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USING THE RIGHT MODULUS OF ELASTICITY TO GET THE BEST GRADES OUT OF SOFTWOOD TIMBER SPECIES IN GREAT BRITAIN

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ABSTRACT: The correct characterisation of modulus of elasticity in bending is fundamental for timber grading, especially when that property limits the allocation to a strength class. The aim of this paper is to empirically determine the relationship between EN 408 local and global modulus of elasticity using data from six British-grown conifer species and to investigate the recent change in EN 384 for the calculation of pure bending stiffness from global modulus of elasticity. For our data, the relationship between local and global modulus of elasticity was almost identical across species and for practical purposes one linear relationship could be used for all species. However our empirical relationship was considerably different to the default EN 384 calculation. Use of our empirical determined conversion substantially improves grading yields on this dataset compared to the default EN 384 calculation. We additionally investigated the effect of sample size on determining the conversion and found that using substantially less than 450 pieces required in EN 384 would be adequate in our case. A preliminary investigation of whether a special conversion according to the standard is required could be performed with fewer test pieces, but further testing is required to ensure safe grading.

KEY WORDS: Modulus of elasticity, Grading, Structural timber, Strength classes, Yields, European standards

1 INTRODUCTION

There are three properties that determine the strength grading of a population of timber: bending stiffness (or modulus of elasticity, E), bending strength (or modulus of rupture, f_m) and density (ρ). In order to attain a strength class, such as those specified in EN 338 [1] (Table 1), values of these three properties for a population of timber, subject to some adjustments [2], must at least meet the required values for that strength class.

Table 1: *Characteristic values for strength classes C14 to C24*

		EN 338 strength class					
Property		C14	C16	C18	C20	C22	C24
$f_{m,k}$	N/mm^2	14	16	18	20	22	24
E_{mean}	kN/mm^2	7	8	9	9.5	10	11
ρ_k	kg/mm^3	290	310	320	330	340	350

Note: the characteristic value for E must equal or exceed 95% of the value given for the strength class.

EN 408 [3] defines two methods for measurement of E via a four-point bending test. The global measurement of E is determined by the mid-span deflection in relation to the supports (E_{global}), while the local measurement of E is determined on middle third of the beam (E_{local}) by the relative deflection of the middle of the beam (Figure 1). E_{local} and E_{global} are related but there are key differences in what they are measuring.

For E_{local} the deformation is measured at the neutral axis as the average displacement of two transducers placed on each side face. Within this region of the four-point bending test, the bending moment is constant and so it is regarded to be under "pure bending", where, theoretically, the shear effect does not exist. On the other hand, the deformation for E_{global} is measured at the centre of the span, typically from the centre of the tension edge. In this measurement, part of the deformation is due to shearing action between the support and load point.

Regardless of the presence or absence of shear effects, the distribution and nature of defects, and experimental errors, play a key role in test results and therefore the comparison of E_{local} to E_{global}. The smaller deflection and method of mounting make the measurement of E_{local} more susceptible to errors and localised effects in the wood (e.g. splitting). The sensitivity to the exact positioning of the worst defect makes identification of it important, but it is not always easy in practice and what

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is worst defect for strength (the priority) is not necessarily worst defect for stiffness. This also means results can be biased by the length of the specimens and the flexibility of positioning.
The E_{global} measurement is an easier to implement test procedure, less sensitive to experimental error and exact positioning of the worst defect, but it does usually include a component of compression deformation at the supports and loading points which can be quite influential on the results. Nonetheless, the benefits of using E_{global} are considered to outweigh the disadvantages.
E_{global} was added in the 2003 version of EN 408. Previously E_{local} had been the standard method. In addition to the benefits outlined above, this also brought some equivalence with other testing standards (e.g. North America and Australia) [4]. However there remain differences in the associated European standards for timber grading: notably, EN 384 [5] requires that the worst defect be placed centrally in the test. The 0.95 factor applied to the stiffness requirement for the strength class is there to compensate for the difference between testing at the critical section compared to random positioning [2]. As E_{global} became the preferred measurement, the option of using measured E_{local} was proposed to be removed from EN 384:2016, but it was retained primarily to allow use of test data that predates 2003.
The move from E_{local} to E_{global} measurement required an adjustment to bring the measurements in line with previous practice. This was incorporated in the form of an equation that adjusts E_{global} to an equivalent "shear free" stiffness (E_0), representative of what had previously been obtained with the local measurement. This conversion is now provided as equation 7 in EN 384:2016. It is identical to that added to the 2004 version (contemporary to the addition of E_{global} measurement in EN 408:2003). The adjustment was originally formulated on the mean stiffness, but in practice was applied to individual results when calculating machine grading settings. Consequentially EN 384:2016 presents the equation as an adjustment for each specimen, this is given here as equation (1).

$$E_0 = E_{global} * 1.3 - 2690 \text{ (N/mm}^2\text{)} \quad (1)$$

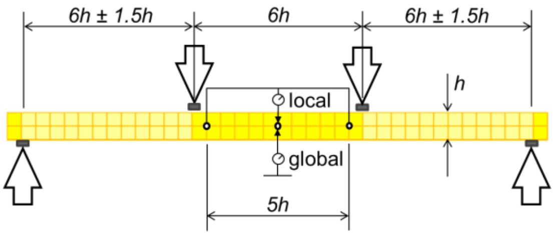

Figure 1. *EN408 test arrangement (for beam of depth h)*

Equation (1) is empirical, based on data from previous tests in Europe. It covers the various factors that govern the difference between E_{local} and E_{global}, including shear deformation. This is why the shear adjustment, added to the calculation of global modulus of elasticity in EN 408:2010, was amended in 2012 to prevent a double adjustment where the results are to be later used for EN 384 procedures.
The ratio of E_{local}/E_{global} and the influencing factors have been researched by a number of authors. Broadly speaking it is thought that the ratio E_{local}/E_{global} is mainly affected by shear deformation for large dimensions specimens, and by the variation of stiffness within the specimen (especially the nature and exact position of the critical defect) for smaller dimension timber[4, 6].
The intrinsic assumption on which equation (1) is based is that the relationship between E_{local} and E_{global} is homoscedastic. This assumption breaks down for two reasons. Firstly, as E decreases because stiffness cannot be negative. Recently, a grading dataset for British spruce (WPCS) [7] had to have two 22x47 specimens removed from the analysis because EN 384 calculated negative E_0. Secondly, this equation is not mechanically consistent because much of the deformation measured by E_{global} is a consequence of the bending for E_{local}. With standard EN 408 spans, and assuming measurements are not affected by experimental error, E_{local} cannot be less than 0.326 E_{global}. (the limiting situation of perfectly stiff shear spans) [8]. This limit of plausibility is breached by the EN 384 equation when E_{global} is less than 2.76 kN/mm^2.
For these reasons, the 2016 version of EN 384 contains a new provision in relation to the adjustment to shear free stiffness: "*If another relevant equation is available from test data, it shall be used instead…This alternative equation shall be established on at least 450 pieces, covering the full range of sizes, sources and quality corresponding to the intended use."* The provided equation is also now limited to softwoods in recognition that hardwoods may be substantially different.
A number of researchers have proposed conversion equations over the years. Some are listed in Table 2. Different studies investigated the nature of the correlation between E_{global} and E_{local} [9, 10], but since the change in EN 408 (2003) researchers have also considered the consequences for grading of using the EN 384 adjustment to E_0.

Table 2*: Studies investigating the relationship E_{local}:E_{global}. E_{local} and E_{local} in kN/mm^2*

Study	E_{local}	E_{global}	Slope	Interc.	R^2
[9]	10.8	9.8	1.18	- 856	0.89
[10]	11.4	10.8	1.13	- 800	0.82
[11]	8.9	8.1	1.1	-225	0.63
[12]	12.4	11.4	1.21	-1421	*
[13]	12.3 to 15.4	11.5 to 14.0	1.28	2300	0.88

* Not reported

A study conducted in Spain [11] with radiata pine (cross section 200x150 mm; E_{global} = 8.11 and mean E_{local} = 8.92 kN/mm^2) questioned the applicability of equation (1) but accepted that differences of timber allocation would be small and in favour of safety compared to the investigated regression ($E_{local} = E_{global} \times 1.1 - 225$, R^2 = 0.63).

In [12] working with high stiffness German timber (E_{global} = 11,4 and mean E_{local} = 12,4 kN/mm^2) from four conifer species of different dimensions, the author recommended not to change the equation in EN 384. It was reported that an alternative equation would fit the data better, but the differences were small so not consequential in their case.

In south Italy [13] a study investigated a linear equation for material from three softwoods and one hardwood. The parameters found for the overall equation was very close to the given in EN 384 ($E_{local} = E_{\mathrm{global}} \times 1.28 - 2300$, R^2 = 0.88) but again the mean stiffness was high (ranged by species from 11.5 kN/mm^2 to 15.4 kN/mm^2). The same study warned that "*the conversion equation used* (in EN 384) *can have an important effect mainly for stiffness limited material"* and so this reduces the yields.

This is a particular concern in the British Isles where British spruce (a combination of about 90% Sitka spruce *Picea sitchensis* and 10% Norway spruce *Picea abies*), produces the vast majority of home-grown structural timber, but is stiffness limited [14] to lower strength class than the European grown timber previously used for most of the research. The British forest industry had previously studied the relationship for specimens of Sitka spruce, and produced conversion equations [15] that are very different to equation (1)

The consequences for grading yields are commercially significant, and hard to appreciate from looking at correlations alone. The characteristic values for E is the mean while for strength and density it is the lower 5th percentile. Increasing the mean of a population requires the rejection of more specimens than increasing the 5th percentile and so grading yields of stiffness limited timber can be affected quite strongly by relatively small changes in mean E. It is therefore necessary to additionally investigate impacts on grading that are a consequence of any conversion.

The first aim of this study is to investigate the relationship between E_{global} and E_{global} on six conifer species in the UK, exploring the new possibility offered in EN 384 of establishing an empirical equation obtained locally at the test facility to calculate modulus of elasticity in pure bending (E_0) from E_{global}. A simplified grading analysis is then used to examine the effects on yield, assuming a perfect grading machine. Secondly, the study explores the reliability of using a smaller number of specimens than the 450 specified by EN 384:2016.

2 MATERIAL AND METHODS

2.1 MATERIAL

To negate effects of varying dimensions of test pieces, the data used here are only for nominally 50x100 mm cross section specimens. The material investigated in this paper comes from six species groups, named with common name, botanical name and the standard four letter code [7] respectively. The main commercial species are larch (*Larix decidua, kaempferi and x eurolepis* - WLAD) and Sitka spruce (*Picea sitchensis* - PCST). For larch, the majority of the material was obtained from normal sawmill production. For the Sitka spruce the majority of the material was obtained during scientific studies. In both cases the sampling covered the mainland Great Britain. These data were used previously as part of datasets for establishing grading machine settings.

The other four species are: western red cedar (*Thuja plicata* - THPL), noble fir (*Abies procera* - ABPR), western hemlock (*Tsuga heterophylla* - THST) and Norway spruce (*Picea abies* - PCAB). This material was obtained from a scientific study investigating even-aged single species plantations. This timber was obtained from 109 trees (28 for western hemlock and 27 for the rest of species) spread across three growing environments, representative of the southern, middle and northern latitudes in Great Britain. Specimens were mostly produced following radial transects centred on the pith (Figure 2). Additional specimens were also produced from the remaining parts of some of the Norway spruce logs.

The dataset used here consists of 252 pieces of larch, 194 of Sitka spruce, 138 of western red cedar, 127 of noble fir, 150 of western hemlock and 233 of Norway spruce. The total number of specimens is 1094. Hereafter when referring to data: full population is used to describe the 1094 pieces and species datasets are referred to by their four letter prefix.

Figure 2. *Processing of a western red cedar log with a portable sawmill.*

2.2 MECHANICAL TESTING

All timbers were kiln dried and conditioned in a controlled environment prior testing in four point bending according to EN 408:2012. Global modulus of elasticity (E_{global}) and local modulus of elasticity (E_{local}) were measured simultaneously (Figure 3). Following testing, a 50 mm length sample was cut near the failure point of each specimen to determine moisture content according to EN 13183-1 [16]. The measured global and local modulus of elasticities were then adjusted to 12% moisture content in accordance with EN 384 [5]. The fixed equation (1) given in the current EN 384 was used to convert the E_{global} to E_{local}, hereafter referred to as E_{EN384}.

Figure 3. *Transducers for measurement of E_{global} (bottom face) and E_{local} (side faces).*

2.3 ANALYSIS

The statistical analysis was made using R software version 3.2.5 [17].

2.3.1 Relationship between E_{local} and E_{global}

The means of E_{global} and E_{local} where compared with a paired *t*-test. E_{EN384} was compared to measured E_{local} in the same way. We then investigated whether the linear relationship between E_{local} and E_{global} changed according to dataset (set) by conducting an analysis of variance (ANOVA) on a General Linear Model which had the form:

$$E_{local} = \alpha_0 + \alpha_1 E_{global} + \alpha_2 set + \alpha_3 E_{global} set + \varepsilon \; (2)$$

Where α_0 is the regression coefficient of intercept, α_1 is the regression coefficient of slope, α_2 represents the additive effect of the dataset studied and α_3 is the interaction term between E_{global} and dataset and ε is residual error not explained by the model. Specifying the model in this way allows us to investigate the effect of the dataset on the regression parameters of slope and intercept by examining the significance of α3 and α2 respectively.

2.3.2 Influence of models on grading yields

A linear regression was used to derive an empirical conversion of E_{global} to a general modulus of elasticity in pure bending (E_{PBG}), with E_{local} acting as the dependent variable and E_{global} as the predictor.

The characteristics values by dataset and for the full population were calculated using all of the measured and derived values of *E*. The basic grade (i.e. the strength class that was obtained with 100% grading yield) was examined, as well as the yields obtained for strength classes C16 to C24.

2.3.3 Subsample size for relationship determination

In order to investigate the minimum subsample size required to establish an equation from which to predict E_{local} from E_{global} we used a Monte Carlo method. We repeatedly drew random subsample sizes of 25, 50, 100, 200 and 450 pieces from the full population of 1049 pieces. Each subsample size was drawn 5000 times with samples being replaced between each draw. For each subsample draw we:

i. Calculated a conversion equation for E_{PBG} by linear regression for the draw, then used this to calculate E_{PBG} for each piece in the full population of 1049 pieces.
ii. Graded the full 1049 piece population by E_{PBG} to achieve the target $E_{mean.}$
iii. Used the measured E_{local} for the pieces passing grading to calculate a "true" pure bending E_{mean}
iv. Calculated the percentage of the “true” pure bending $E_{0,mean}$ achieved by the process of grading with the converted E_{global}.

Additionally, a conversion equation based a dataset that presented an obviously different relationship between E_{local} and E_{global} was applied to the full population. This is referred to as E_{LOW} and was derived using the THPL dataset. We investigated the yields obtained and compared those with results with E_{EN384} and E_{PBG}. This analysis assumes the measured E_{local} is the true measurement and so yields are relative to the yields obtained when grading based on E_{local}, referred to as perfect grading.

3 RESULTS

3.1 RELATION BETWEEN E_{LOCAL} AND E_{GLOBAL}

The comparison of E_{global} and E_{local} with a paired *t*-test showed that they were significantly different (p =<0.001). The same routine determined significant differences between E_{local} and E_{EN384} ($p < 0.001$).

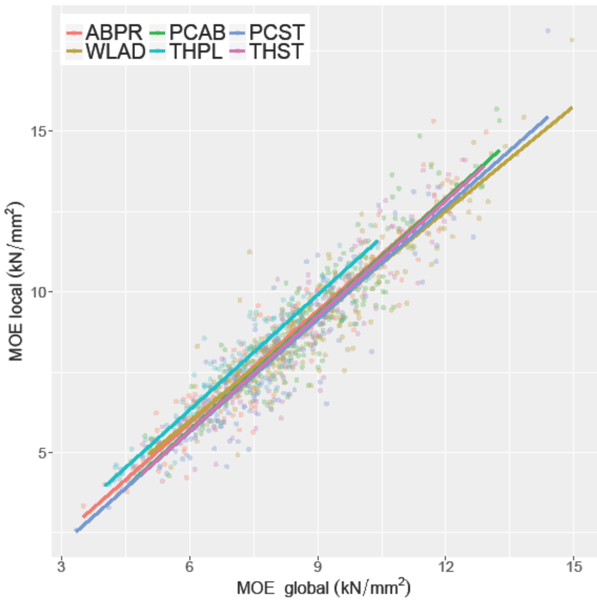

Figure 4. *Relationship E_{global} – E_{local} by dataset. The lines indicate best fit linear regressions independently determined for each dataset.*

Figure 4 shows the relationship between the measured E_{global} and E_{local} for the full population of specimens tested in the laboratory. The effect of each dataset on the relationship between E_{global} and E_{local} was investigated using ANOVA of Equation (2). The rate of change between E_{global} and E_{local} is the same for all the six datasets ($F_{5\ 1082}$ = 2.2, p = 0.057), but there is an additive effect of dataset. In terms of the linear relationship this means that the dataset affects the constant or intercept term ($F_{5\ 1087}$ =14.5, $p < 0.001$) but not the gradient or slope. A multiple comparison of the parameter estimates, using an α = 0.05 level of significance, show that THPL has a slightly higher intercept term than all the other datasets by about 0.5 kN/mm². A small difference existed between ABPR and PCST, where PCST had a lower intercept by about 0.25 kN/mm², but no other differences were significant. Including dataset as an additional explanatory variable in the linear regression had a negligible effect on the overall fit of the regression (R^2 = 0.88, RSE = 0.80 vs R^2 = 0.88, RSE = 0.77). The results in Table 3 show the different regression coefficients and which ones are considered different from each other

Table 3: *Regression coefficients for each dataset. SE = Standard Error*

Coefficients	Dataset	Estimate	SE	Group
Slope	All	1.157	0.013	--
Intercepts	ABPR	-1.024	0.123	A
	WLAD	-1.169	0.086	AB
	PCAB	-1.119	0.087	AB
	THPL	-0.589	0.096	C
	PCST	-1.290	0.088	B
	THST	-1.185	0.094	AB
Equation:	E_{local} = Slope.E_{global} + Intercept			

3.2 INFLUENCE OF MODELS ON GRADING YIELDS

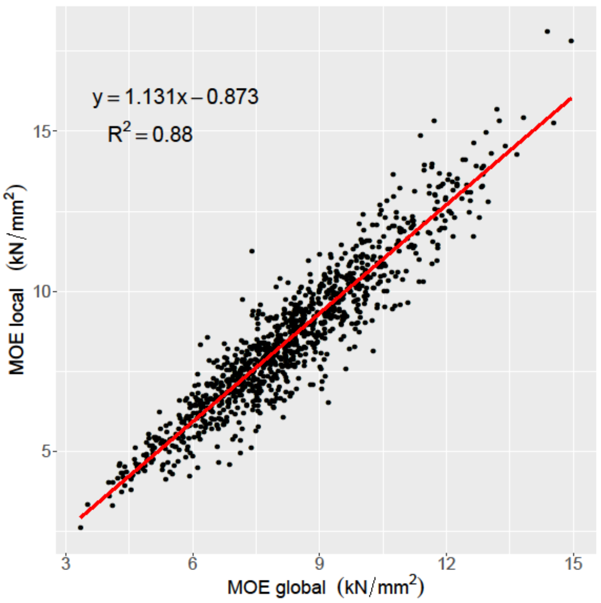

Figure 5. *Relationship between local and global modulus of elasticity.*

A regression line for all the pieces across the full population (i.e. ignoring dataset) was examined (Figure 5), the following relationship was obtained:

$$E_{PBG} = 1.131 * E_{global} - 0.873 \quad (3)$$

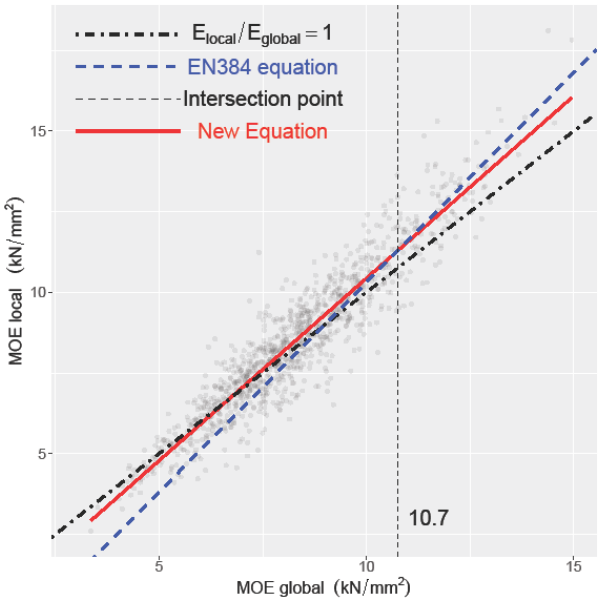

Figure 6. *Relationship between E_{local} and E_{global} as proposed by EN384 and as derived locally in this study. The vertical dashed line denotes the intersection point of the lines described by these relationships. The directly proportional relationship is also shown.*

Proposed relationships between E_{local} and E_{global} are shown in Figure 6. Neither the relationship proposed by Equation (3), proposed here, or Equation (1), proposed by EN384, are directly proportional (i.e. one to one). The relationship proposed here is closer to the one to one relationship. In our data, the ratio of E_{PBG} to E_{global} is less than one for specimens with modulus of elasticity (E_{local} or E_{global}) below 6.7 kN/mm² and greater than one above this value. Comparing the investigated equation (3) and that given in EN 384 (1) we observe in first place that the two lines are substantially different, with the EN384 line having a steeper slope that is unsuitable the material we tested. The straight lines defined by both equations cross at a value of E_{global} equal to 10.7 kN/mm². From this value E_{local} would start to be overestimated when using (1) and below that value it will be underestimated. In this study, 90% of the population is below the crossing point, and so almost all of the specimens will see their stiffness performance undervalued if the equation in EN 384 is used. Similarly, E_{local} is over predicted if (1) is used compared to (3) for values of E_{global} higher than 10.7 kN/mm². In this study only 10% of material was above 10.7 kN/mm² and mostly not far above it. In this range both lines are still close and so the differences are not large.

Table 4 presents the mean (µ) and standard deviation (Sd) values per dataset for the measured and derived modulii of elasticity. The mean ratio of E_{local}/E_{global} is given. The relative performance of the species is not comparable as material comes from trees of different ages.

Table 4: *Characteristics values for each dataset. E_{PBG} empirical equation obtained in the study. E_{local} (N/mm²); S.C. (basic strength class). The datasets are not constructed so as to be representative of differences between species.*

	E_{loc}	E_{glo}	E_{loc}/E_{glo}	E_{384}	E_{PBG}
LADC					
μ	9225	8980	1.03	8985	9280
Sd	2220	1910	0.09	2480	2155
S.C.	C20	C18		C18	C20
PCST					
μ	7830	7880	0.98	7555	8040
Sd	2285	1810	0.11	2350	2045
S.C.	C16	C16		C14[1]	C16
PCAB					
μ	9155	8875	1.03	8850	9165
Sd	1955	1510	0.09	1960	1705
S.C.	C20	C18		C18	C20
THPL					
μ	7455	6950	1.07	6350	7000
Sd	1720	1370	0.08	1780	1545
S.C.	C14	C14		N/A[2]	C14
ABPR					
	7745	7580	1.01	7160	7695
	2425	1965	0.1	2555	2220
	C16	C14[1]		C14	C16
THST					
μ	8615	8465	1.01	8315	8700
Sd	2365	1860	0.1	2415	2100
S.C.	C18	C16		C16	C18

[1] 99% of the population would achieve C16.
[2] The required for the strength class C14 is 6.65 kN/mm².

When comparing the coefficients of variation (Sd/μ) obtained from Table 4, we observe that variation in E_{local} is larger than for E_{global} which is in line with the higher sensitivity of E_{local} to the nature and exact position of the critical defect and the risks of measurement errors. With the exception of PCST, the values of E_{local} are generally higher than E_{global} between 1% and 7%, likely due to the presence of shear effect.
The given characteristic values in Table 4 define the basic strength class achieved for each dataset assuming that stiffness is the limiting property. Table 5 shows the optimum yields (ideal grading machine) obtained when the worst material is sequentially removed from the population so that higher strength classes can be achieved. As a result, the population attained to higher strength classes is progressively reduced. Results from both equations (1) and (3) are reported for comparison.

Table 5: *Yields[5] obtained for the full population tested for C16 to C24 strength classes by using pure bending modulus of elasticity as derived from (1) compared to using the empirical equation obtained in our tests (3)*

Equation	C16	C18	C20	C22	C24
EN384 (1)	100%	89%	76%	63%	40%
Alternative (3)	100%	99%	85%	70%	43%

[5] Assuming stiffness is the limiting property.

Equation (1) and (3) intersect at a value of E_{global} of 10.7 kN/mm². As seen from Figure 6 and Table 5, the largest difference of yields occurs at the range of low values for the pieces studied. The yield of C16 would not be affected as all pieces passed, but C18 would be underestimated when using equation (1). As the stiffness requirements increase, the differences between yields gets smaller. If the results are split by datasets, those datasets with relatively high stiffness are not affected as much as those datasets with lower stiffness. Table 6 compares the optimum yields obtained by using equations (1) and (3).

Table 6: *Yields obtained for a C18 strength class by using E_{EN384} as derived from the standard (1) compared to using the empirical equation E_{PBG} (3). The datasets are not constructed so as to be representative of differences between species.*

Species	EN384 (1)	Alternative (3)
THPL	26%	37%
APCB	65%	76%
PCAB	100%	100%
THST	95%	100%
WLAD	100%	100%
PCST	74%	86%

Assuming stiffness is the limiting property.

The THPL dataset has the lowest modulus of elasticity (Table 4). Consequently this timber is particularly penalised in grading. The ABPR timber would suffer similar problems although it has a higher mean stiffness and so the differences would be smaller. Figure 7 shows the impact in the grading yields of using different equations for THPL and ABPR.

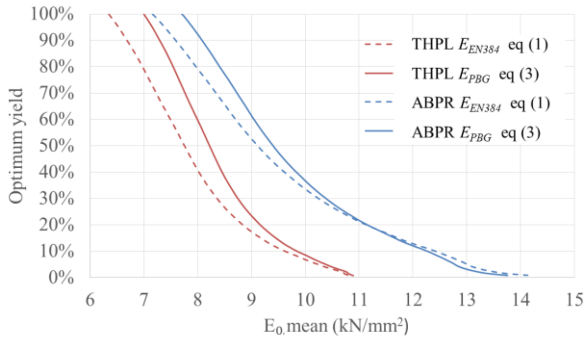

Figure 7: *Graph based on modulus of elasticity for THPL and ABPR samples showing the difference in yields by using equation (1) or (3). $E_{0,mean}$ is the mean of the population achieving the correspondent value.*

If the aim were to grade timber to C16 (minimum characteristic value of 7.55 kN/mm²), the THPL sample would only achieve 57% yield using the fixed equation in EN 384. However, 78% would be roughly attained with the investigated equation. The pattern is repeated for ABPR, but due to the higher stiffness it can be observed that from just below 12 kN/mm² the investigated equation underestimate the performance in bending stiffness which would only involve six specimens.

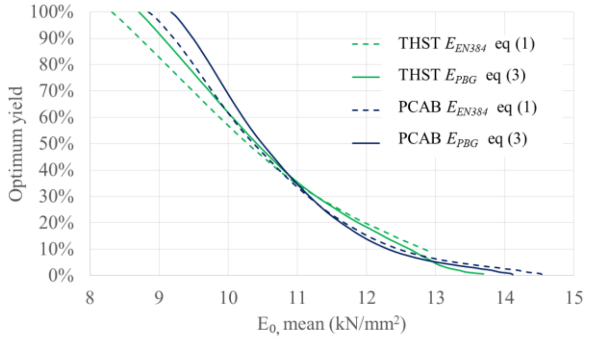

Figure 8: *Graph based on modulus of elasticity for THST and PCAB samples showing the difference in yields by using equation (1) or (3).* $E_{0,mean}$ *is the mean of the population achieving the correspondent value.*

For stiffer material like the THST and PCAB samples (Figure 8), differences are comparatively smaller but yields can still change by more than 10% for the strength class C22 (95% of 9.5 kN/mm^2).

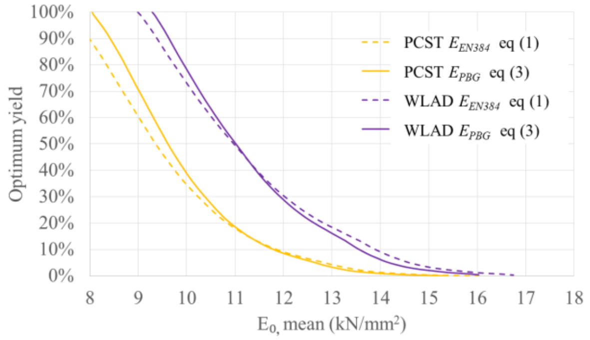

Figure 9: *Graph based on modulus of elasticity for PCST and WLAD sample showing the difference in yields by using equation (1) or (3).* $E_{0,mean}$ *is the mean of the population achieving the correspondent value.*

Figure 9 shows the evolution for WLAD and PCST. As for the other datasets, the graph shows that the use of the equation (1) initially underestimates the yields compared to (3) obtained on these dataset.

3.3 SAMPLE SIZE FOR RELATIONSHIP DETERMINATION

We used a Monte Carlo method to determine if it was possible to establish a conversion from E_{global} to E_{PGB} using less than 450 pieces. This relationship must be reliable in terms of safety. Figure 10 shows how far the mean E_{PGB} of a sample size is from the "true" mean MOE (that calculated using the E_{local} measurements) for 99% of the trials. In other words, only 1% of the time, the calculated mean shear free MOE was further from the "true" shear free mean MOE targeted.

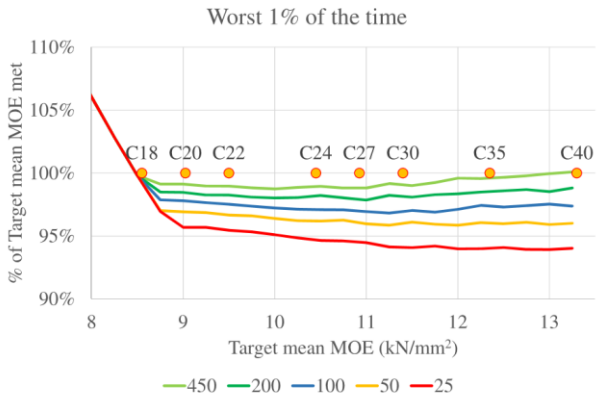

Figure 10: *Simulation of the uncertainty for the mean MOE of different sample size.*

For the C22 strength class an E_{mean} of 9.5 kN/mm^2 is required. If the sample size is 450 pieces, the E_{mean} would be expected to be within 1% of the defined "true" mean E_{local} value measured in the laboratory 99% of the time. If the sample size is 25 pieces, the $E_{0,mean}$ would be expected to be more than 5% less than the true value 1% of the time. The implications of these equations in terms of yields are also investigated and reported in Table 7. The yields for subsamples of size 450 and 100 are those that can be expected to be not exceeded 5% and 95% of the time.

Table 7*: Yield relative to the yield obtained from perfect grading with measured* E_{local}*. Here* E_{PBGxxx} *is derived from random subsamples of 100 (*E_{PBG100}*) and 450 (*E_{PBG450}*) pieces drawn 5000 times.* E_{LOW} *is derived from the coefficients in Table 3 for THPL. For a full description see the text.*

Predictor variable	C14 %	C16 %	C18 %	C20 %	C22 %	C24 %
E_{local}	100	100	100	100	100	100
E_{global}	100	100	94	88	82	70
E_{EN384}	100	100	90	88	88	88
E_{PBG}	100	100	100	99	98	93
$E_{PBG450,5\%}$	100	100	98	96	94	88
$E_{PBG450,95\%}$	100	100	101	101	101	98
$E_{PBG100,5\%}$	100	100	97	93	90	82
$E_{PBG100,95\%}$	100	100	101	104	104	104
E_{LOW}	100	100	101	117	124	131

$E_{PBG450,5\%}$ = yield exceeded 95% of the time, n = 450
$E_{PBG450,95\%}$ = yield exceeded 5% of the time, n = 450
$E_{PBG100,5\%}$ = yield exceeded 95% of the time, n = 100
$E_{PBG100,95\%}$ = yield exceeded 5% of the time, n = 100

Assuming that measured E_{local} is correct and that consequentially the correct yield is 100%, the effects of different E values can be investigated accordingly. In Table 7 any value less than 100 is proportionally and unnecessarily over penalising a timber sample. Any value over 100 is proportionally under penalising a timber sample. Values over 100 are unsafe grading, values under 100 are uneconomical (and unjust) grading. Using E_{global} directly, i.e. without any correction for shear effect/conversion to E_{local}, results in more reliable yields than the E_{EN384} for lower grades. E_{EN384} is more

reliable in terms of yield when the material gets stiffer within the investigated strength classes. E_{PBG}, derived by equation (3), performs much more satisfactorily throughout the range of strength classes investigated.
When equations are established on smaller samples size, some effectiveness is lost compared to E_{PBG} but results are still better than predictions from E_{EN384}. Even when using an equation based on few specimens (100 pieces, $E_{PBG100,5\%}$), grading presents for this study a more realistic yield, although there can be concerns with safety (see above). Finally, if an equation based on the dataset in this study with lowest mean stiffness (E_{LOW}) is applied to a population where the mean stiffness is higher, serious concerns regarding safety will be present. Table 7 shows how E_{LOW} would grade timber too high.

4 DISCUSSION

The properties of British grown timber, mean that it is generally limited to a strength class by its stiffness. In most of Europe timber tends to be limited by density or strength, but there are some species and growth areas where stiffness governs. In such places it is particularly important for primary processors to pay particular attention to any changes to standards for measuring or grading timber stiffness. One such change is the practice of converting the measured global modulus of elasticity to an estimated pure bending, or shear free, modulus of elasticity. The procedure and a default conversion for this is given in EN 384, with the 2016 version including also the possibility to use an own derived empirical conversion using at least 450 pieces. We set out to question the applicability of the default conversion to our material with respect to grading yields and to examine the potential variability of this conversion within our own timber resource. As a secondary aim, we looked at whether the requirement for 450 pieces to derive a custom equation is sufficient, we investigated the effect of sample size on the determination of this relationship.
We began with the investigation of the empirical straight line relationship between measured local (E_{local}) and global (E_{global}) modulii of elasticity and how this might vary for each dataset. The datasets are not intended to be representative of species, or even the species as grown in the UK. Rather these datasets are indicative of different timber samples from Great Britain. The empirical relationship E_{global} and E_{local} was not determined to be significantly different for five datasets, but one dataset (THPL) has a slightly higher intercept term than all the other datasets, though the slope term, which denotes the rate of change of E_{local} per unit value of E_{global} is the same. This means that in that dataset E_{local} in this dataset was systematically higher than E_{global}. There are a number of reasons for why which we may speculate upon but previous researchers have shown that the differences in shear stiffness and bending stiffness within pieces and between the different datasets is a likely factor. Species may or may not also be a factor in this case. It is outside the scope of this study to fully follow these lines of enquiry as further testing and analysis would be required. For us the important thing to consider is that even within one region (Great Britain), just because one relationship can be derived between E_{local} and E_{global} it does not mean that it is constant for all material.
The relative differences in this empirical relationship between the datasets here were very small compared to the differences between the default conversion provided in EN 384, further they were different to those provided previously in the literature ([7], [10], [11] and [13]). Both the default and the literature relationships have all been derived on higher stiffness material. Extrapolating these down to the range of stiffness of our material has the effect of underestimating the true pure bending modulus, which has a large economic penalty in incorrectly classifying our timber according to strength classes. Within the range of stiffness produced by the combined datasets in this study, that would apply to 90% of the timber in this study. Conversely the pure bending modulus of the other 10% is overestimated, which is potentially unsafe. We further investigated the effect of using different sample size and demonstrated that establishing a relationship on smaller samples size is possible using the right conversion. However, grading could become unsafe if the wrong conversion is applied using our own data, where we deliberately applied a conversion derived from one dataset to a sample population that it did not fit. The consequences were that around 30% of timber incorrectly passed the C24 grade. So long as the relationship is investigated appropriately on a subsample representative of the timber being tested this will not happen.

5 CONCLUSION

Using a locally derived empirical conversion from measured global to pure bending modulus elasticity significantly improved the estimation of the true pure bending modulus and grading yields for our material. This satisfies the criteria of being both economically viable and more critically safe. We strongly recommend that it is in the interests of anyone in any region grading timber to investigate this relationship, particularly where their grading yield depends upon it, but also where it may affect design safety. Given our results, it seems possible to reliably investigate this relationship with only 100 pieces, though more pieces will always be safer. We propose that for reasons of safety and economics, the relationship between global and local (pure bending) modulus of elasticity should be empirically investigated and documented for timber populations on a regional basis.

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