AN INVESTIGATION INTO EFFICIENT MULTIPLE COMMAND ORDER PICKING IN HIGH BAY NARROW AISLE WAREHOUSES

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ABSTRACT

With the increasing tempo of business, order picking efficiency becomes of increasing importance. The aim of this research is to find ways of increasing order picking efficiency by decreasing travel time of the picking cycle.

Finding the optimal (shortest) order picking tour can be equated to the task of finding a solution to the Travelling Salesman Problem (TSP). This is notoriously difficult to solve in reasonable time when conventional computers are used.
A number of heuristic algorithms have been developed for solving the TSP, some of these have been specially adapted for multiple command order picking. In this previous work the stacker crane's shuttle was assumed to travel with constant velocities in the horizontal and vertical directions. In this research it is shown that this assumption leads to creating suboptimal tours. The contribution of the error is analytically derived and its magnitude estimated by a simulation experiment.

In previous work it has been shown that the shape of the zones in class based storage affects the travel time for single and dual command order picking. In this research, for the first time, the interaction between class based storage and multiple command order picking is investigated. Three types of zone configurations are modelled and then investigated using simulation in a factorial experiment. The results from the experiment indicate that the zone shape does affect the optimal solution.

The new zone configurations are tested in a case study against existing configurations in a distribution warehouse of Volkswagen - Audi (VAG-UK). This showed that overall improvement in travel time of the new configurations was significant. Computer simulation was used to estimate the individual contribution from zoning and tour construction.

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M. Guenov

DECLARATION

I hereby declare that:

All the work presented in this thesis has been carried out by myself and no part of this work has been submitted in support of another degree or qualification.

signed: ______________

/ Marin Guenov /

date: 30.01.1991

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CHAPTER I

INTRODUCTION

The purpose of this research is to investigate the possibility of increasing the efficiency of multiple command order picking by reducing the travel time of the picker or the picking machine in automated or semi-automated high bay warehouses.

As an introduction, this chapter is organised as follows:
In section 1.1 the place of order picking in the logistics chain is outlined and in section 1.2 the classifications of warehouse organisation and operations involving order picking are described.
The scope and the aims of the thesis are laid out in section 1.3 while key definitions are elucidated in section 1.4.
In section 1.5 a brief description of the organisation of the thesis is presented.

1.1 Order picking - an important part of the logistics chain

There is no internationally accepted standard definition of order picking no matter that the term itself is widely used. In a more abstract form order picking can be defined as: a set of warehousing activities whose aim is to transform the size and assortment of source lots into a desirable size and assortment of destination lots.

In more practical terms, especially when the destination is external to the source, order picking (OP) lies between two sides that often pursue different interests. On one side is the manufacturer or a large supplier who prefers large production and (or) transport lots in order to reduce the corresponding costs. On the other side is the customer (dealer, retailer) who prefers smaller lots but with greater variety of products.

The set of activities involved in OP when processing a customer's order are identification, selection, retrieval, inspection and packaging of the goods.

Currently, with success depending not only on the quality of the product, but the quality and the speed of service, and with increased competition, OP efficiency becomes an increasingly important part of the entire logistics chain.

Since the late seventies a new philosophy called Just in Time (JIT) has been introduced in Europe and America with varied success (Baumgarten [1986]). The main principle behind JIT can be summarised as follows : The exact quantity should be delivered in the right location in a constant quality, exactly on time. Since JIT tends to eliminate sources of variation (eg. demand, lead times, production rates etc.) it is obvious that an implicit purpose is to eliminate the necessity for warehouses which compensated for these variations.

It should be noted that in general warehouses can not be eliminated entirely because of the following reasons (see as well Baumgarten [1986]):

(1) Travel distances in Europe, America and elsewhere are

larger than those in Japan (motherland of JIT), so there will always be a trade off between money spent in stock and money spent for transportation of smaller lot sizes.

(2) With the increasing development of information technology customers' demand for shorter delivery time will always be ahead of the reduced throughput times in manufacturing. Therefore the planning horizon will still exceed the lead times which the customer could accept. With stock orders the necessity for a warehouse emerges again.

However, the influence of JIT is a fact and it does lead to further reduction of lot sizes and an increase in transport frequencies. This applies especially to the most expensive and most voluminous articles with relatively constant demand. There is a trend to a more direct distribution which tends to eliminate one or more intermediate distributions.

The result is that warehouses are becoming generally smaller and more integrated with the manufacturing process and the whole JIT material flow, which means that order picking in particular is expected to respond to a stronger demand for shorter order completion.

1.2 Classification of warehouse operation related to storage, retrieval and order picking

In this section a classification of warehouse organisation and operation related to storage, retrieval and order picking is given. The aim is to show the relation of order picking to organisational and operational factors that influence its efficiency.

The classification presented on fig 1.1 defines the area of this research. A definition or description for each of the entries on the figure is given since these terms are used throughout the thesis and for some of them there are no agreed international standards.

Storage, batching and order picking are often refered to as warehouse organisational and operational policies (see Ashayeri and Goetschalckx [1988]). The objective behind these policies is to optimise the trade off between maximum utilization of storage space and minimum travel time for the storage/retrieval operations.

Storage policy determines the manner by which incoming products are stored and their location in the warehouse. Alternative storage policies are random and dedicated.

Under random storage an incoming pallet is stored in any unoccupied location. Random storage has been shown by Schwarz et al [1978] to be equivalent to the Closest Open Location storage policy where the incomimg pallet is stored into a free location that is closest to the Input/Output (I/O) point of the

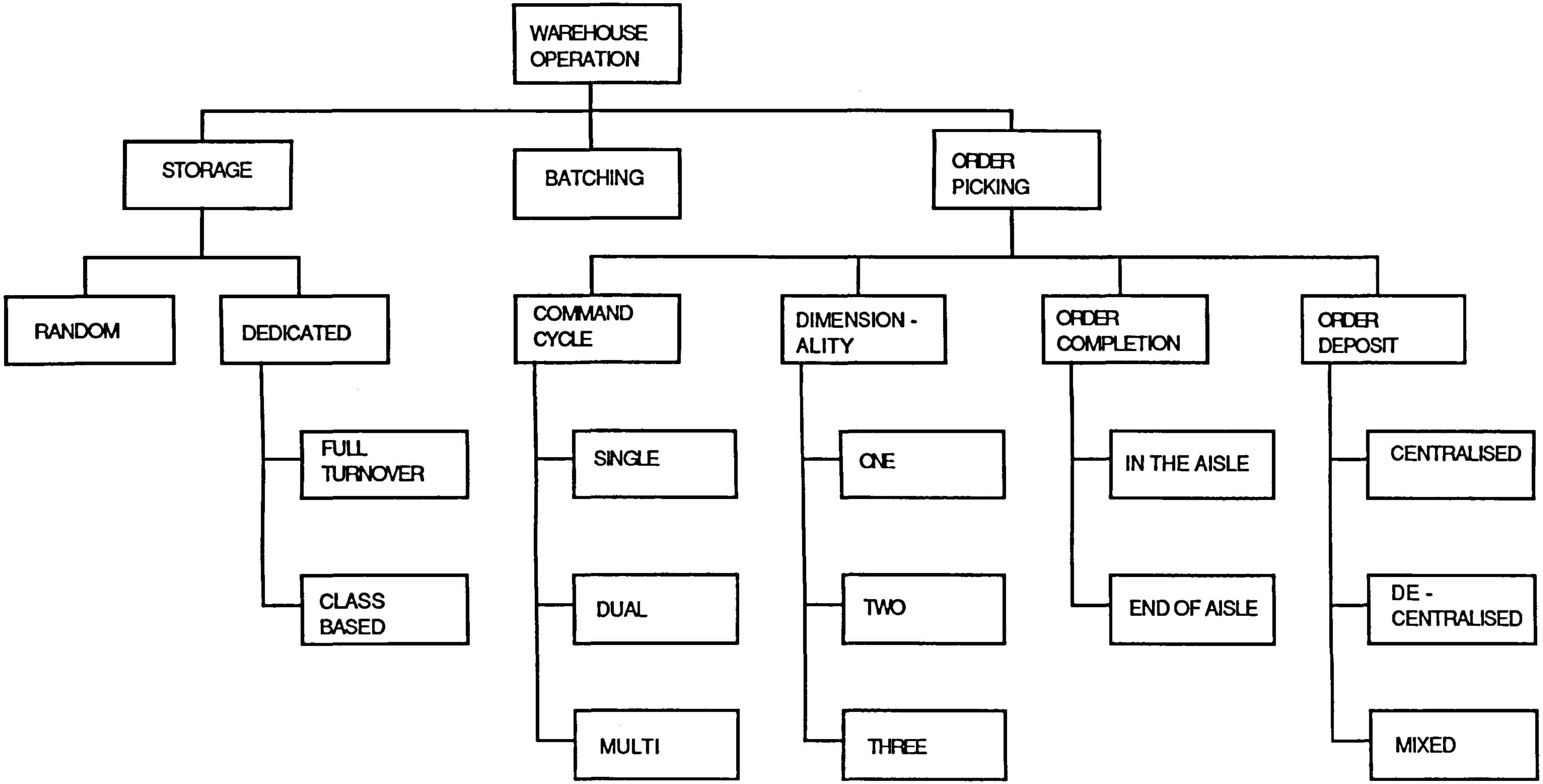


Fig. 1.1 Classification of warehouse operation related to storage and retrieval

storage (rack) area. Under random storage policy maximum storage utilization is achieved but at the expense of longer travel times, as slow movers tend in the long run to congregate near the I/O point.

Under dedicated storage policy products are ranked according to some of their properties such as volume or mass, cost, demand (turnover) etc.. Then a product is stored relative to the I/O point according to its rank.

Under full turnover storage policy every storage location is reserved for a product with a certain turnover, products with largest turnover being stored closest to the I/O point. This policy gains maximum savings in travel time provided that the demand is constant over the planning horizon. At the same time this policy provides least utilization of the storage space.

Under class-based storage policy products are grouped into classes according to their turnover (see Hausman et al [1978]). Number of classes is normally two or three. The highest turnover class is located closest to the I/O point. Within any given class pallets are assigned to locations randomly. The objective behind this policy is a trade off between random and full turnover based storage policy.

Batching policy is the set of rules for splitting and (or) grouping of several customers' orders for picking. The objective is minimizing the travel time and maximising the utilisation of the carrying capacity of the picking machine.

Picking policy in the operational context specifies the sequence and the place in which the items of a customer's order will be retrieved and deposited.

The command cycle specifies how many operations of storage and/or retrieval are executed on a single round trip of the picker or the picking machine.

In a single command cycle only one storage or retrieval is performed in the round trip.

Dual command assumes a combination of one storage and one retrieval or two operations in one cycle. This command cycle can be found in the literature as storage system with interleaving (Graves et al [1977]) or as combined cycle (FEM 9.851 [1978]).

Multi or multiple command order picking includes three or more operations on a single tour.

In some sources single, dual and multi command can be found as single, dual and multi address order picking (see for example Elsayed and Unal [1989]).

Dimensionality of order picking is the number of changed independent coordinates of the travelling stock keeping unit (SKU) or the order keeping unit (OKU), during the picking cycle. A SKU or OKU is normally a pallet, a container or a tote box.

In one dimensional order picking only one coordinate is changed, an example is manual order picking from shelves in a single aisle.

In two dimensional order picking the SKUs (OKUs), move in the plane. An example is a single aisle automated storage/retrieval system (AS/RS). The coordinates that change are the horizontal (the column) and the vertical (the level or the row) coordinate of each storage location.

An example of a three dimensional order picking is lift truck picking in a warehouse with many aisles each containing several horizontal levels (rows) and many columns.

According to where the order is completed order picking is :

(1) In the Aisle - SKUs remain in the rack and the picker and (or) the picking machine carrying the OKUs visits the addresses from which items included in the order should be picked and placed in the corresponding OKU.

(2) End of Aisle - SKUs are transported to a specialised zone or workstations where the picking takes place.

In some of the German literature sources (some of them are referenced in Petkov [1983]), In the Aisle order picking is refered to as a "static" order picking and End of Aisle as a "dynamic" order picking. In other sources (e.g. Bozer et al [1986]) these are called "Picker-to-Part" and "Part-to-picker" respectively.

According to the place the completed order is deposited, order picking can have:

(1) centralised deposit - the OKUs are deposited by the picker to the inspection or packaging area or,

(2) decentralised deposit - OKUs are placed on a material handling (conveyor) system which takes them to the packaging or the shipping area or,

(3) mixed (decentralised with centralised) deposit - it exists in places designed mainly for decentralised deposit but which handle rush orders by centralised deposit.

As it can be seen from fig 1.1 many combinations of the main features of order picking are possible and the choice of the right combination is vital at the design stage, since once built, these expensive systems are relatively inflexible, so it is difficult to change their mode of operation.

According to the level of mechanisation order picking can be classed as:

(1) manual - power and control is provided by the picker. An example is shelf picking or,

(2) mechanised - power is provided partly or fully by the machines with control and some manual operations by the picker. An example is pallet picking, using trucks or,

(3) semi-automated - power and some control is provided by the machines. An example is a miniload system or,

(4) automated - all power and control is provided by machines. There are as yet very few examples of these systems.

According to the information ability, the order picking system can be:

(1) static - all the information needed to determine batching and picking sequence should be available before executing the order and does not change during the execution of the sequence or,

(2) dynamic - the system is capable of handling changing information during execution of a sequence e.g. inserting a rush order in an already defined sequence. Some advanced parts

distribution warehouses work on this basis.

1.3 Purpose and scope of the research

The general purpose of this research is to find ways of increasing the efficiency of multiple command order picking by reducing the travel time of the picker or the picking machine in automated (semi-automated) high bay warehouses.

Most of research in the last decade has favoured End of Aisle order picking which assume single or dual command cycles. The main reasons for this were :

(1) These systems could be more easily automated at lesser cost so they can be highly efficient.

(2) Ergonomic constraints can be relaxed - if these systems are automated then the limitations for accelerations and decelerations become purely mechanical.

However, there are reasons (see Bachers et al [1988]) that in many cases now and in the future multiple command or In the Aisle order picking will be justified. The main reasons are:

(1) Multiple command order picking offers higher performance (throughput) especially in cases where customers' orders consist of a greater variety of items, e.g. parts distribution warehouses.

(2) Research in the fields of vision, material handling, information technology and artificial intelligence has made considerable progress. This along with the general trend of cheaper and more powerful micro processors will probably have a great

impact on the automation of order picking.

The task of sequencing locations to be visited in multiple command order picking is a planar case of the Travelling Salesman Problem (TSP) which is notoriously difficult to solve in a reasonable time because of its NP (non polynomial) completeness (Syslo et al [1983]).
For the last years more powerful computational procedures have been developed (see Parker and Rardin [1983]) but the short computational times have been reported using super computers.
No matter that multi command order picking resembles TSPs with up to 30-40 nodes, direct application of exact methods for solving the TSP is still impracticable for two main reasons:

(1) Powerful computers are still too expensive.

(2) There exist approximate methods which achieve reasonably accurate solutions (within 3% of optimality) in polynomial time.

It is one of the aims of this research to evaluate approximate (heuristic) TSP algorithms which could be particularly suitable for multi command order picking. In previous research (Bozer et al [1986], Goetschalckx and Ratliff [1988]) these have only been tested assuming that the picking machine travels with constant velocities in horizontal and vertical directions. It is a subject of this investigation to analytically derive the possible error when accelerations and decelerations have been neglected. The magnitude of this error will be estimated by simulation using real travel times taken from a stacker crane manufacturer.

The second major purpose of this work is to investigate the little studied interaction between picking and storage policies in multiple command order picking. In particular the effect of different zone configurations of the picking face on the optimal picking tour in a rack when class based storage policy operates.

In the majority of work tackling application of combinatorial optimisation problems to material handling and especially order picking, simulation is mainly used to verify the models. It is believed here that especially when people are involved (as in the semi-automated case of multi command order picking), the best verification of any model is to test it in the real world. Thus the third major task of the research is to check the theoretical and simulation results in a real warehouse in a form of a case study. This will consider the actual problems and the ways of overcoming them in the event that the new techniques prove to be better than the existing ones.

1.4 Definitions

In this section some of the main terms, used throughout the thesis are defined.

A high bay warehouse consists of one or several aisles. An aisle as shown on fig. 1.2-a is a corridor in which the picking machine (stacker crane) travels along. Associated with the aisle are the rack storage locations of either side. Each side is called a picking face (fig. 1.2-b) or a rack. It assumed here, that at the left bottom corner of each rack the Input/Output (I/O) point is situated. I/O point corresponds to the term Pickup/Delivery (P/D) point.

A location is a rack cell which accommodates a storage keeping unit (SKU). It is assumed that each location contains only one SKU. Throughout the thesis "address" or "point" are used as synonyms of location.

A rack is said to be square in time if the ratio of its vertical to horizontal velocity is equal to the ratio of its height to length. Considering the notation on fig. 1.2-b, the rack is square in time if $V_y/V_x = H/L$. In this case, considering that stacker crane's mechanisms for horizontal and vertical movement can work simultaneously, the direction of the resultant absolute velocity vector is parallel to the rack diagonal. In other words any line segment that is parallel to the diagonal can be traversed by a combined movement of the two mechanisms.

For single and dual command cycle it has been proven (see Hausman et al [1976], Graves et al [1977] and Bozer and White [1984]) that square in time rack minimises the travel time of the trip.

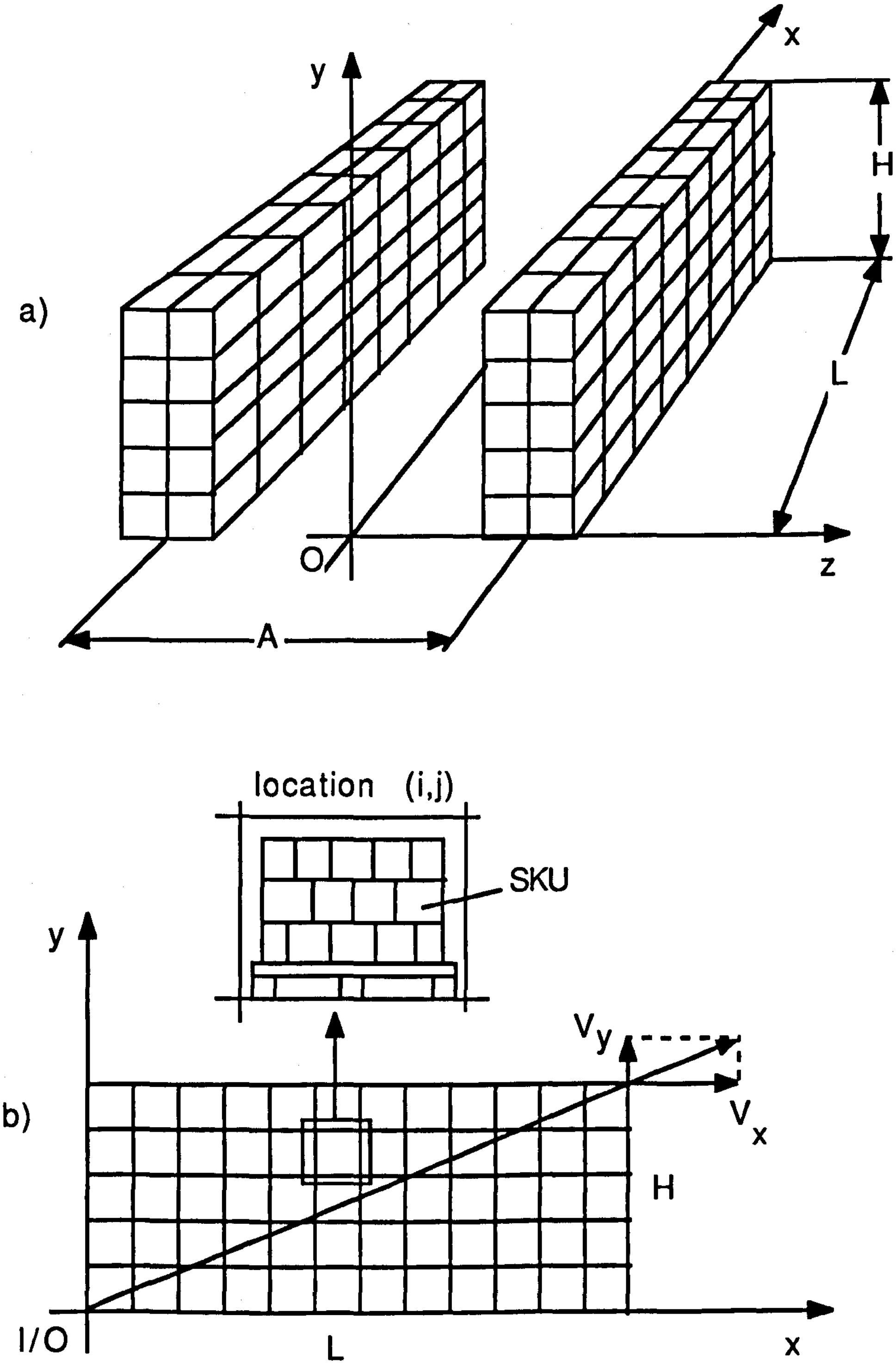


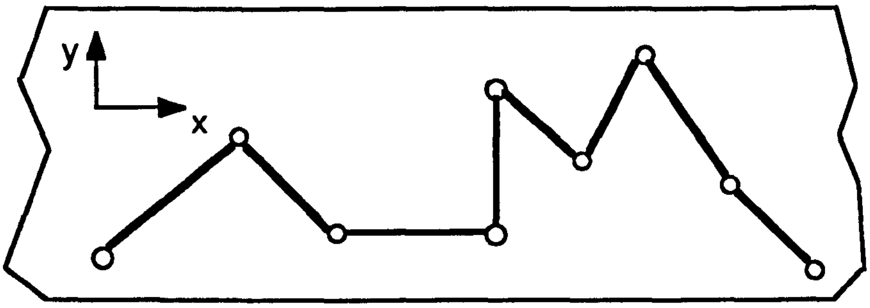
Fig.1.2 An aisle in a high bay warehouse; a) pictorial view; b) front view of a rack

Since it is assumed that the stacker crane shuttle travels simultaneously in horizontal and vertical directions the travel time between two points A and B will be the larger of the times in horizontal or vertical direction. Considering the notation in fig. 1.3-c, $t_{AB} = \max\{ t_{AB(x)}, t_{AB(y)} \}$. Such a travel metric is known as L_∞ or Tchebyshev metric (norm).

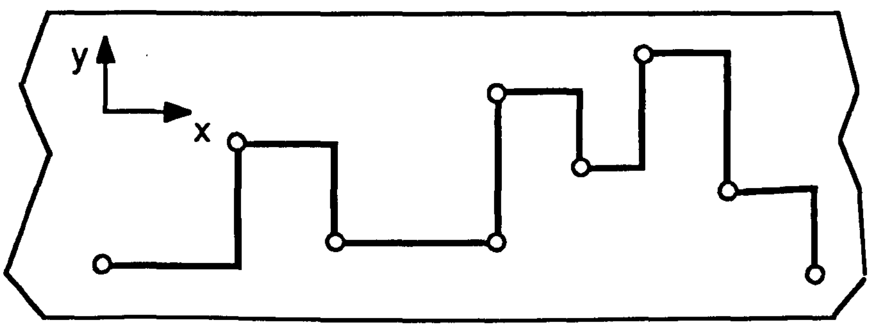
Other metrics are: the Euclidean (L_2) metric, shown on fig. 1.3-a which is the well known travel metric of all road vehicles, and the rectilinear metric. An example of the rectilinear (L_1) metric (fig. 1.3-b) is the reading head movement of some information retrieval devices or micro chip insertion devices.

As it was noted in section 1.3 multi command order picking is a planar case of the Travelling Salesman Problem (TSP). TSP can be stated as follows: Given the distances between all pairs of addresses in an area, find a tour visiting all addresses exactly once, such that the total traversed distance (total travel time) is minimised. In network theory terms the TSP is to find a minimum-weight (Hamiltonian) cycle in a given weighted, complete graph. The formulation of the problem, based on the quadratic assignment problem will be given in chapter III.

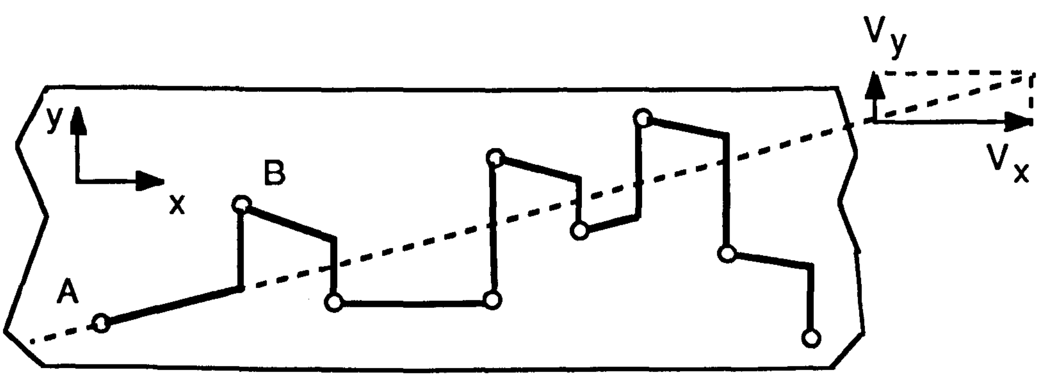
There is a generalisation of the TSP called the Stacker Crane Problem (see Jonson and Papadimitriou [1985]) where the desired tour must contain certain edges and must traverse them in specified directions. In other words, it means that each load picked at an address i must be delivered at a destination address, j, with no intermediate combinations of pickup and delivery allowed. The goal is to minimise the total cost (length)



a) Euclidean metric



b) Rectilinear metric



c) Tchebyshev metric

Fig 1.3 Travel metrics.

of the tour, which possibly contains repeated addresses.
If each pick up location is assumed to be arbitrarily near to its corresponding destination, the TSP arises as the limit.
As it is seen the name of the generalised problem is not chosen because of a specific application due to a special stacker crane's feature.

Other appropriate definitions and assumptions are given in the corresponding chapters.

1.5 Organisation of the thesis

The thesis is divided into six chapters, each representing a stage of the research.
Chapter I was devoted to defining the area, purpose and scope of the research as well as some basic terminology used throughout the thesis.
A review of some of the fundamental and recent publications related to the subject of this research is undertaken in Chapter II.
In Chapter III, the impact of stacker crane's dynamic (accelerations and decelerations) on the optimal picking tour is investigated.
The interaction between multiple command order picking and dedicated storage policies, in terms of minimising the travel time of the picker or the picking device is investigated in Chapter IV.
A case study, conducted in a parts distribution warehouse of

Volkswagen-Audi (VAG-UK) is presentesd in Chapter V. In this chapter the applicability of the theoretical and simulation findings to the real world is considered.

The main conclusions and recommendations of this research are presented in the final chapter .

CHAPTER II

LITERATURE REVIEW

In this chapter a literature review of some of the fundamental and recent publications related to the area of this research is undertaken. Literature sources underpinning specific aspects of the research are discussed in the corresponding chapters.
The chapter is divided into sections which follow the classification presented in section 1.2, considering warehouse design and operation.

2.1 Warehouse design

Literature sources dealing with warehouse design concern layout, dimensionality, selection of level of automation, material handling and information systems and span a wide range from inventory to human factors.
Order picking efficiency (single, dual or multi command) is a central problem in warehouse design, since order picking throughput reflects on the number of pickers, picking machines or systems and the costs associated.
In the publications on warehouse design, order picking is considered along with other major components of the system, such as material handling and control systems, available land and so forth, in order to minimise the total cost with respect to the required throughput.
A good introductory review on warehouse design optimisation is

presented by Ashayeri and Gelders [1985]. The authors review the recent application of analytical, simulation and heuristic methods and they conclude that since simulation alone can not handle the vast number of input variables, the most practical approach is to develop a simplified analytical model first. Then dynamic aspects of the system, such as distributions of arrival and departure times, queues and so forth which have been simplified or neglected in the analytical model are modelled by simulation.
This approach is followed in this research because of the model complexity and the large amount of data required for statistical comparison of alternative procedures.
In some complex cases one may argue that it would be more practical if simulation and analytical techniques are used iteratively.

Warehouse design is a complex and difficult task because of the multitude of different factors to be considered simultaneously. There are a few models that reflect more realistically the complexity of warehouse design.
One of the first algorithms for designing an automated high bay warehouse was given by Zollinger [1975]. Along with the sequence of determining rack and warehouse dimensions, number of aisles and stacker cranes, some valuable considerations of various costs and evaluation of the overall system were given.
Karasawa et al [1980] formulated warehouse design as a non-linear mixed integer optimization problem. The objective was to minimise the total cost of the warehouse. Decision variables are warehouse dimensions and number of cranes. This was subject to

two major constraints :

(1) sufficient crane capability to meet the required services,

(2) adequate capacity (volume) of warehouse to meet the anticipated stock.

In formulation of the objective function however, cost of manpower, service, maintenance and control, information and material handling (apart from stacker cranes cost) were not considered. This limited the applicability of the model.

Ashayeri and Goetschalckx [1988] describe a systematic approach for analysis and design of order picking systems. They include an evaluation of the external (marketing channels, demand and replenishment patterns) and internal (warehouse policies) factors which affect the strategic planning of the system. Then through iterative design procedures different aspects of the system such as layout evaluation, warehouse policies and level of automation are determined.

More recently Park and Webster [1989-a] developed an optimisation procedure for design of three dimensional palletized storage systems. Factors that are simultaneously considered are the input and output patterns of product flows, rack structure, storage policy, handling equipment and movement in the aisle. They also incorporate the economics of alternative storage systems, including capital investment costs, operational and maintenance costs. Design alternatives are compared using the criteria of travel time of the handling equipment, amount of land required and overall cost. When several alternative solutions are close in their expected equivalent total costs, a stochastic dominance approach is used to determine the best overall system.

They give as an example a case study, where alternatives including

Automated Storage/Retrieval Systems (AS/RS), narrow aisle lift truck and counterbalanced lift truck systems are examined.
The authors claim that their model is more complete than any one currently available. It should be noted that the model is restricted to single and dual command order picking and there are no alternatives considered for random and dedicated storage.

2.2 Warehouse operation

2.2.1 Storage

Order picking and storage policies are related in their aim to increase the picking throughput by minimising travel time of the storage and retrieval operations
One early work analysing the intuitive storage rules was done by Heskett [1963,1964] where he introduced the Cube-per-Order Index (CPO) rule for stock location. He outlines four basic factors determining the placement of stock, namely:

- compatibility, e.g. items that could damage each other are considered incompatible and should not be stored together;
- complementarity, e.g. items that are normally ordered together such as a bolt and a corresponding nut. It is desirable that they are stored closely to each other;
- popularity, i.e order frequency of an item;
- volume (bulk) of an item.

The author states that the key factors are the popularity and the

volume and tries to combine them into a single factor (CPO) that can be used in the minimisation of order picking and storage costs. CPO is defined as the ratio of an item's total average volume in stock, to the item's order frequency. Items are ranked in ascending order of their CPO and those ones with lower indexes are placed closer to the order shipping area.
Practical guidance is given for implementation of CPO, and through examples it is shown that using the CPO index rule minimizes travel time and order selection cost for a warehouse with either only order assembly area or both order assembly and reserve storage area.

Kallina and Lynn [1976] summarized Heskett's work pointing out that the CPO index rule is a quantitative trade off between the dual objectives of placing closest to the order shipping area those items taking up least space and also those items which are most popular. The authors note that the overall problem of minimising the total variable cost of stock location and movement is complex compared to the reduced Heskett CPO problem. However, they state that the CPO rule can be a starting point when trying to minimise overall costs, not just order picking costs alone. The authors report on a practical application of the CPO index rule in a canned food distribution warehouse. The savings of order picking costs were in the range of 5-10% over the considered alternatives to CPO i.e. popularity and volume (bulk) of the items.
More recently Malmborg and Krishnakumar [1987] investigated cost formulation of dual command order picking in a conventional warehouse in the form of an assignment problem. Most important

amongst the assumptions are rectilinear travel and fixed inventory levels. It is shown that the CPO Index rule minimises the cost of dual command order picking for that type of warehouse. Assuming rectilinear travel restricts the results to lift truck application. The assumption for fixed inventory levels fits the linear programming model at the expense of the model's accuracy.

Christofides and Collof [1971] investigated a problem arising in warehouse operation when due to the seasonality of the products or other reasons, demand for certain items fluctuates. Most often this requires dynamic rearrangement.
The authors developed a two stage algorithm for minimizing the cost (travel time) of rearranging the items, on the assumption that a temporary auxiliary storage with limited capacity is present. Given the initial and final state of the rearrangement, items are grouped into subsets of one (i.e. no move for that particular item) or more locations that form cycles.
In the first stage, the minimum cost (time) for each cycle is found including the start and the end point of the vehicle. An algorithm very similar to Dijkstra's algorithm (see Christofides [1975] or Syslo et al [1983]) for finding the shortest path through set of points in a graph is used to find the initial arrangement.
In the second stage all cycles are sequenced using dynamic programming to give minimum overall cost (time) for the rearrangement.

A series of three consecutive papers on storage policy were published by W.Hausman, S.Graves and L.Schwarz and are referred to as Hausman et al [1976], Graves et al [1977] and Schwarz et al

[1978]. They introduce the idea of a class based storage as a trade off between the full turnover storage and random or closest open location storage. The authors claim significant savings in travel time at the expense of a possible increase of rack area. The increase in the rack area is in the region of 3% for two class to 5% for three class storage. In these papers expected travel times for single and dual command cycles are obtained and they are discussed in sections 2.2.3.1 and 2.2.3.2.

Park and Webster [1989-b] developed further the work of Hausman et al [1976] and Graves at al [1977] who defined square in time L-shaped boundaries for class-based storage in a rack.
Park and Webster investigated a cubic-in-time storage system, in which storage space was considered to be a cube. One of the bottom edges of the cube is assumed to be the main aisle, at one end of which, the main I/O point for the storage system is located. The I/O (P/D) points of aisles, in which the stacker cranes operate lie on the edge of the cube representing the main aisle. Two-class storage and single command order picking were considered.
The idea behind cubic-in-time storage is that in each aisle the L-shaped square in time boundaries of the fast movers' class are kept. However, they decrease in size proportionally to the distance of the corresponding aisle from the main I/O point and in inverse proportion of the ratio of average horizontal to vertical velocity of the main aisle handling equipment. Thus the authors claim, it will take equal time to visit each of the storage locations starting from the main I/O point. A comparison (using an example) is made for a storage with constant L-shaped boundaries and cubic-in-time

storage and it is stated that the cubic-in-time storage requires less travel time. Overall percentages however, are not given. The idea of cubic-in-time storage reflects a more theoretical case. Two main arguments for this are:

i. Means of transport in the main aisle are either a conveyor system or a system of lift trucks. In the first case loads travel only in the horizontal plane. In the case of lift trucks, even though simultaneous travel in horizontal or vertical directions may be possible, only one motion at a time is permitted for safety reasons. Moreover the change of the vertical coordinate of the load in the main aisle is negligible compared to the distance travelled. This affects one of the coefficients used in a formula which determines the partitioning of the storage into two classes in each aisle as a function of distance from the main I/O point. This coefficient is a ratio of average main-aisle horizontal speed to the vertical speed of the handling equipment and seems to be irrelevant, assuming the above considerations.

ii. The fact that the zone of the fast movers decrease in size with the distance from the main I/O point would lead to over utilization of some of the stacker cranes and under utilization of others. This, if not taken into consideration can create additional queues in the system.

Bozer [1985] investigated the "pick versus reserve" problem which is assigning part of the rack or self contained remote area from the main storage only for picking operations. The aim is to improve picker's efficiency and to reduce inventory related costs. In terms of optimizing picker's throughput a conclusion is made

that picking area should be minimized in both (with or without remote picking area) cases.
Some considerations of pick versus reserve problem are given descriptively by Petkov [1983] in particular, when to use a remote picking area. The order picking factor is defined as a ratio between the order picking throughput to the throughput of the entire warehouse. It is stated that when the order picking factor is large, remote area should be used but at the expense of increased investments for material handling and storage.

Goetschalckx [1983] investigated various rack storage policies. The objective was to minimize picker's travel time (i.e. to increase picking throughput) for single command cycle. When analysing existing policies the author notes that if all products are replenished at the same time or storage requirements for each product stay constant during the planing horizon, then product turnover based dedicated storage remains the optimal policy. However, such policy in practice would give same priority to all units of a product (item). Therefore two units of a product with a large replenishment batch will stay for different lengths of time in the warehouse while occupying locations that are similar in desirability.
The shared optimal policy developed by Goetschalckx is based on the duration of stay of each unit passing through the system. At the time of arrival of a unit its duration of stay is generated by multiplying the quantity on hand of the product by the product's demand inter-arrival time. The shared optimal storage heuristic is claimed to generate travel times on average 30% less than those of dedicated storage with savings in rack size up to 50% of the

rack size over dedicated storage. However, this heuristic reflects a deterministic case where all the required information is considered available which is hardly the case in practice. Goetschalckx developed another heuristic with known material movements, a precognitive heuristic. In this case items are sorted by increasing departure time. If items have equal departure time they are sorted in increasing order of their arrival time. The units are then assigned in that order to the closest open location to the rack's P/D (pick up/delivery) point.

For more practical situations with imperfect information of product arrivals an adaptive heuristic was developed. It is based on the long term average material flow. Groups of items with successive duration of stay and the same (or similar) average number of arriving items per period form a zone. Then the heuristic behaves as the optimal shared storage policy.

Goetschalckx claims that travel time savings are 70% of the savings of turnover dedicated storage.

2.2.2 Batching

As was noted in chapter I, batching rules specify the combinations of several orders for simultaneous picking in order to ensure an optimal trade off between minimum travel time of the picker and maximum utilization of machine's carrying capacity.

Elsayed [1981] and later Elsayed and Stern [1983] proposed several order batching algorithms based on combinations of three steps. First a seed order is chosen with either minimum or maximum locations to be visited or minimum or maximum items in the order. The second step uses congruency rules, where a candidate order for inclusion in the batch is ranked according to how many common (or close) to the seed order's locations there are in the candidate's order.

The third step includes updating of the batch after inclusion of a candidate order. The authors concluded that none of the algorithms are superior to the other ones because the stacker carrying capacity constraint has significant impact on the optimal solution.

Further development of the above work was done by Elsayed and Unal [1989]. They base the new heuristics on one of the following approaches:

i. a combination of two orders (a "large" and a "small" one) that give maximum savings in travel time forms the seed of the batch and more orders are added while checking against constraints such as vehicle carrying capacity.

ii. calculations of savings of total travel time for every possible combination of two orders are made. The class of the combinations is increased until vehicle capacity is reached.

The first one of the above two approaches is reported to give best results.

A more specific case of batching in a conveyorised order picking system is investigated by Armstrong et al [1979]. The system consists of picking and accumulation conveyors with a number of gates for ensuring the specified batch processing discipline. The system is modelled as a mixed integer linear programming problem with the object of minimizing, the total time for completion of k-1 batches since one of the main assumptions is that a next batch could enter the system before the completion of the previous one. There is no explicit report on benefits of the application of the model.

2.2.3 Picking

An important consideration during the design stage of order picking systems is the choice between In-the Aisle (ITA) and End-of-Aisle (EOA) order picking.

Bozer [1985] concluded that EOA order picking generally yields a higher throughput level if the mean pick time is relatively long (compared to travel time) and the variance of the pick time is relatively small.

Some more practical criteria are cited by Petkov [1985] who defines the average number of visits to a location, until the stored items in that location are consumed, as a ratio:

$$q = a/m,$$

where a is the number of items in the location;

m is the number of items of the same unit in the order (sub batch).

Petkov uses this ratio to advise on the choice of the type of order picking:

When $q > 5$, ITA order picking is preferred because it would save multiple retrievals of the same SKU.

When $1 < q < 3$, EOA order picking is preferred because the small number of picks per cycle would increase both average travel time between the corresponding stops and pick up times in the aisle, thus decreasing the throughput of the system.

The interval $3 < q < 5$ is not determined.

Obviously q is a statistical value which is often difficult to set, due to the high variability of demand. Observations on various products with variable demand should be made in order to determine q for each of them and then for the whole system.

2.2.3.1 Single command

Hausman et al [1976] introduce continuous representation of square in time rack when trying to analytically derive the expected travel time for single command order picking with random (closest open location), full turnover and their originally proposed class based storage.
Results for the expected travel depend on the item turnover distribution which in turn is based on ABC analysis.
ABC curves are approximated with the function:

$$G(i) = i^s, \quad 0 < i <= 1 \text{ and } 0 < s <= 1,$$

where i is the cumulative percentage of inventoried items and s is a parameter. Thus G(i) represents cumulative percentage of demand rate (items per period of time).
The expected travel times for class based storage are derived as functions of s and R (R_1 and R_2 for three class storage), R being the partitioning value or boundary between the classes. The formulae are bulky and are not presented here.
Analytical comparison shows that class based storage gains up to 85% of the travel time savings of full turnover storage policy which is considered to be the ideal case.
However, by simulation it was estimated that if the skewness

of the ABC curve increases (i.e. s -> 0, or a small percentage of items represents a large percentage of demand) the continuous model becomes less accurate. The authors still claim that the potential savings in travel time remain significant.

More recently Bozer and White [1984] determined the expected travel time for single and dual command cycle and randomized storage. They introduced shape factor for a continuous representation of a rack which is not necessarily square in time. Assuming that the horizontal and vertical coordinates of an address are independently generated, they determine the joint probability of the coordinates as a function of the shape factor and hence the expected single command travel time is:

$$E(SC) = \frac{b^2}{3} + 1,$$

where b is the shape factor, which lies between 0 and 1. Definition and critical analysis of the shape factor are presented in Chapter III.

2.2.3.2 Dual command

Graves et al [1977] derived analytically the expected travel time for dual command cycle for random (closest open location), full turnover and two and three class based storage with FCFS (first come first serve) discipline for the retrievals. Their paper is a continuation of the work of Hausman et al [1976] (see

2.2.3.1) for single command.

The analytical findings are verified by simulation and it is concluded that, as for single command, for the case of dual command cycle random storage is the slowest in terms of round trip travel time. The savings for two and three class storage over random storage are reported to be in the range of 16-35%, depending on the skewness of the corresponding ABC curve. The authors note however, that the savings of travel time for class based storage are at the expense of possible 3-5% increase of storage space because of the reduced rack utilization under two and three class storage.

Schwarz et al [1978] examined by simulation the results obtained by Hausman et al [1976] and Graves et al [1977]. In their simulation model they relaxed some of the assumptions made for the analytical model. For example the length of stay for each item is no longer considered as constant, but as a random variable with distribution depending on the parameter s (see 2.2.3.1) indicating the skewness of the turnover distribution. The utilization of stacker crane and rack is again considered non constant. The dual command cycle depends on the queue for the retrievals i.e. dual cycle is performed only if the queue for the retrievals is non empty.

It is concluded that the general improvement gained for class based storage is slightly smaller than the results reported for the analytical findings. This is chiefly due to the fact that in reality the rack system is discrete and retrievals are not mandatory for dual command as imposed in the analytical model. The authors still claim that class based storage is an optimal trade off between random and full turnover storage policies.

Bozer and White [1984] derived the expected travel between the storage and retrieval location in dual command cycle from the joint probability of the differences between the corresponding horizontal and vertical coordinates of the storage and the retrieval locations. The probability functions of coordinate differences in horizontal and vertical directions are obtained from the distribution of the ranges of pairs of sampled, uniformly distributed points in a plane. The authors utilise order statistic results assuming that the coordinates of the storage and retrieval addresses are independent in horizontal and vertical direction respectively.
Adding the expected travel time for single command to the expected travel time between the storage and retrieval locations, for the expected travel time for dual command cycle they obtained:

$$E(DC) = \frac{4}{3} + \frac{b^2}{2} - \frac{b^3}{30}.$$

The authors note that when b=1, i.e. rack is square in time, the expected times for single and dual command confirm the results obtained by Hausman et al [1976] and Graves at al [1977].

A special case of the dual command cycle when the stacker crane can operate with two pallets is investigated by Jaikumar and Solomon[1986]. It could be said that this is more "quasi quadrucommand " order picking, since the machine is capable of doing a sequence of two storage/retrieval operations in one cycle. The authors claim that such a system can substantially reduce travel time over dual command cycle. It is noted however that the increase in throughput would be at the expense of the increased cost of the stacker cranes.

2.2.3.3 Multiple command

Multiple command order picking, is as are many other transportation and scheduling problems (see Lenstra and Rinnooy Kan [1975]) a planar case of the Travelling Salesman Problem (TSP). In spite of its simple formulation it is very difficult (computationally expensive and time consuming) to find an exact TSP solution especially for larger problems. For this reason, along with attempts to improve the exact methods, a number of heuristic procedures for solving TSP in polynomial time have been developed.

A detailed quantitative investigation of several heuristics was done by Golden et al [1980]. Authors divide heuristics into three major categories:

i. tour construction procedures;
ii. tour improvement procedures;
iii. composite procedures.

Tour construction procedures generate an approximately optimal tour from the distance matrix.

Tour improvement procedures attempt to find a better tour given an initial tour. The authors examined branch exchange heuristics namely 2-optimal, 3-optimal and k-optimal (see Lin

[1965] and Lin and Kernighan [1973]) algorithms.
In these algorithms 2, 3 or k (k > 2) links of the tour are replaced by the corresponding number of links that have not been involved in the tour to form a new tour. If the new tour is better it is stored. The procedure terminates at a local optimum (minimum) when it is not possible to improve the tour by further link exchange. Then it is said that the tour is 2, 3 or k-optimal (see also Syslo et al [1983]). The larger the number of link exchanges the better the solution but it is at the expense of increased computational effort.
In the k-opt procedure the number of links that are exchanged starts from two and is increased by one until certain stopping conditions are satisfied. The entire process is repeated until every address has been used as a starting point and no further improvement can be found.
Composite procedures construct a starting (often a feasible) tour from one of the tour construction algorithms and then attempt to find a better tour using one or more of the tour improvement procedures.
Since exchange procedures require a symmetric cost matrix investigation for tour improvement and composite procedures was done for the symmetric TSP.
The main findings of Golden et al [1980] were that:
tour construction procedures are often within 8% of the optimal solution and an initial tour created by the Convex Hull algorithm (see chapter III) and improved by 2-opt procedure is often within 3% of the optimal solution. This is a similar result for all composite procedures. However, the authors make an important point that "no conclusions could be reached as to

which tour construction procedure is best to use in a composite procedure".

There are some geometric approaches for constructing the tour. Norback and Love[1977] for example use the "Largest angle" and the "Most eccentric ellipse" methods. Norback and Love point out the ease of constructing tours manually using the above methods. However, these procedures often generate suboptimal tours with crossing links, so they are used to construct the Convex Hull of the points in the plane and then a single point insertion procedure is applied to complete the tour. The heuristics are shown to give satisfactory results for large travelling salesman problems.

In a more recent review Golden and Stewart [1985] examine a modification of the 3-opt procedure developed by I.Or in his doctoral dissertation (referenced in the paper). The authors believe that this modification works extremely well. This procedure considers only those link exchanges that will form a string of one, two or three currently adjacent addresses being inserted between two other addresses in the tour. Thus by limiting the number of exchanges to be considered the procedure requires significantly fewer calculations than the 3-opt procedure.

Golden and Stewart conclude that Or-optimal exchange produces final results that are dependent on the quality of the tour produced in the construction phase.

A new composite heuristic based on the procedures: Convex hull, Cheapest insertion, Largest angle selection and Or-opt exchange was examined. It is claimed that this heuristic outperforms all heuristics compared in the investigation.

Another approach used to construct the tour is the Space filling curves method introduced by Bartholdi and Platzman [1982,1988-a,b]. They report that the algorithm is very fast but the accuracy is 25% less than that of the optimal tour. Shorn [1985] used this method as a tour construction procedure in a composite heuristic which includes 2-opt and 3-opt improvement phases. The reported results are closer, but still poorer than the results of the best performing (Chebhull and 1/2 Band) heuristics investigated in the study.

Some of the heuristics for solving TSP have been used or adapted for the Tchebyshev (L_∞) metric. For example Bozer et al [1986] mainly summarising the work of Shorn [1985] presented and evaluated several heuristics and tour improvement procedures designed for order sequencing in multiple command order picking. Evaluation is based on picking tour quality and run time. They reported that Convex Hull and 1/2 Band heuristic (see Chapter III) plus 2-opt improvement phase perform best. The Convex Hull required most computational time. Since run times are reported in milliseconds, run time differences do not appear to be very significant.

Goetschalckx [1985] and later, in an almost identical paper Goetschalckx and Ratliff [1988] compare Convex Hull, Convex

Hull plus "free" insertion procedure or Chebhull (see Chapter III) and 1/2 Band heuristics. Comparison is made on tours with up to twenty five nodes. The authors conclude that there is no significant difference in effectiveness (accuracy) and efficiency (computational time) between the heuristics. They claim that on average all heuristics are within 0.5% of the optimal solution (given by Little's Branch and Bound algorithm for solving TSP). The run time for all heuristics for up to twenty five locations is reported to be within a second, using a PC.

It should be noted however, that all of the above works dealing with applications of approximate TSP algorithms for order picking consider constant velocities and the average performance of each is obtained by simulating relatively small problems of 5,10,15,20 and 25 addresses which explains why some of the reported results are so close to the optimal.

2.3 Machine performance

Machine performance is an important, complex factor in achieving high order picking throughput. It consists of three major sub factors that affect the throughput:

i. vibration decay time;

ii. velocities;

iii. accelerations and decelerations.

These are interrelated and are discussed in more detail in Chapter III.

There is not much published literature on these specific mechanical and control problems of stacker cranes since this is normally company confidential information.
However, there are several sources, published mainly in Eastern Europe.
Petkov et al [1974] developed a methodology for dynamic calculations of stacker cranes. The mathematical model describes a six mass equivalent system.
P. Karaivanov [1984] gives an analytical method for calculating the middle speed of three speed travel mechanisms for stacker cranes considering minimum travel on middle and creep speeds which ensures maximum throughput.
Guenov and D. Karaivanov [1987] investigated the possibility of decreasing the amplitude of vibration of the stacker crane tower during retardation, when applying variable brake torque.
In a text book on stacker cranes, published by Zertzalov et al [1986], a variety of problems are discussed, including performance data, design of the metal structure and the mechanisms, considering the Russian and European (FEM) standards for stacker cranes.

2.4 Summary and Conclusions

Some of the recent literature sources related to this study were reviewed.
Because of the multitude of different factors involved in warehouse design, researchers have concentrated their efforts

in more specific aspects of warehouse optimisation. With the increased computer power and the latest developments in simulation, there is a tendency towards a more complex approach to warehouse design and optimisation.

This research aims to further develop some of the outlined ideas for increasing multiple command order picking efficiency by applying certain operational research optimisation techniques to tour construction procedures in order to reduce the travel time of the picker. Some of the assumptions such as constant velocities will be reconsidered in order to find the difference in model accuracy.

The research devoted to dedicated storage policies in combination with single and dual command cycle will be extended to multiple command order picking.

Literature sources, directly related to specific aspects of this research will be critically discussed in the corresponding chapters.

CHAPTER III

TRAVEL TIMES AND MACHINE PERFORMANCE IN MULTIPLE COMMAND ORDER PICKING

In this chapter the effect of machine performance on order picking throughput is investigated.
In section 3.1 a stacker crane is described as a dynamic system and the major mechanical factors that affect machine throughput are discussed. The impact of accelerations and decelerations (i.e. real velocity-time patterns) on the optimal multi command order picking tour is investigated in section 3.2. The magnitude of the error involved in constructing the optimal tour when constant velocities are used and accelerations and decelerations ignored, is derived in this section. A designed experiment using computer simulation is described in section 3.3, the aim of which is to estimate the average value of this error and its variance for order picking tours with up to thirty five addresses. The experimental results are also presented in this section. Conclusions are drawn in the last section.

3.1 Stacker crane as a dynamic system

As was pointed out in section 2.3, machine performance plays an important part in achieving higher order picking throughput.
The most significant factor that affects the machine's throughput is stacker crane shuttle's amplitude decay time.
To explain this, consider a stacker crane as a two mass

equivalent system (see Guenov and Karaivanov [1987]) as shown on fig. 3.1.

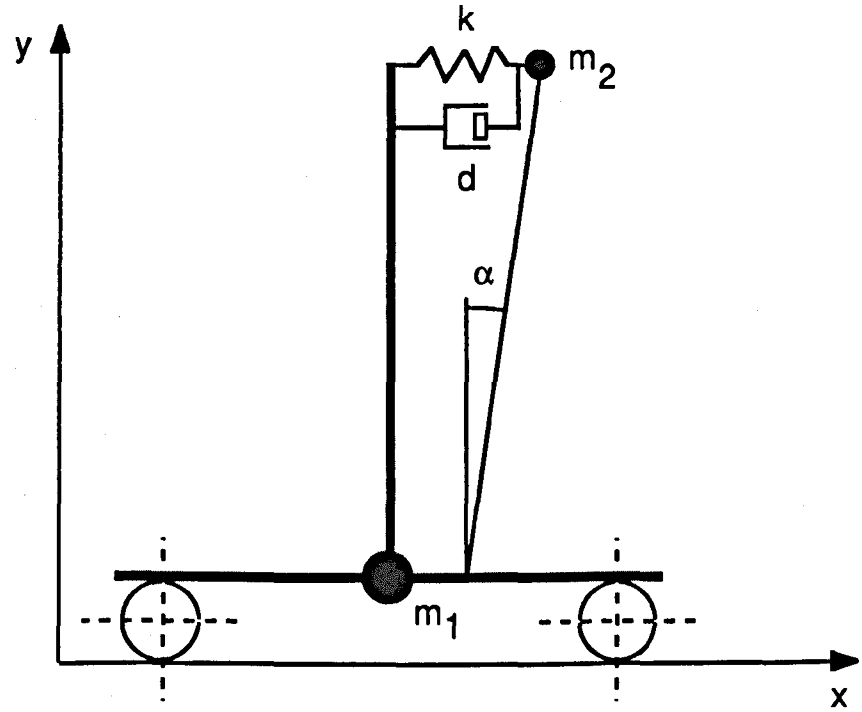


Fig. 3.1 Two mass equivalent model of a stacker crane

The notations on fig. 3.1 are:

m_1- the equivalent mass representing the masses of the chassis, tower and travelling mechanism, all related to the chassis,

m_2- the equivalent mass representing the masses of the load, shuttle and tower, all related to the top of the tower,

k- stiffness of the structure (the tower),

d- damping factor of the structure,

α– deflection of the tower.

During a transient period (accelerating or decelerating) the inertia forces cause the stacker crane's tower to deflect and, because of the tower's elasticity, to oscillate. When the stacker crane stops in front of a location for a storage or retrieval operation, a pause is required, before the amplitudes of oscillation become small enough to allow the shuttle to be inserted into the location without risk of an accident. This pause or decay time is a downtime which depends on the magnitude of the maximum amplitudes and the intensity of the oscillation energy absorption (dissipation). Hence the decay time can be reduced by reducing the maximum amplitudes or increasing oscillation energy absorption.

The maximum amplitudes depend on the magnitude of the inertia forces and the stiffness of the system. Inertia forces in turn, depend on the mass (m_2 in this case) and the accelerations (decelerations).

Thus to reduce the maximum amplitudes one would stiffen the the construction and (or) change the character of the inertia forces, in this case the character of the retardation. An example for the second option is applying a variable brake torque (Guenov and Karaivanov [1987]).

When the more expensive but more easily controlled DC motors are used in the travelling mechanism it is possible to control the smoothness of the velocity-time curve, thus decreasing the value of the decelerations in the final phase before stopping.

The rate of oscillation energy absorption can be increased by installing special dampers in appropriate places in the structure of the stacker crane (Petkov et al [1984], Zertzalov et al [1987]).

The effect of increasing stacker crane velocity is to increase throughput. However, this only applies up to a certain level above which increasing velocities does not improve performance.

Consider the travel of the crane between two locations in the horizontal or vertical direction, as shown on fig. 3.2. It is assumed for the sake of simplicity that there is no creep speed and that the absolute values of accelerations and decelerations are equal.

The travelled distance, S is the area under the trapezium on the figure i.e.:

$$S = \int_0^T V(t)dt = VT - \frac{V^2}{a},$$

where, a is the acceleration (deceleration);

V is the velocity; T is the travel time.

Hence the travel time is:

$$T = \frac{S}{V} + \frac{V}{a}.$$

In order to find the velocity which minimises the travel time, the first derivative of the time with respect to V is set equal to zero:

$$\frac{dT}{dV} = -\frac{S}{V^2} + \frac{1}{a} = 0$$

Hence for the optimal velocity :

$$V_{max} = \sqrt{a.S}$$

The functional relation between V_{max} and time resembles a triangle (fig. 3.2)

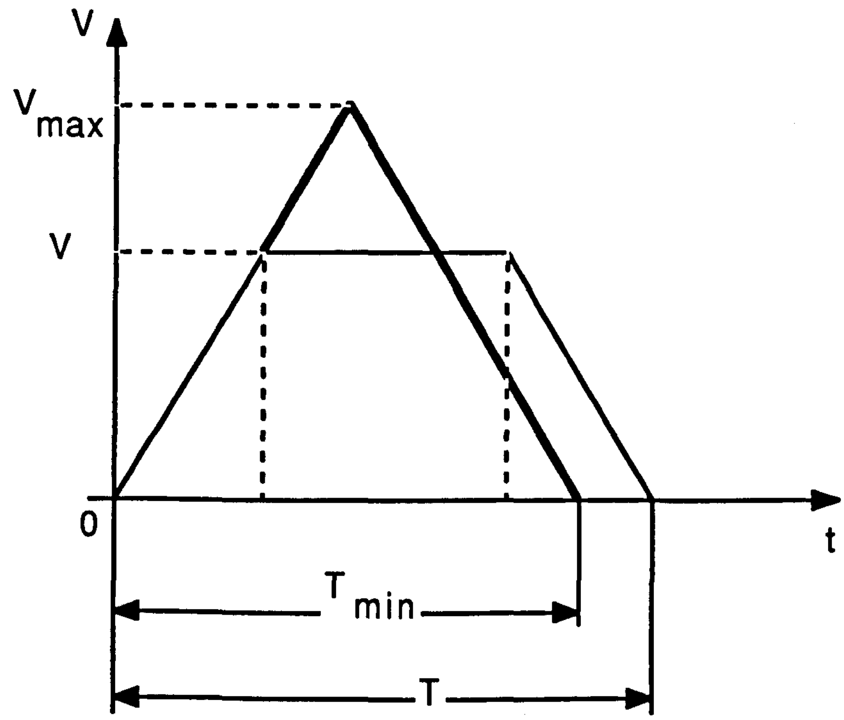


Fig. 3.2 Optimum velocity-time graph.

In other words the minimum time taken to travel between two addresses will be achieved if the stacker crane accelerates until the maximum speed is reached and after that decelerates for the second half of the journey.

It is observed from the last formula that V_{max} can be increased only if the value of the accelerations (decelerations) is increased. This imposes limitations however, since as it was noted earlier high accelerations cause large amplitudes of oscillation and long decay times which in turn decrease the throughput.

Another limiting factor for the horizontal accelerations is the condition for enough road friction in order to avoid slipping during the transient periods. It is for these reasons that in practice the maximum horizontal accelerations and

decelerations are kept below 0.6 m/s^2 (Zertzalov [1986]).
In semi-automated warehouses, where V_{max} is fixed, S represents the expected distance between two locations for single, dual or multi command cycles.
In warehouses with computer controlled stacker cranes V_{max} is calculated for each distance so the travel time is always kept to a minimum.
Another factor that affects order picking throughput is the accuracy of positioning of the stacker crane in front of the location. A creep speed (see fig 3.6) is used at the end of the braking period in order to compensate for the variations of the decelerations due to variations of the load, track resistance and the brake torques. In some semi-automated cases, a second (middle) speed is introduced in the horizontal direction in order to reduce a prolonged movement on creep speed when V_{max} is high, thus increasing machine throughput (Karaivanov [1984]).
The mechanical improvements of stacker cranes for increasing the order picking throughput are beyond the scope of this research. Instead optimal routing is pursued. The effect of the transient periods on the optimal multiple command order picking tour is investigated in the next section.

3.2 <u>The impact of accelerations and decelerations on the optimal tour</u>

As was noted in sections 1.4 and 2.2.3.3 finding the optimal multiple command order picking tour is an application of the

Travelling Salesman Problem (TSP).

All researchers (such as Shorn [1985], Bozer et al [1986] and Goetschalckx and Ratliff [1988]) that tried to adapt heuristics for finding solutions for the TSP when Tchebyshev norm is applied, assumed constant velocities of motion in horizontal and vertical direction. In this section the magnitude of the error involved in the optimal solution, when constant velocities are used, is derived. Before that, brief descriptions of some of the most efficient tour construction and improvement procedures (Bozer et al [1986] and Goetschalckx and Ratliff [1988]) that have been investigated for multiple command order picking are presented.

3.2.1 Band Heuristic

The rack is divided symmetrically into two halves (bands or layers) along its length, as shown on fig. 3.3.

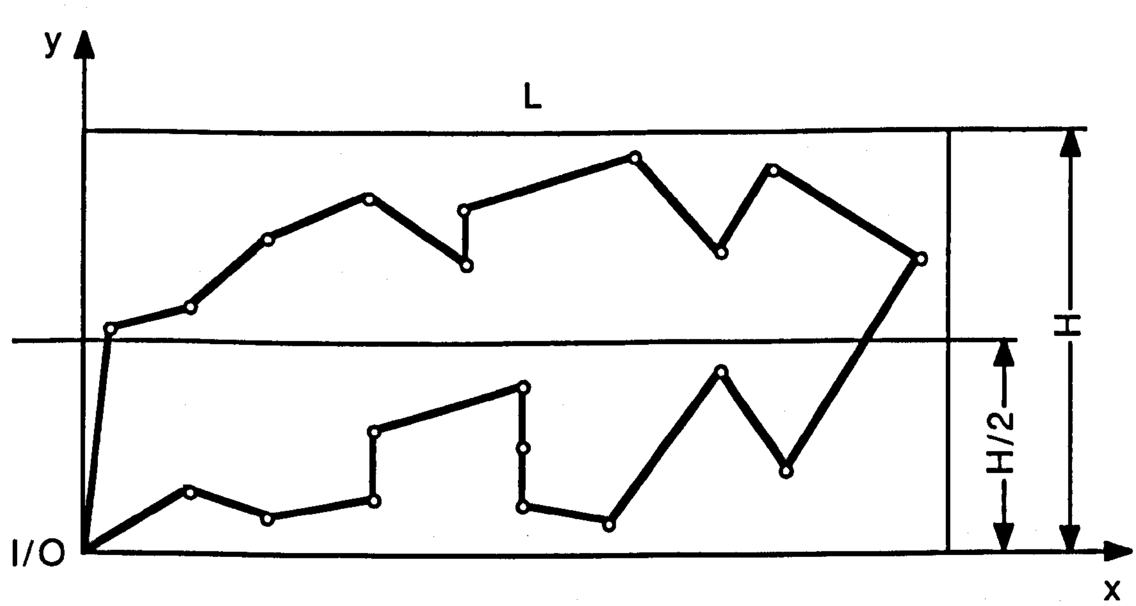


Fig. 3.3 Band heuristic

All addresses in the lower layer are visited in ascending order of their x coordinate, then the addresses in the upper layer are visited in descending order of their x coordinate, to form the tour.

A more detailed description of this algorithm is presented in Appendix A.

In practice when the rack is high, it is divided into a larger even number of bands, most often four. The optimal number of bands was investigated by Daganzo [1984] and Bozer [1985], but their investigation has a practical value only for the case when a large number of addresses are visited.

3.2.2 Chebhull heuristic

The Chebhull heuristic is an adaptation of the Convex Hull heuristic proposed by W.R.Stewart (see Golden et al [1980] and Allison and Noga [1984]) when Tchebyshev norm is used.

Chebhull was first proposed by Goetschlackx [1983, 1985] and it includes one extra step called "Free insertion" or "Optimal insertion" phase, which is described below. The sequence of the main steps included in the Chebhull algorithm are:

Step 1. The initial sub tour is obtained by linking the addresses with extreme coordinates (fig. 3.4-a).

Step 2. The convex hull of all addresses is constructed using an algorithm proposed by Akl and Toussaint [1978] (fig. 3.4-b).

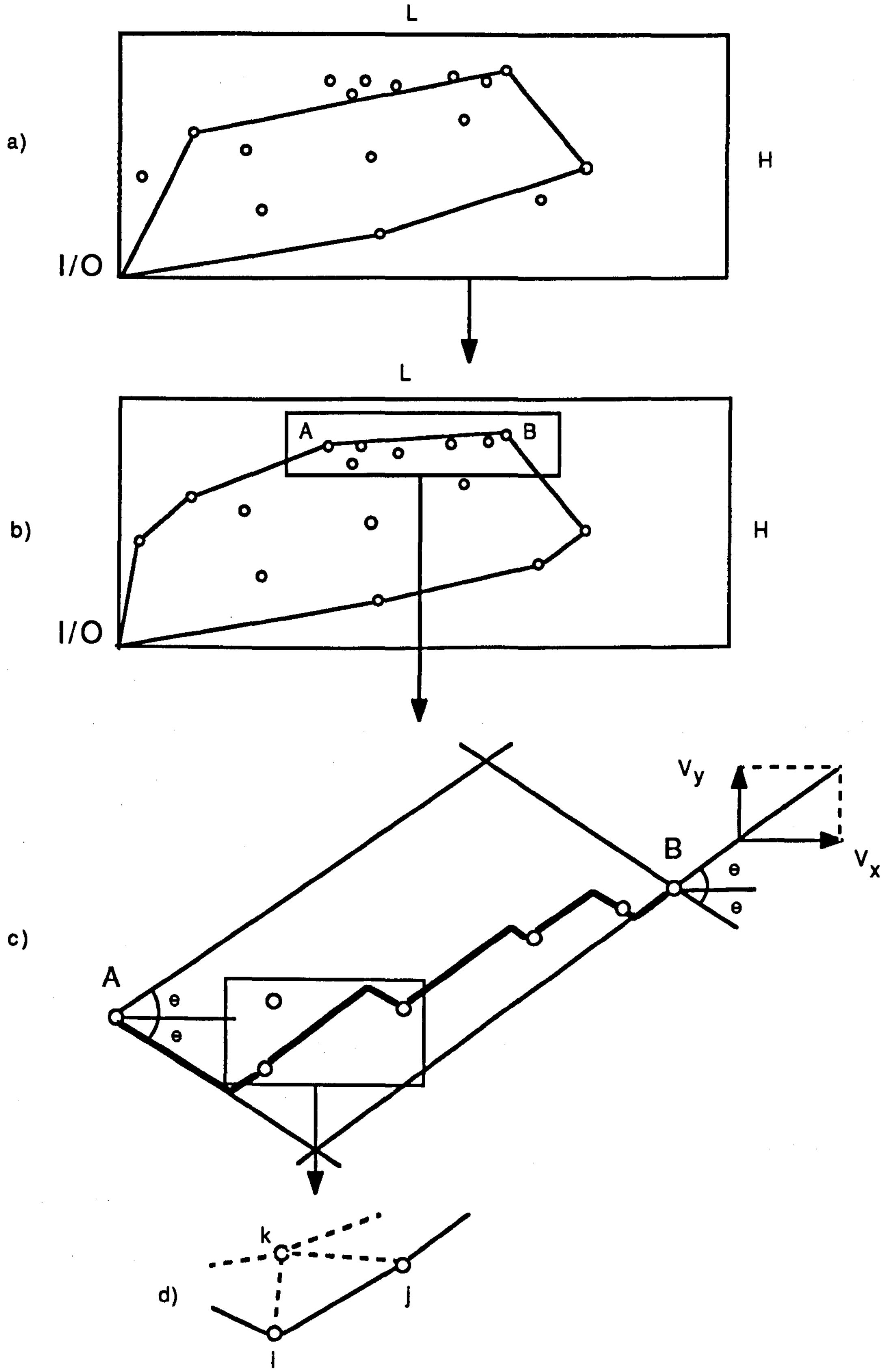


Fig. 3.4 Main steps of Chebhull algorithm

Step 3. This is the Free (Optimum) insertion phase proposed by Goetschalckx [1983,1985]. The purpose is to insert as many addresses as possible between two consecutive points on the convex hull without increasing the travel time between the points. Let e = arctan(V_y/V_x). Then any point in the parallelogram constructed by using the two consecutive convex hull points A and B and the angle e, as shown on fig.3.4-c, which lies on trajectories connecting A and B, parallel to the sides of the parallelogram, can be inserted in the tour without increasing the travel time. This is because trajectories connecting A and B which are parallel to the sides of the parallelogram represent the combined motion, resultant from the simultaneous work of the mechanisms for horizontal and vertical travel. Therefore the time to travel from A to B obeys Tchebychev travel.

To insert as many points as possible without increasing the travel time requires determining the longest path between points A and B. This problem is solved here by applying a recursive procedure "long_path" (see Appendix A).

Step 4. In this step (see fig.3.3-d) each of the remaining points are inserted one at time between two consecutive points on the partial tour in such a way as to minimise the total length of the tour constructed so far (see Golden et al [1980] and Allison and Noga [1984]).

3.2.3 Two optimal tour improvement procedure

This procedure is an implementation of the two optimal local search algorithm developed by Lin [1965] (see also Lin and Kernighan [1973] and Syslo et al [1983]).

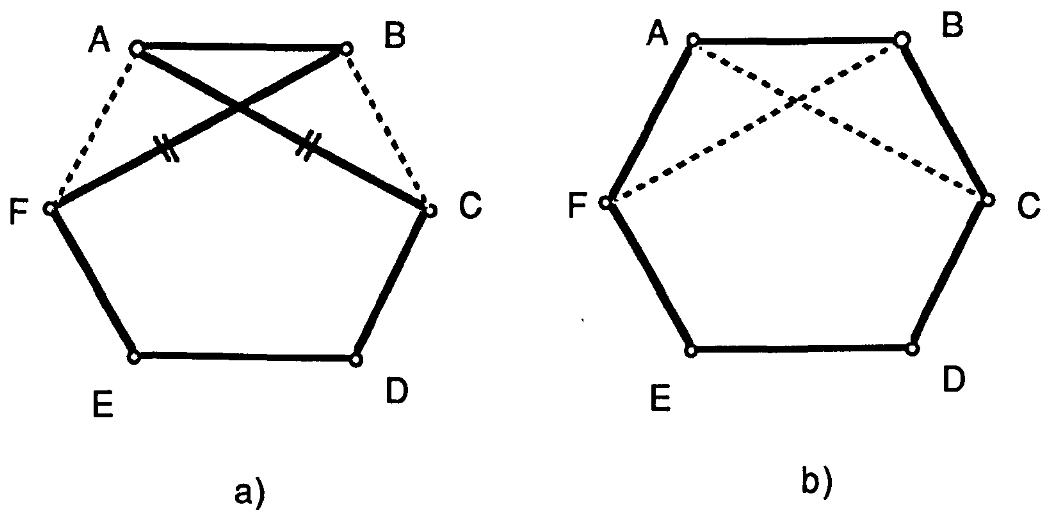


Fig. 3.5 Two optimal exchange

The procedure operates on an initial tour. Two links of this initial tour are replaced by two other links that have not yet been included in the tour as shown on fig 3.5-a,b. If the new tour is better (shorter), it is stored. The procedure terminates at a local optimum when it is not possible to improve the tour by further link exchange.

3.2.4 Magnitude of the error when constant velocities are used

Let us assume a stacker crane travelling between two addresses i and j. Typical velocity profiles are shown on fig. 3.6, where the bold lines show the case of travel between two addresses, taken as an example. If the accelerations and decelerations are neglected, then the machine is assumed to travel with constant (in this case maximum) velocities, V_{xmax} and V_{ymax} in horizontal and vertical direction respectively.

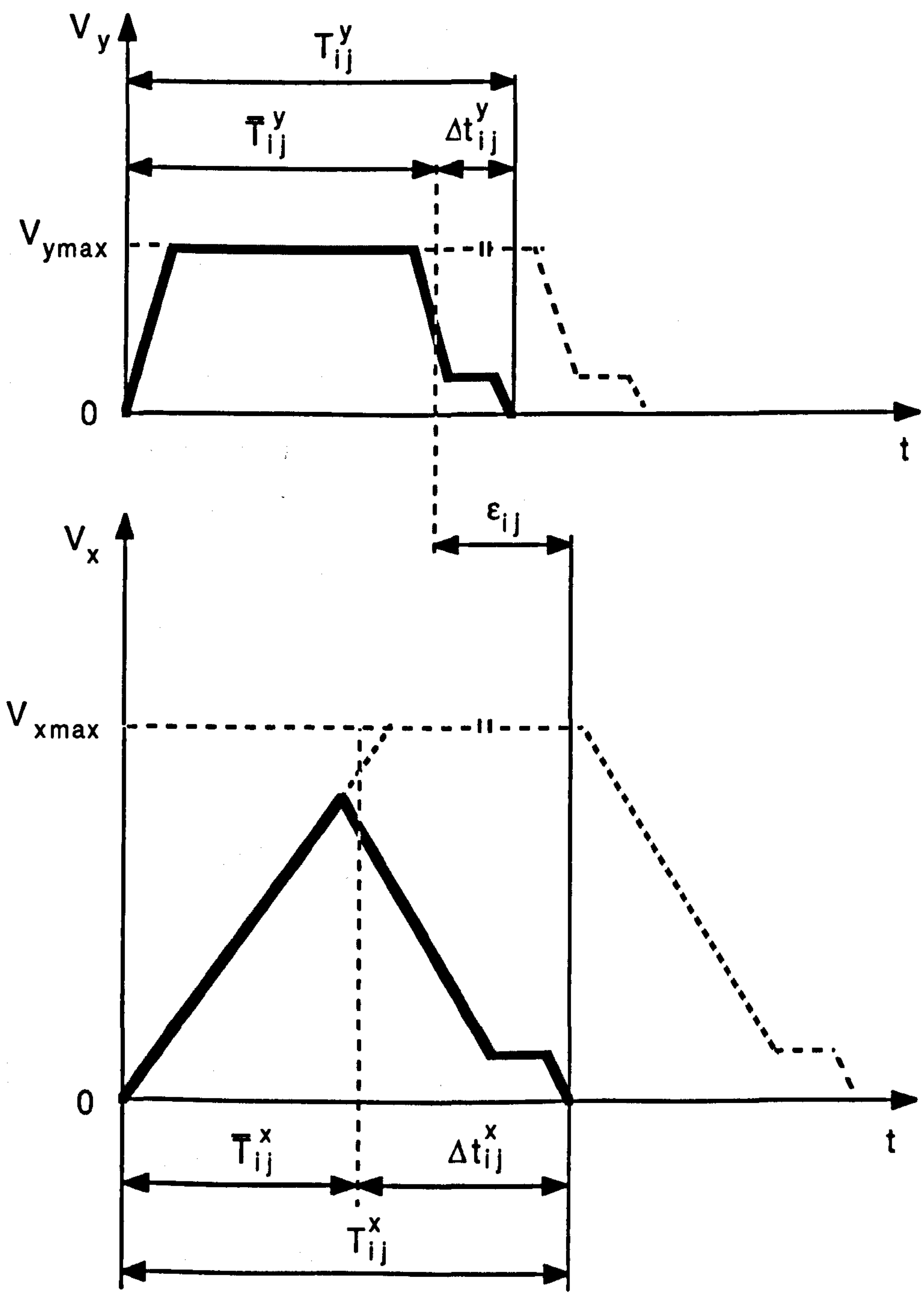


Fig. 3.6 Typical velocity profiles

Assuming constant velocities when calculating travel time between two addresses, the magnitude of the error introduced in the horizontal direction is:

$$\Delta t_{ij}^{x} = \left\| \left| \int_{x_i}^{x_j} \frac{dx}{V_x(x)} \right| - \frac{|x_j - x_i|}{V_x} \right\| , \qquad (3.1)$$

where $V_x(x)$ is the velocity in horizontal direction as a function of distance, V_x is the assumed constant velocity in horizontal direction and x_i and x_j are the horizontal coordinates of addresses i and j respectively.

In expression (3.1) the integral part is the time when real velocity is used and the part that is subtracted is the time when constant velocity is used.

Similarly for vertical direction:

$$\Delta t_{ij}^{y} = \left\| \left| \int_{y_i}^{y_j} \frac{dy}{V_y(y)} \right| - \frac{|y_j - y_i|}{V_y} \right\| . \qquad (3.2)$$

Using the notation in fig. 3.6 equation (3.1) becomes:

$$\Delta t_{ij}^{x} = \left| T_{ij}^{x} - \overline{T}_{ij}^{x} \right| \qquad (3.3)$$

and equation (3.2) becomes :

$$\Delta t_{ij}^{y} = \left| T_{ij}^{y} - \overline{T}_{ij}^{y} \right| . \qquad (3.4)$$

The smaller the distance between the addresses the larger the relative error because the acceleration and deceleration periods are larger proportions of the total travel time.
Furthermore there are no standards or recommendations for determining the constant velocities for multiple command order picking. For example the FEM 9.851 [1978] standard uses maximum velocities when calculating single and dual command cycle times; others use average velocities.
In general average velocities are statistical values dependent on the maximum velocities, accelerations (decelerations), average number of picks, rack shape (dimensions) and are unique for each warehouse.
The absolute error involved in calculating the travel time between addresses i and j when constant velocities are used is now estimated when the Tchebyshev norm is applied. This is done by comparing the travel times obtained for constant and real velocities respectively.
When constant velocities are considered, the travel time between addresses i and j is:

$$\dot{t}_{ij} = \max\left\{ \overline{T}^x_{ij}, \overline{T}^y_{ij} \right\} , \tag{3.5}$$

and when the real travel times in horizontal and vertical direction are considered:

$$t_{ij} = \max\left\{ T^x_{ij}, T^y_{ij} \right\} . \tag{3.6}$$

In practice however there will be cases (see also fig. 3.6) when:

$$\overline{T}^y_{ij} > \overline{T}^x_{ij} \text{ while } T^y_{ij} < T^x_{ij} .$$

The absolute error if link ij is included in the tour will then be:

$$\varepsilon_{ij} = \left| T^x_{ij} - \bar{T}^y_{ij} \right| ; \quad \varepsilon_{ij} >= o. \tag{3.7}$$

In order to find the magnitude of the total error for the whole order picking tour, consider the assignment problem formulated for the lower bound of the Travelling Salesman Problem (see for example Christofides [1975]).

The linear assignment problem for a graph with a cost (travel time) matrix $C = [c_{ij}]$, when constant velocities are used is stated here as:

Find 0-1 variables ω_{ij} so as to minimise:

$$\bar{z} = \sum_{j=1}^{n} \sum_{i=1}^{n} c_{ij}\omega_{ij}, \tag{3.8}$$

subject to

$$\sum_{i} \omega_{ij} = \sum_{j} \omega_{ij} = 1, \text{ for all i and j} = 1, 2, ..., n; \tag{3.9}$$

and

$$\omega_{ij} = 0 \text{ or } 1. \tag{3.10}$$

Equations (3.8)-(3.10) together with the additional constraints that the solution must form a Hamiltonian circuit represent a formulation of the TSP.

In equation (3.10) $\omega_{ij} = 1$ if link ij is included in the tour, otherwise

$\omega_{ij} = 0$. In the above formulation it is assumed that $c_{ii} = \infty$ as an additional constraint removing the possibility of circuits of cardinality 1 from appearing in the solution of the assignment problem.

The total travelling time of the tour will then be:

$$z^* = \sum_{j=1}^{n}\sum_{i=1}^{n}(c_{ij} + \varepsilon_{ij})\omega_{ij}, \tag{3.11}$$

constraints as above.

Thus the magnitude of the total error is:

$$\Delta z = \left| z^* - \bar{z} \right| . \tag{3.12}$$

All heuristic procedures which utilise construction or insertion procedures based on the Tchebyshev norm will incur errors due to the reasons outlined above. Correction for this error especially when the tour consists of a large number of addresses might have important ramifications for crane utilisation and order sequencing.

The magnitude of this error and its effect on the order picking tours is explored in the next section by using computer simulation in a designed experiment.

3.3 Empirical findings

A designed factorial experiment was conducted in order to estimate the value of the error derived in section 3.2.4. In this section a description and results of the experiment are presented. An important factor used in the experiment is discussed first.

3.3.1 Rack shape factor or velocity vector

The shape factor was introduced by Bozer and White [1984] for a continuous representation of a rack which is not necessarily square in time. It was defined as follows:

$$b = \min\left\{ \frac{T_x}{T_y}, \frac{T_y}{T_x} \right\},$$

where:

$$T_x = \frac{L}{V_x} \text{ and } T_y = \frac{H}{V_y}.$$

In the above expressions L is the rack length, H is the rack height, V_x and V_y are the velocities in the horizontal and the vertical directions respectively. It is obvious that:

$$0 <= b <= 1,$$

and when b = 1, the rack is said to be square in time.
On the other hand the parameter:

$$a = \frac{H}{L} \cdot \frac{V_x}{V_y},$$

also called the velocity vector was defined by FEM 9.851 [1978] and represents the shape of the rack and the direction of the absolute velocity when the stacker crane's shuttle moves simultaneously in horizontal and vertical directions.

When a = b = 1, (see fig. 3.7) the direction of the absolute velocity coincides with the diagonal of the rack and in this case the rack is square in time.

The values of the velocity vector, recommended by FEM 9.851 [1978], which give realistic rack shapes are for "a" lying between 0.5 and 2.0.

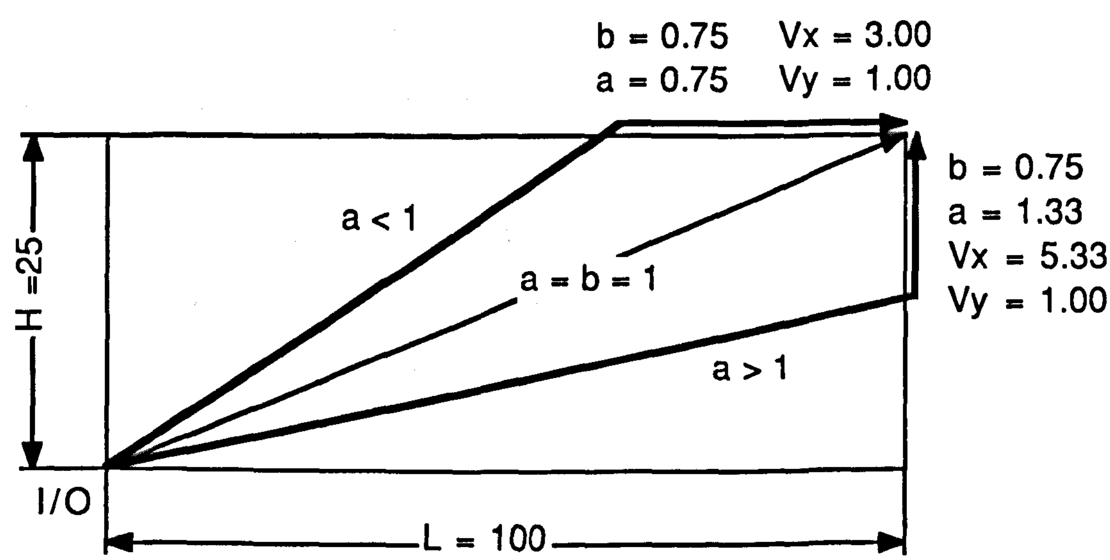


Fig. 3.7 Difference between rack shape factor and velocity vector

The rack shape factor (b) was initially used by Bozer and White [1984] when they derived the expected travel times for single and dual command cycles (see 2.2.3.1 and 2.2.3.2). Later it was used by Shorn [1985], Goetschalckx [1985, 1988] and Bozer et al [1986] when tour construction heuristics for multiple command order picking were tested. The rack shape factor was used, altering rack dimensions while keeping the ratio of the velocities constant.

It is should be noted here that a single value of the shape factor can correspond to two different racks in terms of Tchebyshev travel norm. For example if two racks with the same dimensions are considered (see fig. 3.7), say L = 100 units and H = 25 units and V_x/V_y = 1/3 for the first rack, and V_x/V_y = 1/5.33 for the second one, then both racks have the same shape factor b = 0.75. However, the difference between the corresponding horizontal velocities is more than 43%. Therefore heuristics that apply the Tchebyshev norm in their construction and insertion procedures may produce different results for the "same" shape factor.

In order to avoid confusion, the velocity vector "a" is preferred in the following simulation experiment, as a factor which represents the rack shape.

3.3.2 Description of the experiment

The aim of this experiment is to determine the magnitude of the error when multiple command order picking tours are constructed when it is assumed that the stacker crane's shuttle travels with constant velocities in horizontal and vertical directions respectively.

A full factorial experimental design (see Montgomery [1984]) was implemented, with factors and levels as shown in table 3.1

Table 3.1 Factors and levels in the factorial experiment

No	Factor	Number of levels	Values (Notations)
1.	picks per cycle	6	10, 15, 20, 25, 30, 35
2.	heuristic (algorithm)	5	Band, Cheb, Band_Two, Cheb_Two, BABTSP
3.	velocity vector	4	0.5, 1.0, 1.5, 2.0
4.	velocity type	2	Const. (max), Real

The notations in table 3.1 for factor No.2 represent the following methods:

Band- Band heuristic;

Band_Two- Band heuristic plus two optimal improvement procedure;

Cheb- Chebhull heuristic;

Cheb_Two- Chebhull heuristic plus two optimal improvement procedure;

BABTSP- Branch and Bound algorithm for solving the TSP.

The above heuristics were chosen for this experiment since, as was noted in section 3.2 they are amongst the most efficient tour construction and improvement procedures that have been investigated for multiple command order picking assuming constant speeds for the stacker crane.

Accuracy of each of the heuristics is compared against the results obtained from an exact method for solving the TSP. The Branch and Bound method for solving the TSP (see Syslo et al [1983]) was used. It is based on Little's et al [1963] algorithm, where the lower bounds of the tour are obtained by reducing the cost (time) matrix. This method was chosen because it is relatively easy to convert into computer code and it performs well (see Little et al [1963]) for both symmetric and asymmetric Travelling Salesman problems.
Here it should be noted for the sake of accuracy that the TSP in multi command order picking is asymmetric since during the tour, the added loads change travel times during acceleration and deceleration periods. However, since the mass of the stacker crane is significantly large, compared to that of the load, the effect is considered to be negligible for the purpose of this experiment. Therefore the symmetric TSP will be used.

The experiment was conducted assuming a rack length of 35 units. Thus by varying the velocity vector, rack heights were kept in the range of 5 to 20 units.
Addresses were randomly generated from a uniform distribution, as random storage policy was considered.
A manufacturer's data was used for the separate travel times in horizontal and vertical directions. Data was provided for machines with maximum velocities, $V_{xmax} = 63$ m/min, $V_{ymax} = 18$ m/min, vertical acceleration of 0.5 m/s^2 and horizontal acceleration of 0.25 m/s^2. Accelerations are assumed equal to decelerations.

The software was designed in a way that algorithms were first applied with the time matrix based on the maximum horizontal and vertical velocities. Then the routes obtained for the constant (maximum) velocities were recalculated using the real times, which were read from files. Finally the heuristics and the exact algorithm were applied with the time matrix constructed from real times. This allowed tour cost for each method to be obtained from a route calculated with constant (maximum) and real velocities respectively and the comparison to be based on real times.

Run time was not included as a factor in the experiment, since it was known from Bozer et al [1986] and Goetschalckx and Ratliff [1988] to be fairly small for the heuristics, but was reported to indicate possible differences when real velocities were used.

All heuristics and the exact algorithm were coded by the same person (the author) in C programming language and ran on a number of IBM PS/2 computers.
The suite of the C programs developed are described in Appendix A. In this appendix structured charts are presented, pseudo code is given for some of the modules and appropriate references for the others.
During the development of the software extensive testing was carried out. The statements within each module and the module itself was tested by comparing output against test input. Finally the complete software was tested using manually

constructed examples in which the solutions were known.
These manually constructed test examples were formed in a way to ensure that all the relevant paths were examined.
For the time consuming Branch and Bound algorithm, run time limits of twelve hours were set. If the run time was exceeded, the program terminates and was restarted with a different seed for the random generator.
Multiplying the levels for all factors included in the experiment gives 240 combinations which were replicated five times, thus giving 1200 different runs in total.
The results are presented in the next section.

3.3.3 Results from the experiment

Results were analysed with the aid of the GENSTAT [1980] statistical package. The analysis of variance indicated that all factors including time were significant as were interactions between factors other than time.
Summaries of the means for different levels are presented in tables 3.2 and 3.3.

As seen from tables 3.2 and 3.3, results from the experiment confirm the efficiency of Band_Two and Cheb_Two, when applied with real times. The means of the tour costs (times) for these heuristics are in the range of 3.5% above the optimal solutions produced by BABTSP.

It is shown that the larger the Tchebyshev norm contribution to tour construction, the larger the percentage error when constant (maximum) velocities are used. This is demonstrated on fig 3.8, where the percentage difference (error) of the tour costs obtained for constant and real velocities for Band_Two, Cheb_Two and BABTSP is given as a function of number of picks. It should be noted that fig. 3.8 illustrates the sensitivity of the presented algorithms to real velocities (time) and not how accurate they are.

One unexpected fact appeared after recalculation of the tour costs with real times. It turned out that in many cases the TSP tour is no longer better than the tour created by the best (Band_Two, Cheb_Two) heuristics. This is demonstrated by fig. 3.9 which shows the percentage difference of the recalculated tours from the real optimal TSP tour. In fact in 69 percent of the cases, the recalculated TSP tour cost turned out to be larger than the recalculated tours obtained from Band_Two or Cheb_Two heuristics.

Table 3.2. Tour time in seconds as a function of number of picks.

picks	Band		Band_Two		Cheb		Cheb_Two		BABTSP	
per cycle	const	real	const	real	const	real	const	real	const	real
10	168.52	168.52	146.41	146.51	148.17	147.41	146.72	145.95	147.50	145.95
15	210.75	210.75	179.90	179.45	181.89	180.04	180.36	179.03	180.68	178.80
20	263.24	263.24	218.40	218.40	226.77	221.93	220.66	216.73	221.66	216.26
25	314.03	314.03	250.38	250.38	258.24	252.38	253.57	248.67	254.07	247.07
30	365.86	365.86	286.93	286.93	293.99	286.70	291.94	284.04	290.33	277.63
35	415.05	415.05	314.54	314.54	324.77	314.33	322.60	311.20	321.56	307.99

Table 3.3. Tour time in seconds as a function of rack shape.

velocity vector	Band		Band_Two		Cheb		Cheb_Two		BABTSP	
	const	real	const	real	const	real	const	real	const	real
0.5 (5)	195.36	195.02	188.55	188.21	194.30	189.14	193.30	187.65	192.45	186.63
1.0 (10)	246.15	246.88	217.33	218.07	223.87	218.22	221.69	216.49	223.57	215.66
1.5 (15)	322.33	322.37	249.72	249.46	255.19	251.10	251.50	247.04	251.34	242.80
2.0 (20)	394.45	394.01	275.43	274.99	282.53	276.73	277.40	272.57	276.50	270.72

Note: Number in brackets represents rack height, obtained for the corresponding velocity vector.

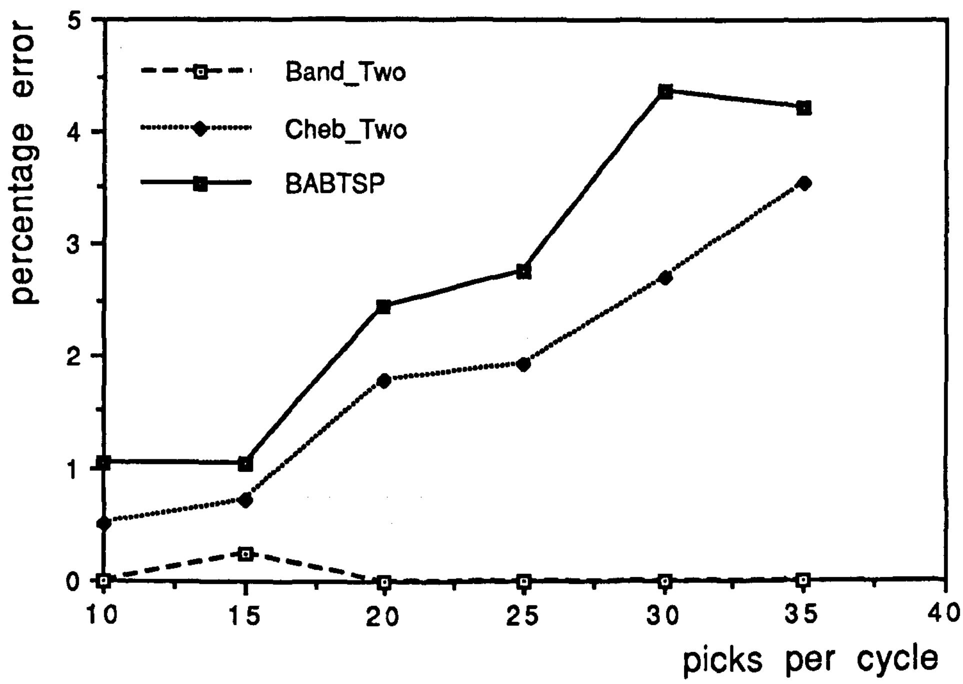


Fig. 3.8. Sensitivity of the algorithms to change from constant to real velocities.

Run times for the heuristics were in the range of 3 seconds for all factor combinations.
For 30 and 35 picks only around 25 to 20 per cent of the runs were within the time limits of twelve hours set for BABTSP. Fortunately, no correlation between run time and tour cost was found.

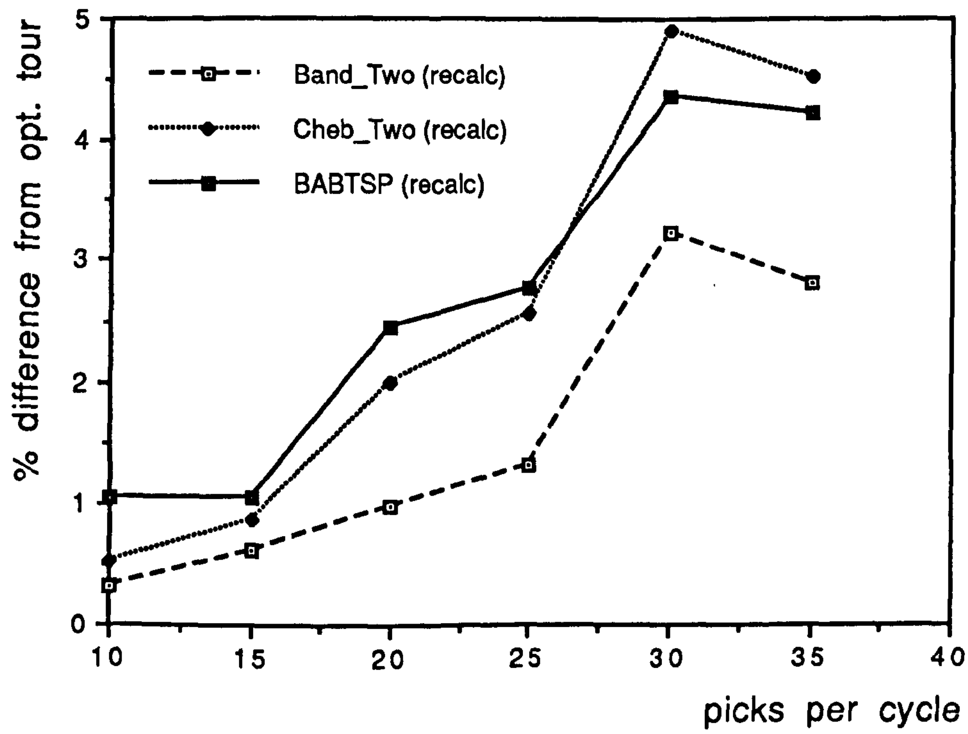


Fig. 3.9. Performance of the algorithms with constant velocities compared to the real optimal tour (BABTSP - real).

From tables 3.2 and 3.3 it can be observed that the Cheb heuristic creates a very good initial tour which explains the relatively small contribution of the two optimal procedure to the final quality of Cheb_Two. On the contrary, the two optimal procedure significantly improves the accuracy of Band_Two.

3.4 Summary and conclusions

In this chapter the effect of stacker crane performance on multiple command order picking was investigated.

The magnitude of the error, involved in constructing the optimal tour when accelerations and decelerations are neglected, was derived and its magnitude estimated from a factorial experiment for up to 35 picks.

The error is significant and increases in approximately a linear fashion and is above 3% when the average number of picks is more than 20 per cycle. This should be taken into consideration in practice, where large orders are common.

When recalculated with real times, the BABTSP tour obtained for constant velocities is often more costly than that of the more efficient heuristics. Therefore when the Tchebyshev norm with constant velocities is applied, the BABTSP tour cannot be considered optimal, no matter that the difference is within 5% up to 35 picks per cycle.

Band_Two and Cheb_Two were confirmed to be very efficient, both in terms of accuracy of the constructed tours and the speed of execution. Band_Two should be preferred because of its simplicity.

The experiment was conducted for only one set of horizontal and vertical velocities. Further research using other velocity sets is required since the velocity ratio is important, when the Tchebyshev norm is applied in tour construction procedures. Furthermore other velocity sets will provide ground for investigation of other rack dimensions, which in turn affect pick density.

The investigation so far was restricted to random storage. The interaction between picking and dedicated storage policy is investigated in the next chapter.

CHAPTER IV

INTERACTION BETWEEN MULTIPLE COMMAND ORDER PICKING AND CLASS BASED STORAGE POLICY

4.1 Introduction

Previous investigations into picking and storage policy have been confined to studies of either class based storage and single and dual command picking or random storage and multiple command order picking.

Since it has been demonstrated by Hausman et al [1976], that zoning of the picking face significantly reduces travel time, it is the purpose of this chapter to investigate, for the first time, the interaction between multiple command order picking and class based storage policy.

The associated work in this area began in the mid-seventies with introduction of class based storage and the concept of square in time rack (definitions of these were presented in Chapter I).

A square in time rack has been shown by Hausman et. al. [1976] and Bozer and White [1984] to be the optimal shape for single command order picking and random storage policie. Hausman et. al. also show, that this holds for turnover based dedicated storage and two and three class based storage policies. The L-shaped boundaries (see Chapter II) of the zones accommodating the corresponding classes are square in time too.

For dual command order picking it was found by Graves

et.al.[1977] that "square L" boundaries are near optimal. The authors claimed that the estimated round trip time was within 3% of the lower bound of the expected round trip time. However, the optimal boundary was not determined.
From simulation results Bozer[1985] claimed that a square in time rack minimises the expected length of the shortest path for multi command order picking when random storage policy is used. As the number of picks increases the tour length becomes less sensitive to the rack shape.

When modelling routes in multiple command In-the-Aisle order picking, the following assumptions were made (see also Chapter I):

1. The picking area is a two sided aisle. Since the time for loading and unloading is relatively small compared to travel time and does not affect picking sequence, it is ignored. Then all locations to be visited are projected over the plane of one of the racks.
2. The Input / Output point is located at the bottom left corner of the rack and each cycle starts and terminates at the I/O point.
3. The picking machine (stacker crane) moves simultaneously in horizontal and vertical directions, i.e. the Tchebyshev metric is applied.
4. Rack is square in time.
5. All storage locations are the same size.

Work in this chapter is divided as follows.

The expected value of the picking tour under the Tchebyshev norm is investigated in section 4.2. This is used as a basis for modelling the zone shapes that are expected to minimise the travel time of the picker. Formulation of the model and defining three types of zone shapes in combination with the Band heuristic plus two optimal improvement procedure are presented in section 4.3.

A factorial experiment was carried out to test the proposed heuristics. Description of the experiment and results from the computer simulation are presented in section 4.4, while conclusions are drawn in the last section.

4.2 The expected value of the picking tour travel time under Tchebyshev norm

4.2.1 Background

Attempts to obtain the expected travel distance among a number of random points (addresses) in a region began in the early forties.

The expected value, D of the shortest path among N points in a region of area, A was found by Ghosh [1948] to be:

$$D = 1.27\sqrt{A.N} .$$

The lower bound for this was found by Marks [1948] to be:

$$D = \sqrt{\frac{A}{2}} . \frac{N-1}{\sqrt{N}} .$$

The above formula gives the lower bound for the Open TSP, where the salesman starts at a point, visits each of the other points exactly once and finishes at the last visited point. The addresses in the journey are N-1, excluding the starting one. For N points (closed TSP) the same method would produce the lower bound:

$$D = 0.71\sqrt{A.N} \ .$$

Elion et al [1971] using an approach similar to that of Ghosh [1948] derived the general formula for the expected value of the TSP tour:

$$D = K.\sqrt{A.N}, \text{ as } N \to \infty. \qquad (4.1)$$

For the Euclidean metric, they obtained, by using simulation, K = 0.75.

Daganzo [1984] showed analytically for a rectangular area of length, L and width, l, that $K = f(\delta l^2)$, for both Euclidean and Rectilinear metrics, where δ is the density of points per unit area. The coefficient K reaches its minimum of 0.9 when $\delta l^2 = 12$. However, the formula over predicts the expected length of the shortest tour as it was derived using a suboptimal tour building strategy (Band heuristic).

Elsayed and Unal [1989] investigated the expected travel time for multi command order picking and random storage.

The derivation of it is a straight generalisation of the work done by Bozer and White [1984] (see chapter II) for single and dual command cycles.

The expected time is obtained as a function of the rack shape factor, b and number of picks, n:

$$E(T) = \frac{b^2}{3} + 1 + (n-1)\left\{ n(n-1)b^n\left[\frac{2n}{2n-1} - \frac{(b+1)(2n-1)}{2n} - \frac{1}{n} + b(\frac{2n-2}{2n+1} + \frac{1}{n+1})\right] + \frac{n-1}{n+1} \right\}$$

The authors claim that results produced by the formula are within the range of the results produced by computer simulation using random generation of addresses and Little's algorithm for solving the TSP.

However, only one set of rack dimensions and one set of horizontal and vertical velocities are used. This poses the same problem discussed in section 3.3.1 that one shape factor does not represent a unique combination of rack dimensions and velocitiy ratios. Therefore for two racks with the same shape factor but different velocity ratios the above formula will produce the same result. This is in contradiction with the role of the Tchebyshev norm in constructing the tour.

4.2.2 <u>One directional trip in a strip and the Tchebychev norm</u>

To determine the optimum width of a strip is the basic idea behind the Band heuristic (see 3.2.1), where the area (rectangle) is divided into m strips and points (addresses) are linked in a tour by traversing the strips in a serpentine pattern.

Daganzo [1984] derived the optimal strip width for Euclidean and Rectilinear metrics to be:

$$w^* = \sqrt{3/\delta} \ , \qquad (4.2)$$

where δ is density of points per unit area.

To follow the same procedure for the Tchebychev metric, consider an infinitely long strip with width w as shown on fig. 4.1. Dimensions and distances will be expressed in time units.

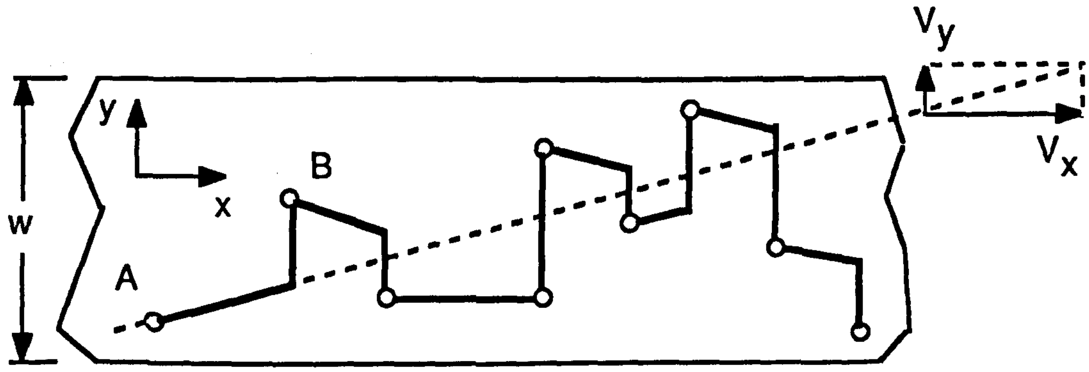


Fig. 4.1 Tchebyshev travel in a single strip

Points are scattered randomly, independently and uniformly in the strip. Density, δ is number of points per unit area (area being measured in squared time units).

Because of the Tchebyshev metric, (fig. 4.1) travel time between two points A and B would be the larger of the travel times along the horizontal and the vertical direction i.e.:

$$t_{AB} = \max \{t_{AB(x)} , t_{AB(y)}\} .$$

Let Y be the random distance between two consecutive points across the width of the strip. Then Daganzo [1984] states:

$$\Pr\{Y > y\} = (1 - y/w)^2, \qquad 0 <= y <= w . \tag{4.3}$$

The expected distance across the vertical direction is :

$$E(Y) = w/3 \quad . \tag{4.4}$$

This is because the distribution of distance across the width of the strip is the same as the distance between two random points on a line segment of length w.
If X is the random distance between two consecutive points along the length of the strip:

$$Pr\{X > x\} = e^{-\delta wx} , \qquad x >= 0 . \tag{4.5}$$

This is because of the Poisson process locally formed by the positions of the independently, randomly and uniformly scattered points on the strip (see Daganzo [1984]). Then the expected value of the horizontal distance between two consecutive points is :

$$E(X) = 1/\delta w . \tag{4.6}$$

The expected total length of a path consisting of N points is :

$$D_W = N.d_W , \tag{4.7}$$

where d_W is the expected distance between two consecutive points.
For the Tchebyshev travel (ignoring the stops) the expected minimum of d_W (D_W respectively) will be reached, if points are visited with maximum utilization of the simultaneous movement in both horizontal and vertical directions. This will

be achieved if the condition:

$$E(X) = E(Y), \tag{4.8}$$

is satisfied for any two consecutive points in the tour. Then from (4.4) and (4.6) the optimal width of the strip for the Tchebyshev metric is found to be :

$$w^* = \sqrt{3/\delta} \tag{4.9}$$

Note that this relation has the same appearance as (4.2), the formula obtained by Daganzo [1984] for Euclidean and rectilinear metric. For the Tchebychev metric, Bozer[1985] obtained by simulation:

$$w^* = \sqrt{3.024/\delta} .$$

Again from (4.4) and the condition $E(x) = E(y) = d_w$, one obtains:

$$d_w = w^*/3 \ . \tag{4.10}$$

For a rectangular zone of 'A' square time units, point density $\delta = N/A$, and width w, substituting (4.9) in (4.10) and then (4.10) in (4.7), one obtains:

$$D_w = \frac{\sqrt{3}}{3}\sqrt{A.N} \sim 0.58.\sqrt{A.N} \tag{4.11}$$

which is the expected tour length in the strip.

However, for a rectangular area which consists of several strips of width w^*, equation (4.11) would not be a lower bound for the expected tour length since Band heuristic, as was noted above is a suboptimal strategy. The same applies if the area consists of not a whole number of strips or its width is less than w^*. Fortunately, zones which accommodate high turnover classes in class based storage are relatively small compared to the entire rack area, so they can easily be modelled as single strips within the rack.

4.3 Formulation of the model

In this section the shapes of the zones of the fast mover classes are modelled as two parallel adjacent strips, similar to the methodology adopted by Daganzo [1984].

4.3.1 One directional travel along two parallel, adjacent strips

Classes in class based storage are based on product turnover and reflect the ABC phenomenon for inventory. The ABC curve (see 2.2.3.1) is a function of cumulative percentage of demand versus cumulative percentage of inventoried products.
In this research ABC classes are represented by the notation A(a) / B(b) / C(c), which means that 'A' percentage of the total demand comprises of 'a' percentage of total inventoried products. A, B and C stands for the corresponding classes in descending order of their turnover, and A+B+C = a+b+c = 100.

For example 80(10) / 15(20) / 5(70) means that for class A, 80 percent of the total demand is represented by 10 percent of all inventoried items. It is clear that for a three class storage system, which is considered here, the notation of only A and B classes defines the system.

Since extremes of the rack is the boundary of the C class (zone), attention will be concentrated on modelling the "fast mover" classes A and B where the major part of the tour is conducted.

Consider a trip along two adjacent parallel strips of infinite length with points randomly uniformly and independently scattered in each strip. One of the strips has width w_1 and the other w_2 (fig. 4.2).

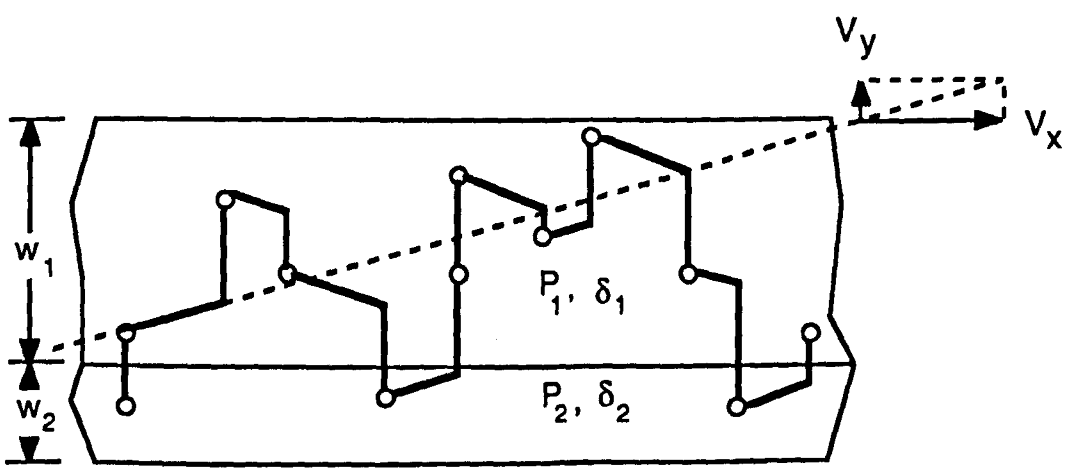


Fig. 4.2 The two strip model

The densities of points in the strips are δ_1 and δ_2. P_1 and P_2 are the probabilities that a point in the tour is in the first or in the second strip respectively.

The expected distance between two consecutive points in the horizontal direction follows directly from equations (4.5) and

(4.6):

$$E(X) = 1/(\delta_1 w_1 + \delta_2 w_2) \qquad (4.12)$$

The expected distance between two consecutive points in the vertical direction is now derived.

The distribution of distance between two points in the vertical direction for the two parallel adjacent strips is the same as the distribution of distance between two random points on two adjacent line segments of lengths w_1 and w_2, lying on the same line.

If P_1 and P_2 are the probabilities as defined above, the joint probability density function of a pair of random points on the line segments is:

$$f(y_1,y_2) = \begin{cases} P_1^2/w_1^2 & 0 <= y_1 <= w_1, \; 0 <= y_2 <= w_1 \quad (\mathrm{I}) \\ P_1P_2/w_1w_2 & w_1 <= y_1 <= w_1+w_2, \; 0 <= y_2 <= w_1 \quad (\mathrm{IV}) \\ P_1P_2/w_1w_2 & 0 <= y_1 <= w_1, \; w_1 <= y_2 <= w_1+w_2 \quad (\mathrm{II}) \\ P_2^2/w_2^2 & w_1 <= y_1 <= w_1+w_2, \; w_1 <= y_2 <= w_1+w_2 \quad (\mathrm{III}) \end{cases} \qquad (4.13)$$

Let $G(y)$ denote the probability that the distance between the two points $|y_1 - y_2|$ is less than or equal to y i.e.:

$$G(y) = Pr| y_1 - y_2| <= y .$$

Since $f(y_1,y_2)$ is a discrete function, $G(y)$ is obtained by graphical integration of equation (4.13) (regions I - IV in fig. 4.3) to be:

$$G(y) = [(2w_1 - y)y.P_1^2/w_1^2 + y^2.P_1P_2/w_1w_2 + (2w_2 - y)y.P_2^2/w_2^2] +$$

$$+ [(y - w_1).2P_1P_2/w_2 + (2w_2-w_1-y)(y-w_1).P_2^2/w_2^2] +$$

$$+ (2w_1 + w_2 - y)(y - w_2).P_1P_2/w_1w_2 .$$

The joint probability density function $g(y)$ has the form:

$$g(y) = g_1(y) + g_2(y) + g_3(y) ,$$

where :

$$g_1(y) = (w_1 - y).2P_1^2/w_1^2 + y.2P_1P_2/w_1w_2 + (w_1 - y).2P_2^2/w_2^2$$

$$g_2(y) = 2P_1P_2/w_2 + (w_2 - y).2P_2^2/w_2^2$$

$$g_3(y) = (w_1 + w_2 - y).2P_1P_2/w_1w_2$$

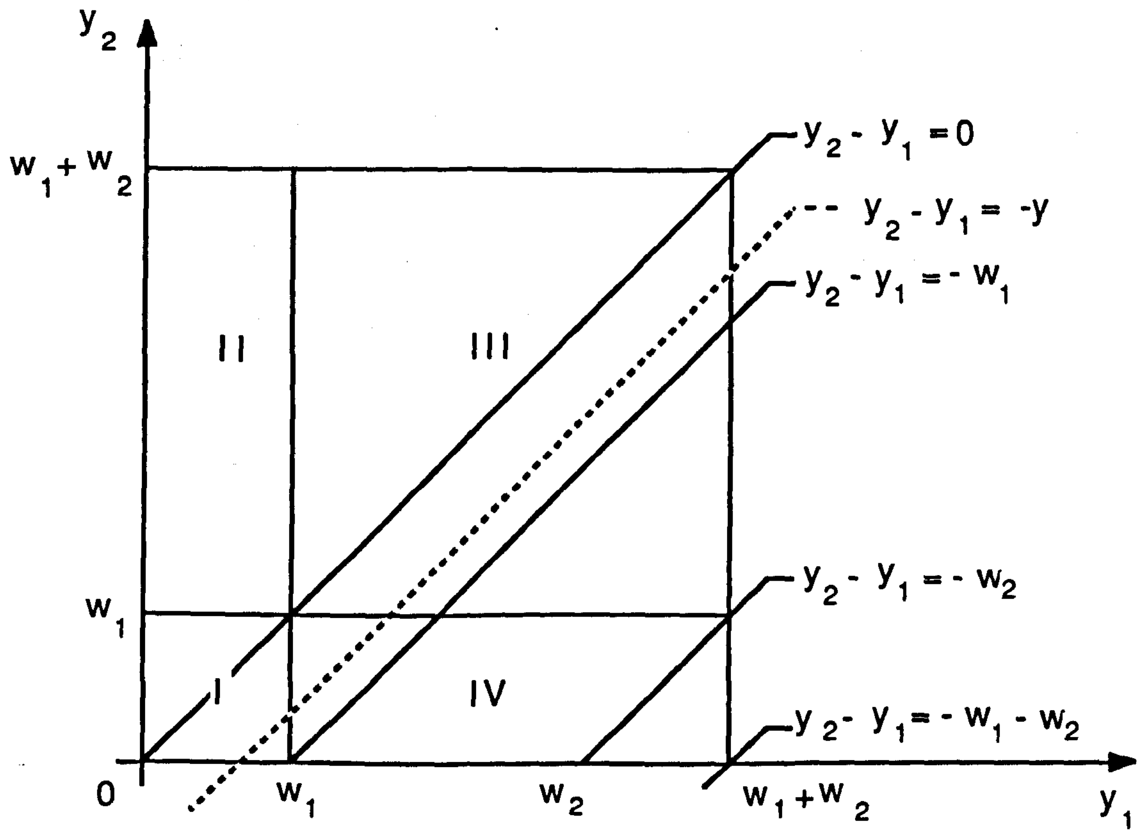


Fig. 4.3. The four regions in which $| y_1 - y_2 | <= y$.

Thus the expected distance E(Y) in the vertical direction is then:

$$E(Y) = \int_0^{w_1} g_1(y) y \, dy + \int_{w_1}^{w_2} g_2(y) y \, dy + \int_{w_2}^{w_1+w_2} g_3(y) y \, dy \quad ,$$

and after the integration:

$$E(Y) = P_1^2 w_1/3 + P_1 P_2 (w_1 + w_2) + P_2^2 w_2/3 \ . \tag{4.14}$$

Assume that strip 1 represents zone A and strip 2, represents zone B and the corresponding optimal widths w_1 and w_2 are

obtained separately for each strip by equation (4.9).
In order to minimise the expected distance between two consecutive points for both strips combined, the condition of equation (4.8) (E(X) = E(Y)) should again be applied . Since the density in the first strip is much higher than in the second, one may set equation (4.12) equal to (4.14), setting w_1 as a parameter for the time being. The obtained quadratic equation is solved for w_2 and has the form:

$$pw_2^2 + qw_2 + r = 0 \tag{4.15}$$

where:

$$p = (P_2^2 + 3P_1P_2)\delta_2$$

$$q = (P_1^2 + 3P_1P_2)\delta_2 w_1 + (P_2^2 + 3P_1P_2)\delta_1 w_1$$

$$r = (P_1^2 + 3P_1P_2)\delta_1 w_1^2 - 3$$

The equation has one real root w_2^* , when $r < 0$ i.e :

$$w_1^* < \sqrt{3/(P_1^2 + 3P_1P_2)\delta_1} \ ,$$

and using equation (4.9) :

$$w_1^* < w_1 / \sqrt{P_1^2 + 3P_1P_2} \qquad (4.16)$$

Since P_1 and P_2 correspond to the percentages of total demand for class A and class B, with class (zone) C ignored, then for the practical range of ABC curves one can write $0.7 <= P_1 <= 0.8$ and $P_1 + P_2 = 1$ and assuming equation (4.16), w_1^* should be between 5 and 6% less than w_1 in order to obtain the real root w_2^*.

In practice w_2^* would be obtained by setting w_1^* approximately to $(2/3)w_1$. This has little effect on the accuracy of the model, since in reality the rack is a discrete (cellular) structure, requiring w_1^* and w_2^* to be integers. Rounding up w_1^* makes the approximation acceptable.

Also Daganzo [1984] noted that if the strip width differs from the optimal one by less than 20%, the expected distance is changed by less than 2%.

These results will now be applied when different zone shapes are investigated.

4.3.2 Zone shapes and Band heuristic

Three zone configurations are examined in this investigation. Their design is based on the application of the Band heuristic plus two optimal improvement procedure as picking policy and the optimal strip widths w_1^* and w_2^* as obtained in section

4.3.1.

The first configuration called Band1(fig. 4.4-a) keeps the L shaped, square in time boundaries of zone A and zone B. The dividing line for the Band heuristic is set at half of the height of zone B. This is done, because the percentage turnover of A, B and C zones corresponds to the percentage turnover as defined by the ABC curves.

The same applies for the percentage of inventoried items. Thus the boundary of zone B confines the majority (90-95%) of the locations to be visited and therefore in many cases zone B will resemble a rack (within the actual rack) in which (almost) the entire order picking is done.

If for example, we assume that for a square in time rack with height H (fig.4.4-a), zone A contains 10% of the stored items (therefore 1/10 of the rack area assuming one unit in each location) and that the zone itself is square in time, its height will then be:

$$\frac{H}{\sqrt{10}}.$$

Assuming that zone B contains 20% of the inventoried items plus the 10% of the incorporated zone A, gives approximately 1/3 of the rack area being confined by zone B. Again considering that zone B is square in time, for its height one obtains:

$$\frac{H}{\sqrt{3}}.$$

The dividing line is set therefore at half of this height (fig. 4.4-a) from the I/O point in the vertical direction.

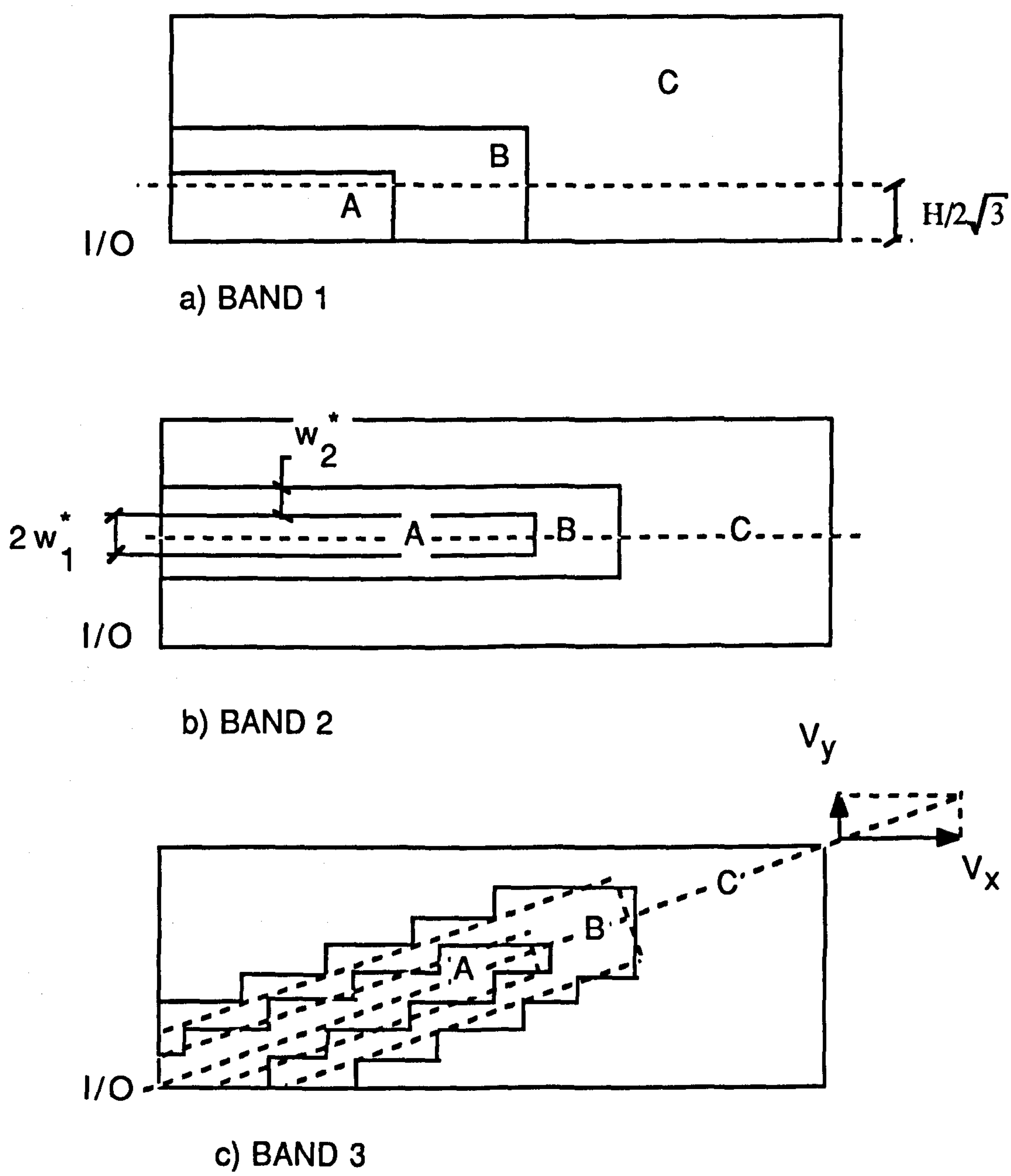


Fig. 4.4 Zone configurations

This is a trade off between w_1^*, w_2^* and the fact that the few locations to be visited in zone C might have considerable weight in the total tour length.

It should be noted that in practice dimensions of the A and B zones, and therefore the position of the dividing line will depend on the type of ABC curve applied to the corresponding picking face.

Band2 (fig. 4.4-b) is the second configuration. Zone A and B are situated as strips symmetrically along the dividing line which halves the rack height. Here the values of w_1^* and w_2^* are fully incorporated as they were derived theoretically.

Band3 (fig.4.4-c) is the third configuration. It is very similar to Band2. Here the dividing line is the diagonal of the square in time rack which is also the direction of the absolute velocity, when the stacker crane's shuttle moves simultaneously in horizontal and vertical directions. It is expected that this configuration would best utilize the combined movements of the picking machine.

In reality this configuration would have stepped shape because of the cellular rack structure. It should be noted that because the strips are inclined on an angle $e=\arctan(V_y/V_x)$, the strip widths become $w_1^*/\cos(e)$ and $w_2^*/\cos(e)$, but in practice e is such that $\cos(e) \sim 1$.

4.4 Model simulation

4.4.1 Description of the experiment

A designed experiment was conducted to determine the influence of each proposed zone configuration on the optimal tour length. A full factorial experimental design was implemented, with factors and levels as shown in table 4.1.

Table 4.1. Factors and levels in the factorial experiment

No	factor	No. of levels	value (notation)
1.	picks per cycle	5	10,15,20,25,30
2.	method(algorithm)	2	Band_Two, TSP
3.	velocity type	2	Const (max), Real
4.	rack length [m]	4	35,40,45,50
5.	ABC curve	2	80(10)/15(20)/5(70), 60(10)/15(20)/25(70)
6.	zone configuration	3	BAND1, BAND2, BAND3

To each of the investigated shapes, the Band heuristic plus two optimal improvement procedure (in table 4.1 noted as Band_Two) and an exact method (Little's Branch and Bound algorithm, see [Syslo et.al.1983]) for solving the TSP were applied.

Five pick levels (see table 4.1), four rack lengths and two ABC curves were used as well.

Since it was shown in chapter III (see as well Guenov and Raeside[1989]) for random storage, that neglecting the accelerations and decelerations may produce suboptimal tours this was taken into account.

Manufacturer's data for horizontal and vertical travel times were used for Vxmax = 63 m/min and Vymax = 18 m/min, horizontal acceleration 0.25 m/s^2 and vertical acceleration 0.5 m/s^2. Accelerations are assumed equal to decelerations.

The algorithms were first applied with the time matrix based on the maximum horizontal and vertical velocities and then the obtained routes recalculated using real times. Then the same algorithms were applied with the time matrix based on real times. This allowed the comparison of tour time obtained for constant (max) and real velocities to be based on real times for each algorithm.

Multiplying the levels for all factors gives 480 combinations, which were replicated five times each, thus giving 2400 different runs in total.

Addresses were generated randomly to represent the percentage turnover of each class and to be uniformly distributed within each zone. This was done by a two stage random generation procedure (see Appendix, B).

Since Little's Branch and Bound algorithm requires substantial computer power, a ten hour run time limit was set. If the run time limit was exceeded, the program terminated and was restarted with different seed for the random generator.

The simulation software used for the experiment in Chapter III was modified for the purpose of this experiment (see Appendix B) and ran on a number of IBM PS/2 computers.

4.4.2 Results of the experiment

Results were analysed with the aid of the GENSTAT statistical package to carry out analysis of variance. All factors were found to be significant . The pick level and the type of zone configuration made by far the biggest contribution in explaining the variance.

Summaries of the mean responses for different levels when real velocities are used are shown in tables 4.2, 4.3 and 4.4. The travel times of the tour for each heuristic and its exact solution as a function of number of picks per cycle and rack length are shown in table 4.2 and 4.3 respectively.

Table 4.2. Tour time in seconds as function of number of picks per cycle

picks/cycle	Band1	TSP1	Band2	TSP2	Band3	TSP3
10	113.41	113.22	130.65	130.40	116.60	116.13
15	145.54	144.02	167.33	166.35	146.72	145.99
20	183.12	181.20	195.88	192.83	181.65	180.16
25	215.15	212.01	225.81	222.57	218.11	215.74
30	248.91	244.40	257.92	251.64	243.22	239.97

Table 4.3. Tour time as a function of rack length

rack length	Band1	TSP1	Band2	TSP2	Band3	TSP3
35 (10)	162.87	161.10	174.55	171.90	159.84	158.09
40 (11)	175.23	173.06	190.30	187.14	172.60	171.06
45 (13)	188.40	185.75	203.88	200.72	190.61	188.67
50 (14)	198.41	195.97	213.35	211.27	201.98	200.56

Note - Number in brackets represents corresponding rack height

These tables indicate that Band1 and Band3 perform almost equally well, while Band2 gives solutions that are at most 14% poorer than the two other configurations (the average difference was 7.3%).

In fig. 4.5 the percentage difference between the exact solutions (TSP1 and TSP3) for the first and the third configuration as function of the number of picks is displayed. From this figure it is observed that the absolute value of the difference is within 3% and the slight trend (as illustrated in fig. 4.5 by the least squares regression line of the BAND heuristic points) suggests that as number of picks per cycle increase, the third configuration (BAND3) will produce a better solution.

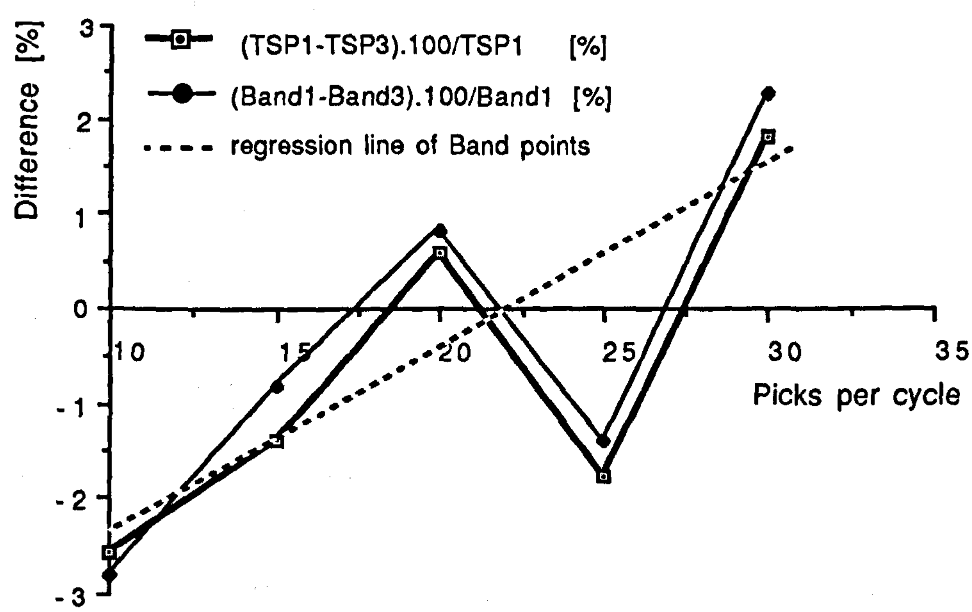


Fig.4.5. Percentage difference between tour times for the first and the third configuration

The percentage difference between the first and the third heuristic follows a similar pattern.
There is a great deal of uncertainty in the percentage differences illustrated in the above figure. This is exemplified by the mean width of the confidence intervals being approximately 8%.
Further investigation is required to specify the character of the curves for more than 30 picks per cycle.
Other findings are that Band1 performs slightly better for the larger racks (more than 45[m] long) and for 80(10)/15(20) while Band3 is best for 60(10)/15(20). These findings are evident from tables 4.3 and 4.4.

Table 4.4. Means of the tour times in seconds for the two ABC curves

ABC curve	Band1	TSP1	Band2	TSP2	Band3	TSP3
80(10)/15(20)	160.99	158.84	185.24	182.89	166.66	165.33
60(10)/15(20)	201.47	199.09	205.80	202.63	195.86	193.86

From tables 4.2-4.4 and fig.4.5 it is seen that each heuristic produces solutions which are on average no more than 3% worse than the corresponding exact solution.

It was found that tours constructed with constant velocities are poorer than those constructed with real ones and the difference conforms with the results given in chapter III for random storage (see as well Guenov and Raeside[1989]).

The reported run times for the heuristics were all within 3 seconds, while for the exact methods the run time limits were often exceeded for 25 and especially for 30 picks. Fortunately, once again as in chapter III, no correlation was found between tour time and run time.

TSP3 requires more computational time either than TSP1 or TSP2. This can be explained by the fact that the third configuration is designed to utilize the combined movements of the stacker crane. This increases the number of tours with similar travel times.

4.5 Summary and conclusions

In this chapter the effect of different zone configurations on the optimal multiple command order picking tour has been investigated.

Zone shapes were modelled in a way to maximise the utilisation of simultaneous movement of the stacker crane's shuttle in horizontal and vertical directions, while keeping the capacity (area) of the corresponding zone as defined by the applied ABC curve.

From the factorial experiment which was conducted it was found that the shape of the zone does affect the optimal solution. Configurations Band1 and Band3 give best performance for square in time racks with I/O point located at the bottom left corner of the rack.

Band3 appears to improve its performance when the number of picks per cycle increases. This implies that the L shaped zone boundaries of Band1 should not be considered optimal for multi command order picking.

Band2 was found to perform worse than the other two configurations, but it should be noted that it may prove to be successful in cases where the I/O point is half way between the two left corners of the rack as exists in practice (this is pursued in chapter V, where a case study is modelled). This would eliminate in many cases the time for reaching the "fast mover" A and B zones from a bottom left corner I/O point at the beginning of the cycle and then reaching the I/O point from A and B zones at the end of the cycle. Further investigation is required to verify this.

This study was confined to one velocity set and square in time racks. Research could be extended to investigate the influence of zone shapes on the optimal tour with different velocity sets and racks that are not square in time.

The configurations that were investigated in this chapter were compared to each other but not to a method that has been proven to be currently the best, since to the author's best knowledge there is not one. The intention is therefore that the proposed heuristics are tested in a warehouse with proven efficient order picking.

In the next chapter a case study conducted in an operational warehouse is presented.

CHAPTER V

CASE STUDY

5.1 Introduction

In this chapter a case study is presented. The case study was designed to estimate the overall savings in travel time, when zoning of the picking face is used in conjunction with the location sequencing (tour construction) algorithm as proposed in chapter IV. The algorithm used was the Band heuristic plus two optimal improvement procedure.

Since no optimal zoning policy for multiple command order picking currently exists, the new zone configurations had to be tested against a combination of storage and picking policies, which are currently being used satisfactorily.

This case study was conducted in a distribution warehouse of Volkswagen-Audi (VAG-UK), which is one of the few attempting to serve its clients (dealers, retailers) within 24 hours of order placement for the whole UK.

In the next section a brief description of the existing storage, batching and picking policies in the warehouse is given. The layout of the picking face is also described. In section 5.3 the necessary preparation work for the experiment is outlined, while in section 5.4 the description of the experiment is presented. Results of the experiment are given in section 5.5 and a summary and conclusions for this chapter are presented in the last section.

5.2 Existing situation

The distribution warehouse of VAG-UK supplies customers with spare parts all over the UK.

There are four types of orders that are processed at the warehouse:

i. factory back orders,

ii. daily orders,

iii. weekly orders,

iv. two weekly orders.

Factory back orders are considered rush orders and require overnight delivery period. Hence, they are processed once they arrive.

Daily orders are delivered on the next day as well as part of the weekly orders when they include back orders.

The weekly and two weekly orders are part of regular transactions, consisting of a large number of different items (called lines) and are delivered between one and ten days after the order placement.

According to their type, orders are grouped into four types of batches for processing:

- "X" and "D" batches consists of back orders for overnight or next day delivery respectively,
- "N" batches consist of daily orders and/or weekly orders which include back orders or cases where the number of lines

per batch is less than 120. Two weekly orders can also form a "N" batch, when the number of lines in the batch is less than 120.

"N" batches are delivered on the next day.

- "S" batches consist of weekly and two weekly orders. They are the largest batches consisting of 120 or more lines per batch. Their delivery period can vary between one and ten days. Here it should be noted that orders of a batch that are picked from the same aisle are called a sub-batch.

The warehouse consists of two main areas: storage and picking but each of them is divided into sub areas according to the physical characteristics of the products. Each of the sub areas is a rack structure served by fork lift trucks, pallet stacker cranes or stacker cranes operating with tote boxes.

The sub area of interest is the rack structure where multiple command order picking of relatively small parts takes place. The system is called DECOMBI and is served by a number of MANESMANN-DEMAG stacker cranes, each capable of carrying six tote boxes, which means that a maximum of six different orders can be included into a sub-batch.

Stacker cranes' maximum and creep horizontal velocities are 80 and 13 [m/s] respectively and the vertical velocities are 12 and 1.9 [m/s] respectively.

The racks are served by a roller conveyor system. The empty tote boxes come in front of the rack on a gravity roller conveyor (see fig 5.1, the arrow "input"), from which they are uplifted by the picker. After the picking cycle is completed, the full tote boxes are placed on another gravity conveyor (in fig. 5.1, the arrow "output") which takes them to the main roller conveyor. Then the tote boxes are transported to the consolidation area.

A typical layout of the picking face is shown in fig 5.1.

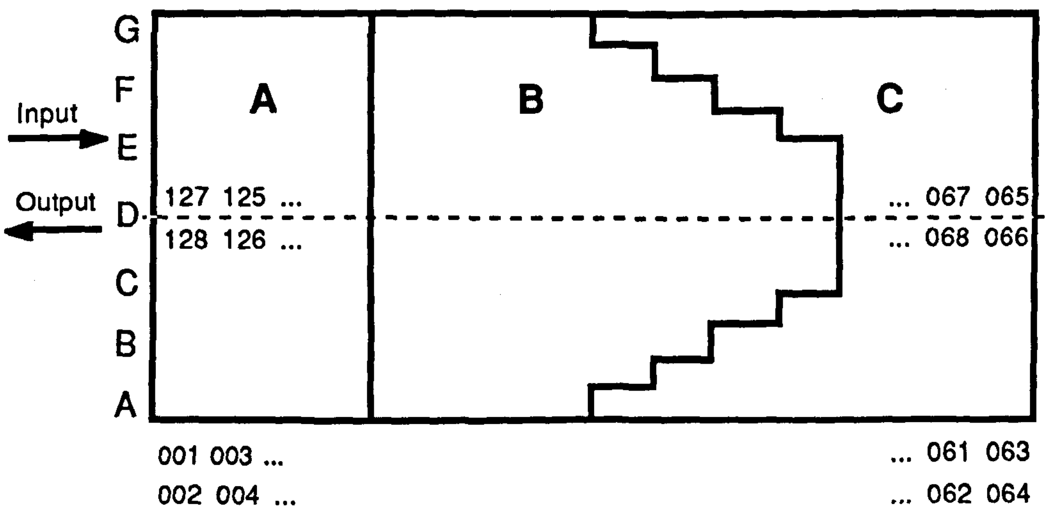


Fig. 5.1. Layout of the picking face at the DECOMBI system.

As it is seen from fig. 5.1 each of the racks consist of seven levels (rows) A-G and 32 columns. Since for each aisle there are two associated racks, the numeration for one of them is implemented in even numbers and for the opposite one in odd numbers. The numeration of the columns for the first three levels (A-C) for each rack is in ascending order, while for the other levels (D-G) it is in descending order (see fig 5.1).

The shape of the zones for each turnover class (denoted with the bold letters A, B and C) is shown on fig 5.1. The shape of zone B is a parabola, which has the stepped form on the figure, because of the cellular structure of the rack.

There is a safety distance at the left side of each aisle, about five meters from the Input/Output points where the stacker crane travels only at a creep speed. This means that about 16% of the total number of locations (and these are fast movers) are visited at creep speed.

Each of the locations is a rack cell of one square meter. In each of the locations there are several identical bins which contain the stored items.

According to the number of bins in a location (therefore according to the size of the bins) there are six types of locations denoted as follows:

a- one open bin per square metre

b- two open bins per square meter

c- three open bins per square meter

h- nine open bins per square meter

s- twelve open bins per square meter

g- forty two compartment bins per square meter

The size of the bin corresponds to the size of the stored item. For example in the smallest compartment bins there are washers, small bulbs, special pins and so forth.

The existing software handles a wide range of operations, including direct links with the suppliers and the customers.

In this case study interest is focused on the part of the software, used for constructing the order picking tours (location sequence) in the racks, and which combines the orders into sub-batches.

The current tour constructing algorithm is almost identical to the Band heuristic, described in chapter III. However, the software does not take into account that lines which lie in locations, directly across the aisle can be picked only with one

stop, thus eliminating unnecessary vertical movements. Only experienced operators searching through their picking lists manage to solve the problem by not following the picking sequence, prescribed by the computer.
In this case study batching policy is not discussed, but it should be noted that the current software, when combining orders in a sub-batch checks if the maximum allowed number of lines is exceeded and if so, "cuts" one or more of the orders from the sub-batch. This leads to a considerable under utilization of the stacker crane's capacity (cube).

5.3 Preparation work

Preliminary work was done mainly to collect the necessary data for the experiment and writing of the simulation software.
The data required included figures for the ABC analysis, the travel times of the stacker crane between any two locations in the horizontal and vertical directions and actual picking lists. Samples and measurements were taken in an aisle, which represented the DECOMBI section of the warehouse during the experiment.
From observation at the existing picking face in the aisle and from samples of "N" and "S" batches it was estimated that 25% of the lines represent 52% of the total turnover (demand)-these are the fast movers. Medium movers are 44% and represent 45% of the demand. Slow movers are 31% of the items and represent only 3% of the demand. Thus using the notation of ABC curves

proposed in chapter IV, the present ABC curve is 52(25)/45(44)/3(31).

The second set of data required were stacker crane's travel times between any two locations in horizontal and vertical directions. Since this set was unavailable the times had to be estimated.

Each travel time between any two locations in the horizontal and vertical directions was timed five times alternating forwards and backwards or upwards and downwards movements respectively. Then the means for each set of measurements formed the required travel times for the experiment.

The weight of the load was negligible compared to the stacker crane shuttle's weight, so it was ignored as a factor that could influence the mean travel times.

The third set of data required for the experiment was a number of picking lists for the DECOMBI section. These were obtained from the company and were edited to represent the actual number of stops and not the number of lines per travel cycle (sub-batch), as only the travel time was recorded.

The simulation software used for the experiments in chapter III and chapter IV was modified for the needs of this case study. The fixed dimensions of the racks and the actual ABC curve were taken into account when generating the new zone shapes and the distributions of addresses in each storage class.

5.4 Description of the experiment

Four different picking face layouts (zone configurations) and two sequencing algorithms were tested in the experiment. The first picking face layout and sequencing algorithm are the existing ones and their combination will be referred to as MK. The other three zone configurations combined with the improved sequencing algorithm (Band heuristic plus two optimal improvement procedure, slightly modified for each zone configuration) are referred to as BAND1, BAND2 and BAND3. Because of the shape of the zones accommodating different turnover classes, the I/O point for BAND1 and BAND3 was considered to be the bottom left corner of the rack (001A in fig. 5.1). For BAND2 the I/O point was located at 127D (fig. 5.1). I/O point is the start and the end point for each cycle.
For MK the Input and Output points are the existing ones (see fig. 5.1) which in practice coincide with the I/O point for BAND2.
During the editing of the existing picking lists for MK one substantial drawback of the existing software was eliminated i.e. the fact that the current software does not take into account that lines from two directly opposite locations in the aisle can be picked with only one stop. If not corrected for, this leads to an excessive travel up and down the rack. Correcting for this fault allowed BAND1, BAND2 and BAND3 to be compared to MK, assuming that an experienced operator carried out the MK picking cycles.

The experiment included routing the stacker crane through a sequence of 10, 15, 20, 25, 30 and 35 stops for each method. An experienced operator was used for each run.
This was replicated five times to provide enough data for statistical analysis.
Multiplying the four methods by the six sets of stops and by five replications of each set give 120 different picking (travelling) cycles whose times were recorded using a stop watch.
Addresses for BAND1, BAND2 and BAND3 were generated randomly to represent the percentage turnover for each class, as specified by the ABC curve, and to be uniformly distributed within each zone.
Travel sequences for BAND1, BAND2 and BAND3 were calculated by computer using the simulation software which was modified to produce picking lists similar to those used at V.A.G.-DECOMBI section, so there was no confusion for the operator.
Samples for the sets of 10, 15, 20 and 25 stops were selected from "N" batches and for 30 and 35 stops from "S" batches.

5.5 Results from the experiment

The results from the measurements are presented in table 5.1-a to g. The means for the results were obtained by GENSTAT [1980] statistical package, using the ANOVA directive.

Table 5.1. Results from the measurements.

a) ten stops per cycle (sub batch)

Stops	Method	Travel time [s] for replication					Mean
		1	2	3	4	5	
10	MK	87.10	60.01	89.79	82.65	83.66	80.64
10	Band1	82.87	80.52	88.71	68.95	79.50	80.11
10	Band2	72.02	84.17	86.34	79.86	73.67	79.21
10	Band3	81.44	77.39	90.43	85.66	81.50	83.28

b) fifteen stops per cycle

Stops	Method	Travel time [s] for replication					Mean
		1	2	3	4	5	
15	MK	120.22	100.02	103.28	123.14	93.54	108.04
15	Band1	99.24	88.79	107.95	90.50	98.26	96.95
15	Band2	93.24	87.33	90.01	95.50	105.12	94.24
15	Band3	98.12	85.20	102.36	104.40	98.78	97.77

c) twenty stops per cycle

Stops	Method	Travel time [s] for replication					Mean
		1	2	3	4	5	
20	MK	119.90	130.21	131.73	131.80	132.55	129.24
20	Band1	111.91	105.90	109.37	112.88	117.16	111.44
20	Band2	122.60	116.36	114.89	112.65	113.63	116.03
20	Band3	117.87	121.13	114.07	114.59	118.53	117.23

d) twenty five stops per cycle

Stops	Method	Travel time [s] for replication					Mean
		1	2	3	4	5	
25	MK	153.22	150.18	163.49	171.10	171.18	161.83
25	Band1	132.37	139.88	135.70	134.45	131.28	134.74
25	Band2	128.20	136.28	146.20	133.38	145.59	137.93
25	Band3	138.53	137.14	132.12	134.82	138.20	136.16

e) thirty stops per cycle

Stops	Method	Travel time [s] for replication 1	2	3	4	5	Mean
30	MK	172.39	168.32	178.32	182.30	155.33	171.33
30	Band1	149.22	147.51	153.61	140.38	147.09	147.56
30	Band2	143.79	146.72	141.28	139.96	149.59	144.27
30	Band3	148.70	149.78	139.76	137.62	146.73	144.52

f) thirty five stops per cycle

Stops	Method	Travel time [s] for replication 1	2	3	4	5	Mean
35	MK	199.76	189.89	208.43	181.97	190.28	194.07
35	Band1	162.90	163.55	159.06	173.20	163.45	164.45
35	Band2	168.89	164.56	168.89	164.23	164.54	166.22
35	Band3	161.35	171.98	164.98	162.49	163.62	164.88

g) mean for each method

Method	Total mean
MK	140.86
BAND1	122.54
BAND2	122.98
BAND3	123.98

From table 5.1 it is clearly seen that MK is the poorest performing method. As the difference amongst the other three methods is within 4%, BAND2 will be considered as a representative of the three new configurations. Moreover the zone configuration of BAND2 fits best the existing location of the I/O points for the racks.

The percentage difference between the mean travel time for MK and BAND2 as a function of the number of stops per cycle is given in fig. 5.2.

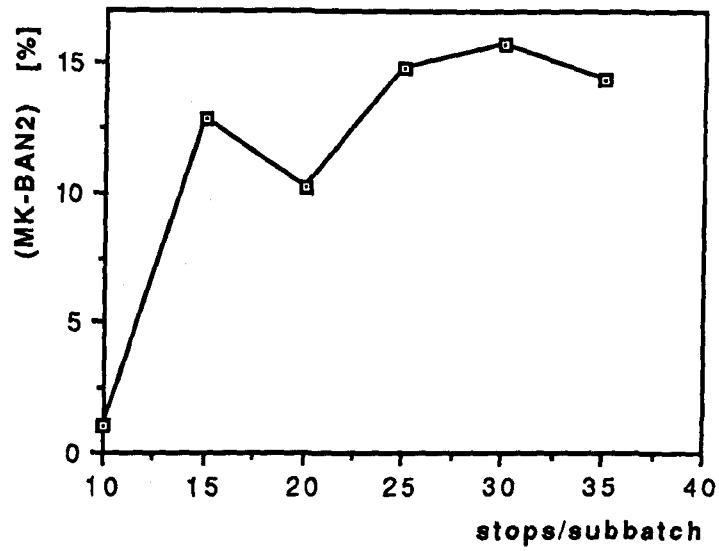


Fig. 5.2. Percentage difference of mean travel time between MK and BAND2.

From the means for each method (table 5.1-g, see as well fig. 5.2) it was estimated that on average Band2 performs 12.7% better than MK.

The separate contributions of the new zone configurations and the two optimal tour improvement procedure can be judged from the results from the computer simulation. Since the computer program was designed to produce separate results for BAND 1-3 with and without the two optimal improvement procedure, these were fed into the statistical package to produce the means.

The means and the percentage difference for BAND1-3 with and without the two optimal improvement procedure are presented in table 5.2.

Table 5.2. Average contribution of the two optimal procedure to

the savings in travel time.

	BAND1	BAND2	BAND3
With two opt. procedure	129.55	126.81	129.75
Without two opt. procedure	134.56	133.26	137.22
Percentage difference	3.72	4.84	5.44

From table 5.2 it is seen that on average, around 5% of the overall savings of travel time are due to the tour improvement procedure and therefore 7-8% are due to the new zone configurations.

The relatively small contribution of the tour improvement procedure to the overall savings can be explained as a result of the relatively small rack height.

In the course of the experiments the picker was asked to share her/his opinion about the procedure. Some mentioned that they felt a bit disorientated. This is explained by the fact that three new methods were tried at the same time. For these the route did not follow the two layer traverse of MK which has been used for years.

5.6 Summary and conclusions

In this chapter a case study, conducted at a distribution warehouse of Volkswagen-Audi (VAG-UK) was presented.
The aim of the case study was to estimate the savings in travel time, when the proposed zone shapes in combination with an efficient tour construction algorithm were used in a class based storage and multiple command order picking system.
Since no optimal zone shapes are currently known, the proposed combinations had to be compared to existing ones in an operational warehouse.
Results from the experiment showed overall savings in travel time, on average more than 12%, when the new combination of storage and picking policy was applied.
From computer simulation it was concluded that on average more than 7% of the overall savings were due to the new zoning and about 5% to the improved sequencing algorithm.
Since the experiment was conducted in a particular warehouse, one can not consider that the savings are universal. However, it is believed in warehouses with higher racks and more skewed ABC curves of the turnover distribution (see chapter I), that the savings in travel time will be even larger. This is because picking tours will be confined mainly in the zones of the fast mover classes and many of the unnecessary vertical movements created by the Band heuristic will be eliminated by the two optimal improvement procedure. Further investigation should verify this.

CHAPTER VI

SUMMARY AND CONCLUSIONS

In this chapter the main findings of this research are summarised and areas, where work can be extended, are pointed out.

The aim of this research was to find ways of increasing efficiency of multiple command order picking by decreasing travel time of the picker, hence ultimately to decrease the cost of the picking.
Finding the optimal tour for multiple command order picking resembled the task of finding a solution of the Travelling Salesman Problem (TSP). This is notoriously difficult to solve in reasonable time, even with the increased and cheaper computer power. For this reason some efficient heuristic methods have been adapted especially for multiple command order picking. Their accuracy was compared to exact solutions of the TSP. In all work, previous to this research, the stacker crane's shuttle was assumed to travel with constant velocities in the horizontal and vertical directions.
In this research the magnitude of the error introduced in calculating the tour, due to the above assumption was derived. A simulation experiment was carried out to estimate the value of this error. It was found that for tours constructed with

constant velocities the error increases approximately linearly with the number of picks (stops) per cycle. The magnitude of this error is in the region of 5%.
An even more important finding was that solutions of the TSP found for constant velocities often produced more costly tours than the heuristics, when these solutions are recalculated for real velocities. This means that when constant velocities are used the exact solution is in fact a non-exact one.

Another major contribution of this research was the investigation into the interaction between the optimal picking tours and the zone shapes of the turnover classes in class based storage.
Three zone configurations were modelled, based on the optimal Tchebyshev travel in two parallel adjacent strips, representing the high turnover classes. From the factorial simulation experiment which was conducted it was found that the shape of the zone does effect the optimal solution. The first and the third of the proposed configurations appeared to give best performance for square in time racks with the I/O point located at the bottom left corner of the rack.
Since no optimal zone configurations are currently known, the originally proposed ones were tested against existing configurations in a distribution warehouse of Volkswagen-Audi (VAG-UK), which is one of the most reputable of its type in the UK. The results from the case study experiment indicated that overall savings in travel time of more than 12% on average are

attainable.

From a computer simulation it was estimated that more than 7% are due to the new zoning and around 5% to the improved tour construction algorithm.

This research has demonstrated that two different aspects of the same problem, of increasing order picking efficiency, i.e. physical and managerial, can be tackled together. Doing so gives greater understanding and a better solution.

There is a scope for further work in this area to:

1. Investigate the effect of accelerations and decelerations on the optimal order picking tour by simulating a wider range of rack dimensions and different velocity sets.

2. Explore variations of the proposed zone configurations for class based storage, especially for different positions of the Input and Output points of the racks. Investigation should also be extended to different rack dimensions and turnover distributions of the stored units, as represented by the ABC curves.

It is recommended that the findings of this research be implemented in high bay narrow aisle warehouses in order to improve multiple command order picking efficiency. Doing so, as has been demonstrated, can amply be justified in economic terms and would contribute to the improved efficiency of the logistics chain.

As was noted in Chapter I order picking is an important part of

the logistics chain and in particular warehousing. The design and implementation of an efficient order picking system therefore must consider a hierachy of interacting factors.
In terms of priorities to improve order picking efficiency, at the highest level are the decisions on the type of order picking (see Chapter I), layout, storage policy and the level of mechanisation (automation).
One vital part of order picking efficiency is the correct analysis of the demand patern of the products from the input and output flows. This is dependent upon the information processing ability of the system.
At the lowest level of the hierarchy is the optimization of the batching (clustering of the orders) and vehicle routing.
The practical application of this work will be most beneficial in warehouses with the following features:

i. automated/semi automated distribution warehouses or warehouses for small parts where the average number of picks (stops) per cycle is more than 8-10, and picking time is relatively small compared to the travel time of the picker (picking machine).

ii. warehouses, where the racks are more than 6-7 m high, which would obtain more of the potential of the improved sequencing algorithm.

iii. warehouses, where either manually or by a computer, a record for item popularity (demand frequency) is kept. This would allow easy transformation of the picking face into classes formed on the basis of the ABC analysis. These into classes formed on the basis of the ABC analysis. These classes would be arranged into the zone shapes proposed in this work. Regular updating of the size of the shapes as a result of regular updating of item popularity would contribute to even more efficient order picking.

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