Value Chain Management: An Illustration using Variability Mapping and Decision Frontier Analysis by Michael Pearson

1.1 A Simple Value Chain: The Stochastic Model and Problem Formulation

A simple value chain is modeled by a contractor (dual operator) making an agreement with a customer (primal operator) to complete a project in a number of stages monitored within a certain agreed time scale. The customer defines the demand (*D*) for services and products at each stage of the project while the contractor determines the way in which these services are supplied (*Q*), including a safety margin ($k\sigma_e$) to ensure customer satisfaction and successful completion on time in the risky environment.

The collaborative objective (assuming normally distributed forecasting errors) is to maximize

E{*Profit*} = *E*{*Contribution from captured demand -Costs of overage - Costs of underage*} $= \mu_1(c_{p_1} + c_{p_2}) - E[(Q-D)^+]\,(c_{o_1} + c_{u_2}) \; - E[(D-Q)^+]\,(c_{u_1} + c_{o_2} + c_p)$

$$= \mu_1(c_{p_1} + c_{p_2}) - \{(k\Phi(k) + \phi(k))(c_{o_1} + c_{u_2}) + (\phi(k) - k(1-\Phi(k))(c_{u_1} + c_{o_2} + c_p)\}\sigma_e \qquad (1)$$

subject to: $E(Q) = E(D) - k\sigma_e$ (or $\mu - k\sigma_e = 0$) (Newsvendor Constraint) (2)

where $^+$ indicates the value of the variable when it is positive and zero otherwise. Also $\mu_1 = E(D)$, $\mu_2 = E(Q)$, $\mu = \mu_2 - \mu_1$ and $\phi(k)$, $\Phi(k)$ are the normal distribution density and cumulative distribution functions, respectively, for safety factor, *k*. The customer and contractor agree to complete certain stages of the project perhaps with the assistance of a Gantt chart. Probability distributions may be associated with the completion of the stages (perhaps by using PERT or other methods). The outcome may lead to certain targets not being met, which results in a series of forecasting errors generated at each stage of the project. σ_e is the standard deviation of these jointly calculated forecasting errors. The contributions to profit of the customer (downstream operator) and contractor (upstream operator) are c_{p_1} and c_{p_2}, respectively, and $c_p = c_{p_1} + c_{p_2}$. The contribution to profit for the contractor is the (nominal) payment associated with completing that stage minus the contractual costs of completing it. The contribution to profit for the customer is calculated in terms of the measurable increase in satisfaction gained from successful completion of that stage. The overage and underage costs of the customer are c_{o_1} and c_{u_1}, respectively, while for the contractor they are c_{o_2} and c_{u_2}. An interesting feature of the problem and the way it is formulated is that the customer's (primal) overage is the same as the contractor's (dual) underage. The associated costs, however, are not necessarily the same. For instance, the customer may put a certain value on a disappointment, such as the failure to deliver on time, which differs from that of the contractor. In this context the contractor may face a financial penalty as drawn up in the contractual agreement, while the customer faces the disappointment (and possible financial loss) when the project defaults at that stage.

1.2 The ‘Mix’ (Overage/Underage) Solution

The mix solution tracks the way in which partners across decision frontiers synchronize their efforts to reach optimality (Pearson, 2008a). In this context the decision frontier is at the tie which exists between the customer and the contractor. The solution (Appendix 1) is described by the equation

$$\Phi(k) + \frac{\partial \sigma_e}{\partial \mu}\phi(k) = Const \left\{ = \frac{c_{u_1} + c_{o_2} + c_p / 2}{c_{o_1} + c_{u_2} + c_{u_1} + c_{o_2} + c_p} \right\} \qquad (3)$$

We see that the constant in equation (3) depends on the project costs and contribution to profit (where $c_p = c_{p_1} + c_{p_2}$) defined in Section 1.1. The LHS of Eqn. (3) has an additional term to the classical fixed variability solution which measures the rate at which error variability changes with respect to the ‘mix’ variable μ. This assists in the identification of the optimal (maximum profit) solution in conditions of increased uncertainty.

1.3 The ‘Global’ (Volume) Solution

In contrast to the local (mix) variable, μ, the global variable, η, is formulated as the sum of expected demand and supply ($\eta = \mu_1 + \mu_2$), which assists in the monitoring of change in the global market place. The global (‘volume of the total expected demand and supply output’) solution is described by the equation (Appendix 1):

$$\phi(k)\frac{\partial \sigma_e}{\partial \eta} = Const = \left\{ \frac{c_p}{2(c_{o_1} + c_{u_2} + c_{u_1} + c_{o_2} + c_p)} \right\} \qquad (4)$$

We see that the constant again depends on the costs and contribution to profit defined in section 1.1. The LHS has a single term which measures the rate at which the error variability changes with respect to the ‘global’ variable, η. This also assists in the identification of the optimal (maximum profit) solution. The two equations (3) and (4) form a dynamic system of stochastic differential equations which trace the optimal solution in circumstances where variability changes and uncertainty increases or decreases over time and with relation to differing contractual and marketing strategies.

2. Phase Planes: An Illustration

We use an example to illustrate the methodology. A city council (customer and primal operator) approaches a contractor (dual operator) to install a single route tram system in a city. The operators agree that the tram system will be installed in 16 two-monthly stages (32 months in all) with monitoring taking place at each stage.

2.1 Local (Mix) Phase Plane

The local phase plane maps the progress of the contractual agreement entered into by the operators during the stages of the project. If the mutually agreed targets are exactly achieved within the risk environment then both operators will be satisfied and the target solution will be achieved (Figure 1). If the path described by the mix plot drifts outside of the area of capability (that is, when there is higher overage or underage than planned) but still remains within the boundary set by the isovalue line (so that, for instance, the contractor fails to meet one of the targets within the time schedule for that stage but still meets the overall contribution to profit target by improvements in other areas) then the two parties may not be over concerned. If, however, the path drifts outside of the boundary described by the profit isovalue line due to high uncertainty and failures in forecasting then there may be a need to implement penalties or renegotiate the contractual agreement.

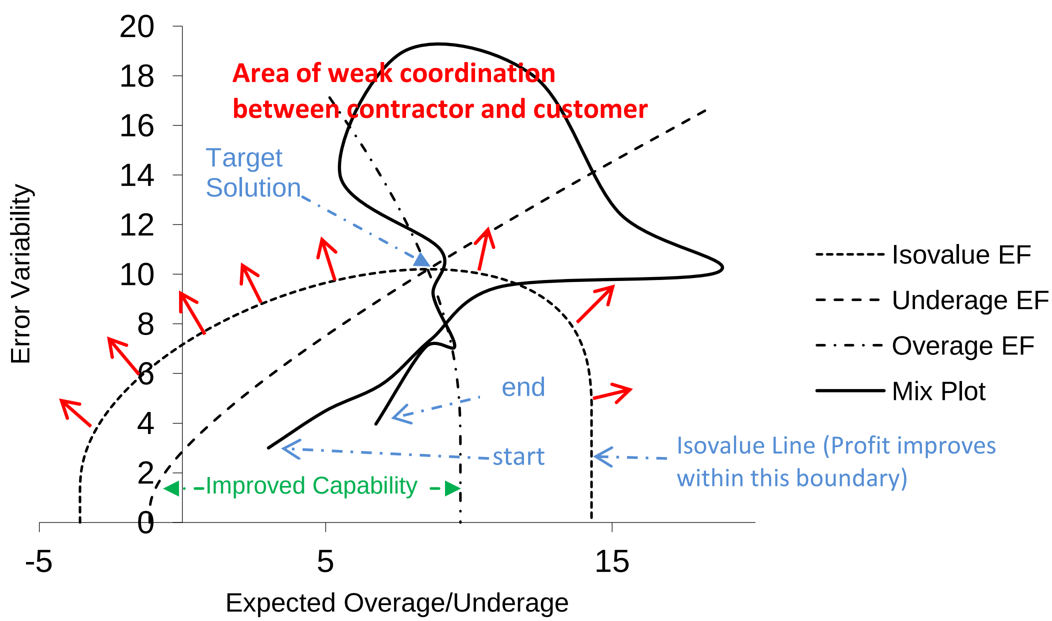

Figure 1 Local (Mix) Phase Plane

Figure 1 illustrates this during the central period of the contract (months 10 to 24). The Figure shows the three efficient frontiers (EF) associated with improvement in profit (isovalue line, Appendix 2), improvement in underage and improvement in overage associated with the risky environment. If the mix plot moves to the wrong side any of the efficient frontiers then the path is moving into a suboptimal position with regard to that requirement mapped in the uncertain environment. This leads to a risk of a failure of coordination and requires adjustment. In our example we see that the project operates efficiently at the beginning and the end of its life cycle, but does not do so during the central period as the path strays into the area of weak coordination so that the profit margins are compromised. The area where the profit improves on the agreed initial target lies within the profit isovalue line. The area where the targets for underage (e.g. insufficient tram rail laid at this stage of the project) and overage (e.g. excess of tram rail laid on unsuitably prepared ground, or exceeding the agreed time scale) lie outside of the underage and overage efficient frontiers and so the progress of the project would be no longer capable with respect to these targets. In our example the central phase of the project reveals a tendency to failures in both of these targets (as well as the profit target) so that there is instability in the chain of operation. To get a more complete picture, however, we should also refer to the global phase plane in Figure 2.

2.2 Global Phase Plane

Figure 2 illustrates the progress of the project in the context of the global risk environment. After several months the contractors discovered problems in meeting the targets agreed at the beginning. Part of the difficulty was the amount of infrastructure work which had to be carried out before laying the tram lines. This led to delays and increased expense. Another factor was the increased uncertainty due to a recession which meant that access to finance was restricted. The efficient frontier determined by reference to Eqn. (4) was crossed by the path in the global phase plane leading to concern about the viability of continuing with the project. As a result the terms of the contract needed to be renegotiated and the schedule changed.

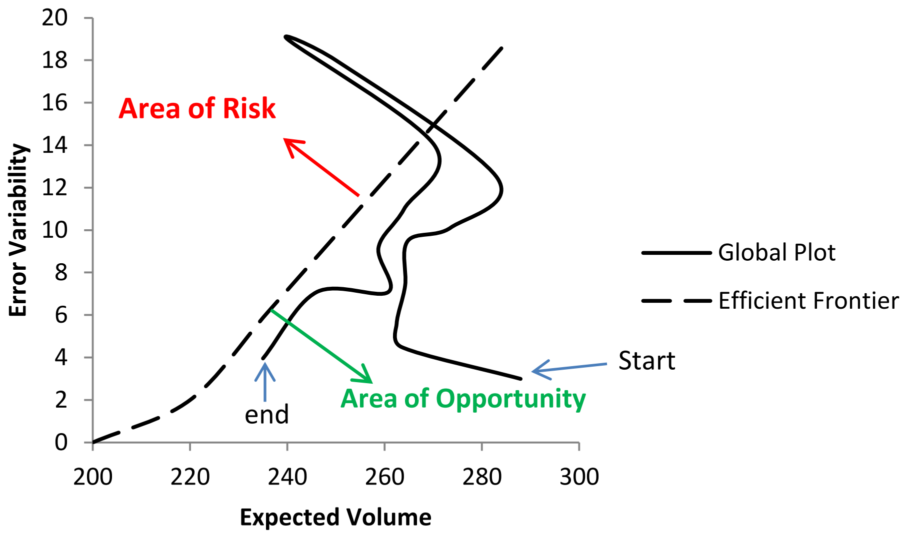

Figure 2 Global Phase Plane

3. Conclusion

The methodology which makes use of variability mapping, phase planes and decision frontier analysis has already been applied in the newsvendor problem and supply chain networks (Pearson, 2003, 2006, 2007, 2008a, 2008b, 2010). The new approach brings rigorous tools for measuring and monitoring coordinated project management between networked operators in both a local and global environment of uncertainty. The methods are also applicable to value chain management where they can be employed to monitor progress in projects open to uncertainty leading to risk but also opportunity. The need to distinguish between risk and opportunity in a scientific way can lead to improved value chain management and better opportunity discernment. The methodology is currently being further developed in order to incorporate dynamic pricing with the ability to identify optimality due to price changes in the value chain.

Appendix 1 (Two-echelon Proof) (Pearson, 2003)

Changing Variability The primal-dual objective function makes use of the primal-dual transformation. We consider the mix variable, which is the difference between the dual (*Q*) and primal (*D*) variables. We define *X*=*Q*-*D* and forecast values $\hat{X} = \hat{Q} - \hat{D}$. We let σ_e be the standard deviation of $e = e_2 - e_1$, where e_1 and e_2 are the primal and dual forecasting error terms, respectively. Then $\hat{X} - X + k\sigma_e \sim N(k\sigma_e, \sigma_e{}^2)$, and $E(X) = \mu = \mu_2 - \mu_1$. *e* will have a univariate normal distribution with mean zero. A similar result follows for the global variable, η. The primal dual transformation is: $\mu = \mu_2 - \mu_1$ and $\eta = \mu_2 + \mu_1$, so that $\mu_1 = (\eta - \mu)/2$, $\mu_2 = (\eta + \mu)/2$, and $\partial\mu_1/\partial\mu = -1/2$, $\partial\mu_2/\partial\mu = +1/2$, $\partial\mu_1/\partial\eta = +1/2$ and $\partial\mu_2/\partial\eta = +1/2$. We are then able to find the maximum value of the objective function (1) subject to the constraint (2) by applying a Lagrange multiplier with decision variables μ, η and *k*. We have

$$L = \mu_1 c_p - \{(\phi(k) - k(1-\Phi(k)))c_p + (k\Phi(k) + \phi(k))(c_{o_1} + c_{u_2}) + (\phi(k) - k(1-\Phi(k)))(c_{u_1} + c_{o_2})\}\sigma_e - \lambda(\mu - k\sigma_e).$$

So that $\dfrac{\partial L}{\partial k} = -\sigma_e\{\Phi(k)(c_{o_1} + c_{u_2} + c_{u_1} + c_{o_2} + c_p) - (c_{u_1} + c_{o_2} + c_p)\} + \lambda\sigma_e = 0$,

and $\lambda = \Phi(k)(c_{o_1} + c_{u_2} + c_{u_1} + c_{o_2} + c_p) - (c_{u_1} + c_{o_2} + c_p)$. Also

$$\frac{\partial L}{\partial \mu} = -\frac{1}{2}c_p - \sigma_e^{\mu}\left\{(\phi(k) + k\Phi(k))(c_{o_1} + c_{u_2} + c_{u_1} + c_{o_2} + c_p) - k(c_{u_1} + c_{o_2} + c_p)\right\} - \lambda(1 - k\sigma_e^{\mu}) = 0,$$

where $\sigma_e^{\mu} = {\partial\sigma_e}/{\partial\mu}$ so that $\lambda = -\frac{1}{2}c_p - \sigma_e^{\mu}[\phi(k)(c_{o_1} + c_{u_2} + c_{u_1} + c_{o_2} + c_p)]$

$= \Phi(k)(c_{o_1} + c_{u_2} + c_{u_1} + c_{o_2} + c_p) - (c_{u_1} + c_{o_2} + c_p)$.

Equation (29) follows. Furthermore

$$\frac{\partial L}{\partial \eta} = \frac{1}{2}c_p - \sigma_e^{\eta}\left\{(\phi(k) + k\Phi(k))(c_{o_1} + c_{u_2} + c_{u_1} + c_{o_2} + c_p) - k(c_{u_1} + c_{o_2} + c_p)\right\} + \lambda k\sigma_e^{\eta} = 0,$$

where $\sigma_e^{\eta} = {\partial\sigma_e}/{\partial\eta}$, which leads to equation (30).

The Hessian determinant is negative definite for $\sigma_e^{\mu\mu}\sigma_e^{\eta\eta} - (\sigma_e^{\mu\eta})^2 > 0$. If there were no clear maximum, then the structure of the Hessian would need to be investigated. It could be that the profit increases without limit as the global output increases. This would happen if σ_e^{η} <0, for instance.

Appendix 2 (Equation of Isovalue Line)

This is derived by substituting the optimal values for the overage (o_v) and underage (u_n) into Eqn. 1:

E(Profit) =

$$\mu_1 c_p - \{(\phi(k) - k(1-\Phi(k)))c_p + (k\Phi(k) + \phi(k))(c_{o_1} + c_{u_2}) + (\phi(k) - k(1-\Phi(k)))(c_{u_1} + c_{o_2})\}\sigma_e$$

$$= \mu_1 c_p - \{(\phi(k) - k(1-\Phi(k)))(c_{u_1} + c_{o_2} + c_p) + (k\Phi(k) + \phi(k))(c_{o_1} + c_{u_2})\}\sigma_e$$

$$= \mu_1 c_p - \{(c_{u_1} + c_{o_2} + c_p)u_n + (c_{o_1} + c_{u_2})o_v\}$$

And so $\mu_1 c_p - E(profit) = \{(c_{u_1} + c_{o_2} + c_p)u_n + (c_{o_1} + c_{u_2})o_v\}$ (App 2.1)

On the isovalue line the LHS of Equations (1) and (App.2.1) are constant. Setting $k = +\infty$ in Eqn. (1),

$$\mu_1 c_p - E(profit) =$$

$$\{(\phi(k) - k(1-\Phi(k)))c_p + (k\Phi(k) + \phi(k))(c_{o_1} + c_{u_2}) + (\phi(k) - k(1-\Phi(k)))(c_{u_1} + c_{o_2})\}\sigma_e$$

$$=$$

$$k\sigma_e\{(\phi(k)/k - (1-\Phi(k)))c_p + (\Phi(k) + \phi(k)/k)(c_{o_1} + c_{u_2}) + (\phi(k)/k - (1-\Phi(k)))(c_{u_1} + c_{o_2})\}$$

$$= \mu\{(\phi(k)/k - (1-\Phi(k)))c_p + (\Phi(k) + \phi(k)/k)(c_{o_1} + c_{u_2}) + (\phi(k)/k - (1-\Phi(k)))(c_{u_1} + c_{o_2})\}$$

$$= \mu(c_{u_1} + c_{o_2}) \qquad \text{(App 2.2)}$$

Equating the RHS of (App 2.1) and (App 2.2) gives $\mu = [o_v + ((c_{u_1} + c_{o_2} + c_p)/(c_{o_1} + c_{u_2}))u_n]$. The second result follows by setting $k = -\infty$ in Eqn. (1).

References

Pearson, M. 2003. “An Equilibrium Solution to Supply Chain Synchronization” *IMA Journal of Management Mathematics,* 14 (3), 165-185.

Pearson, M., 2008a. Decision frontiers in supply chain networks. In: Dwivedi, A., Butcher, T. (Eds.), Supply Chain Management and Knowledge Management— Integrating Critical Perspectives in Theory and Practice. Palgrave MacMillan.

Pearson, M. 2008b “Prioritising Edge over Node: Process Control in Supply Chain Networks” *Journal of Operational Research Society*, 59, 494-502, 2008.

Pearson, M.2006 “The Application of Prediction Capability and Adaptive Target Control to the Newspaper Business” *European Journal of Operational Research,* 168(2), 475-491.

Pearson, M. 2007 “Goodwill Hunting and Profit Sharing: Decision-Making in a Newspaper Chain” *European Journal of Operational Research*, 181, 1593-1606.

M. Pearson, R.Masson, A. Swain. 2010. ‘Process Control in an Agile Supply Chain Network’ *International Journal of Production Economics*, 128, 22-30.