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Using grain size distribution to estimate permeability

Utiliser granulometry pour estimer la perméabilité

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ABSTRACT: This study examines the saturated water permeability using samples of loose and dense 2-fraction coarse sand mixtures. Besides the non-normalised entropy variables, some additional parameters like void ratio or relative density are used to characterize the pore sizes. Two variable pairs the (Cu and d10) and the non-normalised entropy variables (ΔS and S0) are found to have similar role, to describe the mean and the maximum controlling pore diameter sizes, resp.

RÉSUMÉ: Cette étude examine la perméabilité à l'eau saturée à l'aide d'échantillons de mélanges de sable grossier à 2 fractions lâches et denses. Outre les variables d'entropie non normalisées, certains paramètres supplémentaires tels que la densité (trouve important) sont utilisés pour caractériser la taille des pores. Deux variables paires (Cu et d10) and d'entropie non normalisées (ΔS et S_0) ont similaire role pour decrire la moyenne et maximum de contrôle diameter des pore, resp.

Keywords: Grading entropy; saturated permeability; coarse sand; model fitting; specific surface area; harmonic mean.

1 INTRODUCTION

A study has been started within the context of a large research project related to the various physical model parameters of granular matter as a function of the grading curve (Imre et al., 2021, 2022, 2023).

The space of the grading curves with N fractions is isomorphic to the space of the unit-sided simplex with dimension N-1 (Imre et al., 2014). The image of the simplex by the entropy map is the 2-dimensional grading entropy diagram. Instead of the interpolation over the simplex based on j^{N-1} interpolation points (j in each coordinate direction), the interpolation is made on the grading entropy diagram, using about $2j$ such points where the entropy map is unique (eg., on the image points of mean grading curves).

This paper is related to the saturated water permeability prediction in terms of the grading curve. A model discrimination was started to be made. The models contained traditional and new diameter type variables (the specific surface variables, the d_{10} and C_u, the grading entropy parameters), the void ratio and the relative density as a new parameter. According to the results, without the density parameters no acceptably good fit was attainable. Moreover, an acceptably good fit needed both mean pore size and maximum pore size related parameters.

2 MATERIALS AND METHODS

2.1 New permeability variables

The base entropy S_o and its normalised form, A, are defined by Lőrincz (Lőrincz,1986; Singh, 2014) as:

$$S_o = \sum_{i=1}^{N} x_i S_{oi} \text{ and } A = \frac{S_o - S_{o\min}}{S_{o\max} - S_{o\min}} \quad (1)$$

where x_i is the relative frequency of fraction i, $S_{0i} = i$ is the i^{th} fraction entropy (see Table 1), and N is the number of fractions. The entropy increment ΔS:

$$\Delta S = -\frac{1}{\ln 2}\sum_{i=1}^{N} x_i \ln x_i \text{ and } B = \frac{\Delta S}{\ln(N)} \quad (2)$$

The harmonic mean diameter (being similar to d_{10}) and the specific surface area per volume are related to the mean pore size (Imre et al., 2014):

$$d_h = \frac{1}{\sum_{i=1}^{N} \frac{x_i}{d_i}} \text{ and } S_{sv} = 6/[(1+e)d_h] \quad (3)$$

The tested density variables were the void ratio e and the relative density $R_d = (e_{max} - e)/(e_{max} - e_{min})$.

2.2 Data

In this paper, an existing granular database (Imre et al., 2021, Figure 1a) is started to be extended onto the coarse sand range of granular soils. The four fractions: 0.25 – 0.50 mm, 0.5 – 1.0 mm, 1.0 – 2.0 mm and 2.0 – 4.0 mm, are illustrated in Figure 1b.

Constant head permeability tests were made on the samples comprising 15 finite fractal 2-fraction mixtures, with two different sample preparation methods to obtain loose densities (15 samples) and dense densities (45 samples, 3 repetitions).

In Figure 1c, the maximum void ratios were assessed from earlier research of Lőrincz (1986) with different sand materials but the same composition, e_{min} was computed from e_{max}/e_{min} (Imre et al., 2011).

2.3 Regression models

The equations for k_{sat} were as follows, extending the approach of Feng et al. (Feng et al., 2019):

$$k = \exp(C_3)\Delta S^{C_1} S_0^{C_2} \quad (4)$$

$$k = \exp(C_4)\Delta S^{C_1} S_0^{C_2} e^{C_3} \quad (5)$$

$$k = \exp(C_4)\Delta S^{C_1} S_0^{C_2} p^{C_3} \quad (6)$$

where C_1,C_2, C_3 and C_4 are model parameters to be identified, p can be the relative density, specific surface area per unit volume or other variable. The grading entropy parameter pair was interchanged by d_{10} and C_u, the Ren and Santamarina (2017) equation:

$$k = \frac{e^{C_1}}{(S_{sv})^2} \quad (7)$$

where S_{sv} is the specific surface area per unit volume of the soil (mm^{-1}), parameter C_1 is identified.

Table 1. Definition and properties of fraction j.

j	1	23	24
Limits in d_0	1 to 2	2^{22} to 2^{23}	2^{23} to 2^{24}
S_{0j} [-]	1	23	24

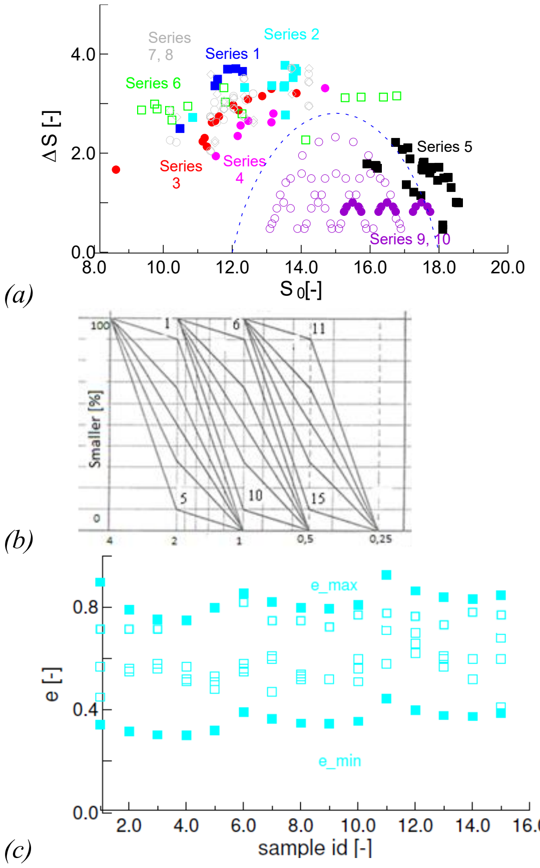

Figure 1. (a) Series 9 in the database (i.e. new samples 1 to 15) in purple closed symbol. Blue, light blue, red, pink, green, black, purple open: old series 1, 2, 3, 4, 5, 6, and planned mixtures, resp. (b) Particle size distributions of the 2-fraction fractal mixtures in Series 9. (c) void ratio for 15 mixtures in terms of id number, shown in Figure 1(b).

3 RESULTS

The k_{sat} of the 15 optimal 2-fraction soil mixtures with finite fractal gradings was between about 0.01 to 1 cm/s. The data were separated considerably for loose and dense samples in terms of specific surface area per volume (Figure 2a) or d_{10} relations.

In contrast, the measured data did not separate within series when the relative density or when the Ren and Santamarina variable (Figures 2b, c) were used. Since one sample was tested at a low relative density value, and three samples were tested at higher relative density values, the repetition of the former could reduce the difference, some further data are needed.

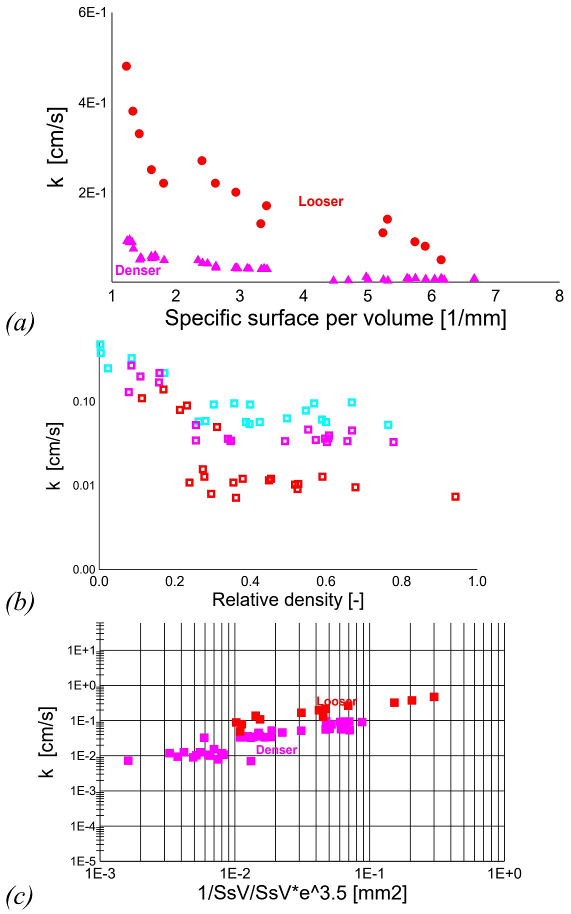

Figure 2. Permeability in terms of (a) specific surface per unit volume and (b) relative density, (c) Ren and Santamarina variable. Colours in (b): sub-series 1, 2 and 3 in turquoise, pink, and red, respectively.

The parameters of the empirical equations were determined using the Gauss Normal Equations approach (Press et al., 2007). The results of the Least-Squares fitting process was evaluated through the coefficient of determination value (R^2) – expressing what fraction of the variance is explained.

Referring to Table 2, by using only entropy parameters for the 2-fraction soils with small C_u, the R^2 is not satisfactory having identical gradings with different density. When additional density parameters were considered (i.e. void ratio e, relative density R_d) the models showed higher values of R^2. The highest values of R^2 were found with the relative density, this result need confirmation with new data.

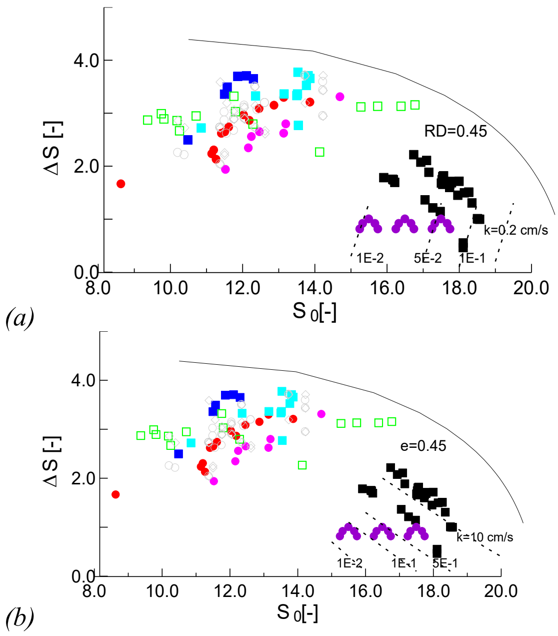

Figure 3. Approximate constant permeability level lines in the edge of diagram (a) assuming constant void ratio e, (b) constant relative density R_d. The shape of the level lines is finalized once series 5 data are completed with R_d.

Table 2. 2-fraction mixtures with R^2 values.

Variable	R^2
Entropy parameters, relative density	0.8847
d_{10}, C_u and relative density	0.8733
d_{10}, C_u with specific surface area	0.7709
Entropy parameters with void ratio, e	0.7701
Entropy parameters, S_{sv}	0.7476
Ren and Santamarina, 2017	0.7362
Entropy parameters only	< 0.5

Approximate k-level lines for series 9 were constructed in the non-normalised diagram using the identified parameters by fixing the R_d and the e values. The constant void ratio and the constant relative density R_d level lines qualitatively differed (Figure 3).

4 DISCUSSION

The grading entropy parameter pair was interchanged by pair d_{10} and $C_{u,}$ (the latter were suggested by Nagy, 2011), and the fit had a similar R^2 as in the case of the grading entropy coordinates.

These parameter pairs were compared by using a series of mean grading curves (Imre et al., 2022). The mean C_u - ΔS relation (Figure 4) is not unique, having two close branches: in $0<A<2/3$ and $1>A>2/3$ (see more in Imre et al., 2022). The base entropy S_o or A has a unique relation with d_{10} (Figure 5). These relations may explain the similar results in Table 2.

The identified exponent of e was greater than 2 in in every empirical equation and for the other parts of the database (Figure 1(a), Imre et al., 2021). The relative density was established from the ratio of e_{max}/e_{min} (Imre et al., 2011, Kabai, 1974) and the e_{max} data of Lőrincz (1986).

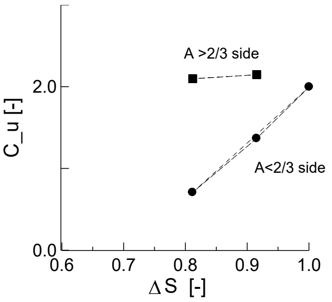

Figure 4. The relation of C_u ΔS, N= 2, all tested data.

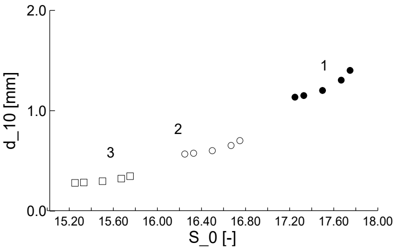

Figure 5. The relation of d_{10} and S_o, N= 2, all tested data.

The e_{max} at fixed A is decreasing with increasing fraction diameter range (since larger fractions have wider diameter range allowing a denser configuration). The void ratio is not related to the maximum pore channel size, being important in describing the k_{sat}. besides the mean pore size.

5 CONCLUSION

Theoretically, to assess for k_{sat}, the relevant problem is to assess the mean and the maximum controlling diameters of the pore channels (Imre et al., 2008).

The permeability variables can be divided into various groups and are either related to the mean controlling pore size variables (Imre et al., 2014) as d_{10}, d_h, S_{sA}, S_0, or to the maximum controlling pore size variables as C_u and ΔS, scaled by the relative density or the void ratio or the $e/(1+e)$ porosity term.

The result of the model fitting was acceptable in case of the tested 2-fraction coarse sands (identical composition, different densities) if one variable from each class was included. The best density parameter was the relative density.

The statement is supported by the facts that the S_0, ΔS and the d_{10}, C_u pairs were similar in mean grading curves relations, moreover these pairs were interchangeable in the equations. Further measurements and research are suggested, e.g., on the role of the the relative density.

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