigid
(C): (A) + support x-springs + y/z-fixed	1.366×10^6	Fixed	Fixed	Rigid	Rigid	Rigid
(D): (A) + support x/y-springs + z-fixed	1.366×10^6	1.366×10^6	Fixed	Rigid	Rigid	Rigid
(E_1): (A) + support z-springs + x/y-fixed	Fixed	Fixed	3.000×10^5	Rigid	Rigid	Rigid
(E_2): (E_1) + higher spring value	Fixed	Fixed	1.100×10^6	Rigid	Rigid	Rigid
(F_1): (A) + support springs in x/y/z	1.366×10^6	1.366×10^6	3.000×10^5	Rigid	Rigid	Rigid
(F_2): (F_1) + higher z-spring value	1.366×10^6	1.366×10^6	1.100×10^6	Rigid	Rigid	Rigid
(G): (A) + deck screws y-springs + "rigid" x	Fixed	Fixed	Fixed	1.000×10^{17}	1.638×10^6	Rigid
(H): (A) + deck screws x-springs + "rigid" y	Fixed	Fixed	Fixed	1.638×10^6	1.000×10^{17}	Rigid
(I): **(A)** + deck screws x/y-springs	Fixed	Fixed	Fixed	1.638×10^6	1.638×10^6	Rigid
(K_1): (A) + support x/y/z-springs + deck x/y-springs	1.366×10^6	1.366×10^6	3.000×10^5	1.638×10^6	1.638×10^6	Rigid
(K_2): (K_1) + higher z-spring value (support)	1.366×10^6	1.366×10^6	1.100×10^6	1.638×10^6	1.638×10^6	Rigid
(K_3): **(K_2)** + adjusted z-spring values (support)	1.366×10^6	1.366×10^6	4.500×10^5	1.638×10^6	1.638×10^6	Rigid
(K_4): **(K_3)** + adjusted z-spring values (support)	1.366×10^6	1.366×10^6	5.500×10^5	1.638×10^6	1.638×10^6	Rigid
(K_5): **(K_4)** + adjusted z-spring values (support)	1.366×10^6	1.366×10^6	5.250×10^5	1.638×10^6	1.638×10^6	Rigid

Table 3.2: Predicted natural frequencies for varied end fixity/joint properties [3]

Floor JJI 2 A	$f_{(1,1)^*}$ **[Hz]**	$f_{(1,2)}$ **[Hz]**	$f_{(1,3)}$ **[Hz]**	$f_{(1,4)}$ **[Hz]**	$f_{(1,5)}$ **[Hz]**
(A): complete floor without support springs	34.79	38.34	50.18	68.35	78.98
(B): **(A)** + support y-springs + x/z-fixed	34.79	38.07	49.97	71.37	78.86
(C): **(A)** + support x-springs + y/z-fixed	27.84	31.58	41.11	60.59	74.43
(D): **(A)** + support x/y-springs + z-fixed	27.84	31.51	41.04	61.79	74.39
(E_1): **(A)** + support z-springs + x/y-fixed	24.10	27.04	44.01	67.84	78.88
(E_2): **(E_1)** + higher spring value	30.87	34.48	45.98	67.36	78.63
(F_1): **(A)** + support springs in x/y/z	21.49	24.40	36.62	59.49	73.70
(F_2): **(F_1)** + higher z-spring value	25.70	29.24	38.59	60.37	73.98
(G): **(A)** + deck screws y-springs + "rigid" x	32.30	34.93	43.48	65.06	77.30
(H): **(A)** + deck screws x-springs + "rigid" y	32.23	35.45	42.83	60.35	72.90
(I): **(A)** + deck screws x/y-springs	31.75	34.42	42.27	60.09	70.88
(K_1): **(A)** + support x/y/z-springs + deck x/y-springs	20.69	23.43	33.01	52.91	66.03
(K_2): **(K_1)** + higher z-spring value (support)	24.29	27.53	34.48	53.71	65.86
(K_3): **(K_2)** + adjusted z-spring values (support)	22.18	25.17	33.46	53.27	65.56
(K_4): **(K_3)** + adjusted z-spring values (support)	22.78	25.87	33.70	53.40	65.70
(K_5): **(K_4)** + adjusted z-spring values (support)	22.65	25.71	33.64	53.37	65.68

** $f_{(m,n)}$: m = mode number in longitudinal direction, n = mode number in transverse direction*

4 CORRELATION OF NATURAL FREQUENCIES AND MODAL SHAPES DETERMINED FROM FEA AND MEASUREMENTS

The correlation of predicted and measured natural frequencies is presented in Table 4.1, showing the absolute values, the absolute difference and errors. Generally, there was little difference between predicted and measured fundamental frequencies with an error of 0.28 Hz or 1.09% on average and also rather low variation in the predicted and measured values of the third principal modes with 1.70 Hz or 4.44%. The frequencies of the second principal modes were under-predicted by 2.52 to 3.20 Hz, with an error of 2.87 Hz or 9.72% on average. The frequencies corresponding to the two highest examined modes were over-predicted by 5.61 Hz and 5.38 Hz on average, or 11.66% and 9.02%, respectively.

Table 4.1: Comparison of predicted and measured natural frequencies [3]

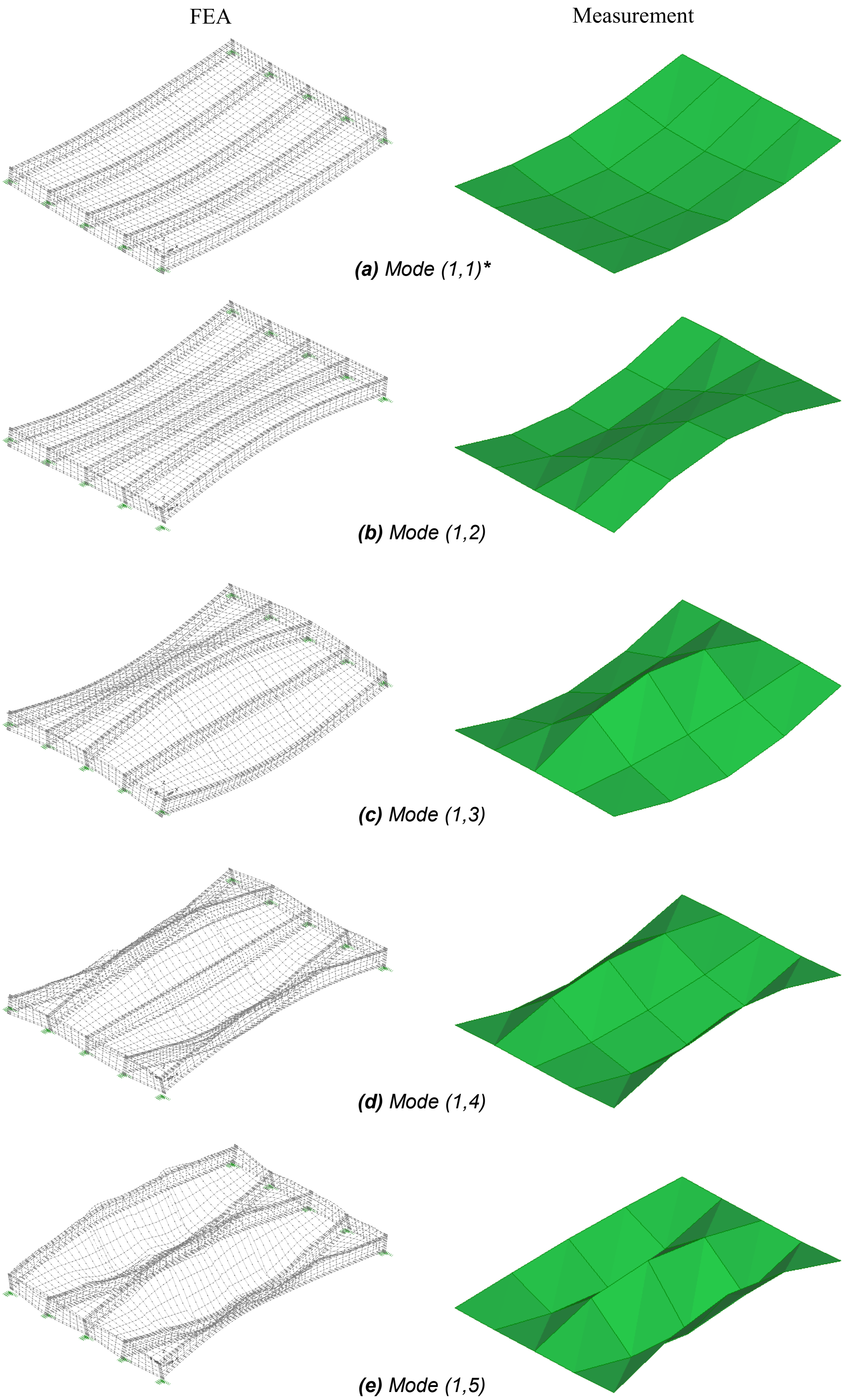
Floor JJI		$f_{(1,1)}$ [Hz]	$f_{(1,2)}$ [Hz]	$f_{(1,3)}$ [Hz]	$f_{(1,4)}$ [Hz]	$f_{(1,5)}$ [Hz]
2 A	FEA Prediction	22.65	25.71	33.64	53.37	65.68
	Measurement	22.66	28.81	34.88	47.65	59.63
	Difference (abs)	**0.01**	**3.10**	**1.24**	**5.72**	**6.05**
	Error (%)	**0.04**	**10.76**	**3.56**	**12.00**	**10.15**
1 A	FEA Prediction	24.44	27.59	36.24	55.45	67.35
	Measurement	24.46	30.11	35.58	47.33	60.72
	Difference (abs)	**0.02**	**2.52**	**0.66**	**8.12**	**6.63**
	Error (%)	**0.08**	**8.37**	**1.85**	**17.16**	**10.92**
2 G	FEA Prediction	24.83	25.76	34.73	53.49	62.79
	Measurement	24.89	28.57	40.17	45.51	57.81
	Difference (abs)	**0.06**	**2.81**	**5.44**	**7.98**	**4.98**
	Error (%)	**0.24**	**9.84**	**13.54**	**17.53**	**8.61**
1 G	FEA Prediction	26.52	27.64	37.65	55.62	64.62
	Measurement	27.29	30.84	39.17	51.38	60.63
	Difference (abs)	**0.77**	**3.20**	**1.52**	**4.24**	**3.99**
	Error (%)	**2.82**	**10.38**	**3.88**	**8.25**	**6.58**
2 J	FEA Prediction	24.15	25.70	34.01	53.42	64.00
	Measurement	23.87	28.58	35.34	49.79	58.85
	Difference (abs)	**0.28**	**2.88**	**1.33**	**3.63**	**5.15**
	Error (%)	**1.17**	**10.08**	**3.76**	**7.29**	**8.75**
1 J	FEA Prediction	25.98	27.58	36.87	55.52	65.88
	Measurement	25.43	30.26	36.86	51.55	60.39
	Difference (abs)	**0.55**	**2.68**	**0.01**	**3.97**	**5.49**
	Error (%)	**2.16**	**8.86**	**0.03**	**7.70**	**9.09**
Mean	**Difference (abs)**	**0.28**	**2.87**	**1.70**	**5.61**	**5.38**
	Error (%)	**1.09**	**9.72**	**4.44**	**11.66**	**9.02**

The FEA detected also modes other than the principal ones presented, which usually included some horizontal movement or rotation with some bending, being sometimes restricted to local areas or individual elements. Those modes may occur, inter alia, due to simplifications in the model, which impact the options of movement. Also modes of second order were identified. For three floors, one of the third or fourth mode occurred twice. There was indeed only little difference in frequency between the original and repeated modes. The first occurring one was selected for representation of the results if the modal amplitudes of the repeated modes were not greater.

The predicted principal mode shapes produced by the FEA correlated considerably well with those determined from experimental investigations. The mode shapes of Floor JJI 2 A are shown in Figure 5.1. The correlation worked as well for the remaining floors, which are therefore not illustrated. The modal shapes from the FEA, the deformed meshes, show more details than the ones obtained from the measurements since they illustrate the full structure compared to only the floor surface the measurements were taken on.

5 EXAMINATION OF VARIATIONS IN MODE SHAPES DUE TO STRUCTURAL MODIFICATIONS

As mentioned earlier, double joists were used for some of the structures, for which the concept is detailed in [3],[12]. The use of double joists can have strong influence on natural frequencies, local deflections and also on the vibrational shapes, which were confirmed by the FEA as illustrated in Figure 5.2. It shows the modal shapes of the odd mode numbers in transverse direction of Floors JJI 2 A with only single joists and Floor JJI 2 G with double joists with wider flanges in floor centre. The anti-node location of the fundamental mode in floor centre of the original Floor 2 A became the location with lowest movement for Floor 2 G due to the use of double joists. Their use can furthermore clearly lower the relative movement of the third and fifth principal mode in floor centre.



(a) *Mode (1,1)**

(b) *Mode (1,2)*

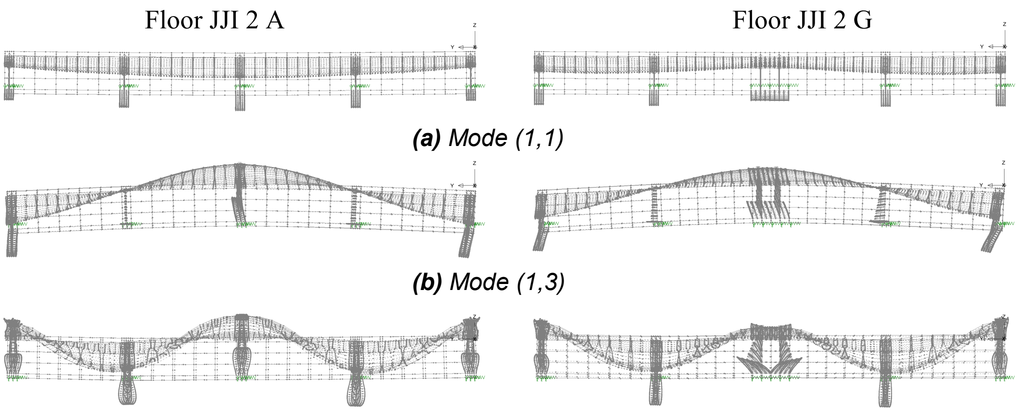
(c) *Mode (1,3)*

(d) *Mode (1,4)*

(e) *Mode (1,5)*

** Mode (m,n): m = mode number in longitudinal direction, n = mode number in transverse direction*

Figure 5.1: *Mode shapes of Floor JJI 2 A obtained from FEA and measurements* [3]



(a) *Mode (1,1)*

(b) *Mode (1,3)*

(c) *Mode (1,5)*

Figure 5.2: *Mode shapes in transverse direction for the first three odd mode numbers of Floors JJI 2 A and G obtained from FEA* [3]

6 DEFLECTION IN FLOOR CENTRE UNDER POINT LOAD

After performing eigenproblem analyses, the static deflection *w* at floor centre under unit point load was examined. The net deflection was obtained under consideration of the movement at the joist ends. The deformed joist mesh of Floor 2 A is shown in Figure 6.1. The remaining parts of the mesh were only set to be invisible so as to better identify the decisive nodes.

The predicted results were then compared to the results obtained from the experimental investigations (Table 7.1) to get further verifications of the models. For the two floors donated "A" the deflections were under-predicted by about 0.2 mm each, or 14.5%. Otherwise, the predictions (nearly) matched the measured results. The average error for all six flooring systems was 0.09 mm or 7.63%.

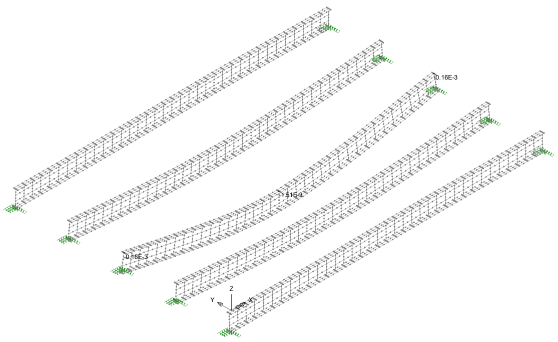


Figure 6.1: *I-joists of Floor JJI 2 A under unit point load at the floor centre* [3]

7 DISCUSSION, CONCLUSIONS AND FURTHER WORK

This paper presented a method to establish a finite element model for the purpose of eigenvalue analyses of floorings systems constructed with timber I-joists, using only one element type. It addressed issues of end fixity and degree of composite action by considering the use of spring stiffness at the supports and at the interface of deck and joists. The slip moduli were calculated under consideration of the serviceability aspect. For the value of withdrawal stiffness, it was referred to the experimental investigations by other researchers. Also the material stiffness attributes were obtained from literature. Finally, the stiffness in vertical direction at the supports was the only property requiring adjustments. This stiffness basically accounted for the withdrawal stiffness of the screws connecting joist ends and supports. However, it may also consider the stiffness of the supporting structure.

Table 7.1: *Comparison of predicted and measured point load deflections* [3]

Floor JJI		***w* [mm]**
2 A	Prediction	1.36
	Measurement	1.59
	Difference (abs)	**0.23**
	Error (%)	**14.47**
1 A	Prediction	1.12
	Measurement	1.31
	Difference (abs)	**0.19**
	Error (%)	**14.50**
2 G	Prediction	0.57
	Measurement	0.57
	Difference (abs)	**0.00**
	Error (%)	**0.00**
1 G	Prediction	0.46
	Measurement	0.42
	Difference (abs)	**0.04**
	Error (%)	**9.52**
2 J	Prediction	0.90
	Measurement	0.85
	Difference (abs)	**0.05**
	Error (%)	**5.88**
1 J	Prediction	0.73
	Measurement	0.72
	Difference (abs)	**0.01**
	Error (%)	**1.39**
Mean	**Difference (abs)**	**0.09**
	Error (%)	**7.63**

The so determined stiffness for the vertical direction to accurately predict the fundamental natural frequency of the base model was then kept constant for the analyses of the other selected flooring structures. The rather high correlation of predicted and measured fundamental frequencies in general and similar deviations in the prediction of the higher natural frequencies of the other floors compared to the base floor confirmed the adequacy of the selected approach. This was further supported by rather very accurate prediction of static point load deflections.

The gradual enhancement of the models showed that consideration of spring stiffness at the supports is required for an accurate eigenproblem analysis. Even if floors are installed between walls in buildings, the condition "fully fixed" for end fixity may not be achieved in such timber structures. Introducing joint elements to represent the effect of fasteners for connecting decking sheets and joists further contributes to an improved prediction of natural frequencies, especially with respect to higher modes. It appears that raised model complexity is required for rebuilding the real conditions in order to further increase the accuracy of predicting higher mode frequencies.

The high correlation of the predicted modal shapes and those obtained from experimental measurements demonstrated that the principal modes can be definitely identified in general, therefore distinguished from other eigenvalues and used for comparison of mode shape variations due to structural modifications. It can thus be concluded that the composition of the model was successful.

The presented results from the six flooring structures under investigation suggest that the developed FE model can be applied to get a general overview of the most critical natural frequencies with particular high accuracy for the first and third principal modes, excellent correlation of predicted and measured modal shapes and sound predictions of point load deflections for timber flooring systems constructed with I-joists. Furthermore, it provides options for parametric studies with respect to the fundamental frequencies and the first five principal mode shapes, due to their accurate predictions, to investigate the impact of structural modifications. Most of this cannot reliably and easily be obtained from hand calculations.

Refinement of the model is needed to improve the prediction of higher natural frequencies, particularly those higher than the third one. An increased accuracy in predicting the second mode frequency can enhance the estimation of natural frequency separation. It could be concentrated on finding enhanced methods for consideration of the composite action of deck and joists and on conducting further physical examinations to more accurately reflect stiffness parameters in the model.

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