COMPUTER-ENHANCED LEARNING IN TERTIARY EDUCATION

DIANA MARY MACKIE

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Abstract

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Diana Mackie

It is widely accepted that mathematics courses for science and engineering undergraduates should aim to develop an enquiring and creative approach to mathematics together with good communication skills. Due to their versatility, computatonal power and graphical capabilities, computers can play a significant role in developing these skills. A review of the development of computer-assisted learning of mathematics established that a new investigative approach could exploit the potential of the computer.

For this project, two comprehensive computer-based learning packages were developed. The content and educational objectives of the packages were determined by consultation with mathematics lecturers. These objectives were to encourage investigative work, to facilitate problem solving and to enhance student understanding of certain algorithms and topics. The packages were evaluated over a four-year period, whilst in regular use in the mathematical sciences laboratories at Napier Polytechnic as part of the curriculum of several degree courses. During the formative evaluation, modifications and improvements were incorporated. The second stage of the evaluation comprised an investigation of the impact of the packages on the mathematics curriculum. In particular, changes in teaching approaches, learning outcomes and student attitudes towards mathematics were studied through observation, questionnaires and interviews. The feasibility of transfer of the materials developed to other higher educational establishments was also examined.

The study identified an increase in the use of graphical methods to explore the behaviour of functions, numerical methods and models, more emphasis on investigative work, and more analysis and interpretation of results. Improved communication skills were also noted. It was deduced that the computer-based approaches adopted had fostered the development of higher cognitive skills, thus leading to an enhanced quality of learning.

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Diana Mackie

Declaration

I hereby declare that:

All the work presented in this thesis has been carried out by myself and no part of this work has been submitted in support of another degree.

D. Mackie

Diana Mackie

CONTENTS

Page

CHAPTER 1 - Introduction 1

1.1 Technology and Mathematics 1

1.2 Computer-assisted Learning in Mathematics 3

1.3 Development of Computer-assisted Learning of Mathematics at Napier 5

1.4 Aims and Scope of the Research 6

CHAPTER 2 - Learning Mathematics with Computers 9

2.1 The Role of Computers 9

2.2 The Development of Computer-assisted Learning in Mathematics 16

2.3 The Computer as a Tool 19

2.4 New Teaching Approaches 23

2.5 Evaluation Studies 27

2.6 Summary 29

CHAPTER 3 - The Learning Environment at Napier Polytechnic 31

3.1 Introduction 31

3.2 Early Use of Computers 32

3.3 Mathematics Laboratories 34

3.4 Software and Worksheets 37

3.5 The Continuing Development 38

3.6 Aims and Scope of the Investigation 40

CHAPTER 4 - The Learning Packages 42

4.1 Design Criteria 42

4.2 The Linear Programming Package 42

4.3 The Differential Equations Package 48

Page

CHAPTER 5 - The Process of Evaluation 54

5.1 Methodology 54

5.2 Classes Selected for Monitoring 58

5.3 Observation of Laboratory Sessions 62

5.4 Student Surveys 62

5.5 Staff Surveys 67

5.6 Use in other Institutions 68

CHAPTER 6 - Formative Evaluation of the Learning Packages 70

6.1 LINPROG 70

6.2 NODES 75

6.3 Mathematical Sciences Laboratories 80

6.4 Student Attitudes 84

6.5 Summary of the Formative Evaluation 89

CHAPTER 7 - Results of the Summative Evaluation 92

7.1 Introduction 92

7.2 Teaching Approach 92

7.3 Emphasis and Content of the Curriculum 99

7.4 Assessment 101

7.5 Student Attitudes 102

7.6 Transfer to other Institutions 112

CHAPTER 8 - Discussion and Developments 117

8.1 The Learning Packages Developed 117

8.2 A Changing Role for the Teacher 119

8.3 Learning Outcomes 120

8.4 Implications for the Curriculum 126

8.5 Developments 127

8.6 Model Curricula 130

BIBLIOGRAPHY 132

APPENDICES

1. LINPROG Worksheets

2. NODES Worksheets

3. B.Sc. Science with Industrial Studies Course Details
   B.Sc. Communications and Electronic Engineering Course Details

4. Student Questionnaires

5. Staff Questionnaires

6. External Questionnaires

7. Summary of Student Interviews, 1986

8. Results of Questionnaires
   SQ0 (1986, 1987), SQ1, SQ3, SQ4

9. Results of Questionnaires
   SQ0 (1987), SQ2, SQN2

10. Staff Interviews, 1989

11. Student Interviews, 1988

12. Results of Attitude Surveys, 1989

13. Results of Questionnaire SQL1

14. Publications

Note: Listings and documentaton of the computer programs developed are available, on request, from the author.

Tables

		Page
5.1	Student surveys	67
6.1	Student comments relating to LINPROG	72
6.2	Staff LINPROG ratings 1986-87	73
6.3	LINPROG evaluation 1989	74
6.4	Reasons for using NODES	77
6.5	Student comments relating to NODES	78
6.6	Staff NODES ratings 1987	78
6.7	NODES evaluation 1989	80
6.8	Suggested improvements to the laboratories	82
6.9	Computer programs used	84
6.10	Student comments relating to computer-aided learning	85
6.11	Ways in which computer packages help	88
6.12	Computer packages as an aid to learning (percentages)	88
7.1	Number of staff using laboratories	93
7.2	Use of laboratories for formal class meetings	93
7.3	Lecturers' reasons for using the laboratories	94
7.4	Reasons for not using the laboratories	95
7.5	LINPROG - student ratings, Dundee, 1988	114
7.6	Customers' reasons for using LINPROG	115
7.7	Purchasers' ratings	115

Figures

		Page
3.1	Mathematical Sciences Laboratory	36
3.2	Menu of Mathematics Programs	36
4.1	Integer Programming Data Sheet	44
4.2	LINPROG - Tableau by Tableau Output	46
4.3	LINPROG - Menu of Options	46
4.4	NODES - Numerical Methods	49
4.5	NODES - Graphical Solution	51
4.6	NODES - Error Analysis	51
6.1	LINPROG - Students' Ratings (percentages)	71
6.2	NODES - Students' Ratings (percentages)	76
6.3	Students' enjoyment of use of computers	87
7.1	Preferred mode of working at computer	104
7.2	Student attitudes towards use of LINPROG	107
7.3	LINPROG as an aid to understanding	109
7.4	Student attitude towards use of NODES	110

CHAPTER 1

Introduction

1.1 Technology and Mathematics

> A "programme of studies must aim to stimulate an enquiring, analytical and creative approach, encouraging independent judgement and critical self-awareness."
>
> CNAA Handbook, 1988

Mathematics is studied at Polytechnics primarily as a tool for use in science, engineering and the management sciences. Many complex problems in industry and commerce can be formulated as a mathematical model whose behaviour can be studied under different conditions, usually with the aid of a computer. The rapid pace of innovation in microelectronics in industry and commerce has resulted in a demand for graduate mathematicians and engineers able to utilise new technology and to tackle a wide variety of unfamiliar mathematical problems with confidence. Various committees have been set up in recent years to study future requirements for mathematical and engineering education.

The report by the Finniston Committee (1980) introduced the concept of an engineering applications (EA) element integrated into all engineering degree courses. One particular aspect of EA aims to develop a student's ability to apply knowledge to the solution of practical problems based upon engineering processes and systems. The Institute of Electrical Engineers interprets this as placing an emphasis on the relevance of theory to engineering systems, including the ability to develop and use mathematical models from which the behaviour of the physical world can be predicted. The acquisition of communication skills, both oral and written, is also highlighted as an important aspect of an engineer's training.

Broadly similar conclusions were reached by both Lighthill (1979) and the Grant Committee set up by the Institute of Mechanical Engineers (Grant, 1985).

Powerful computers now enable increasingly complex engineering problems to be tackled. McQuade (1985) suggests that future engineers will require more not less mathematics and that students must be given a sounder grasp of mathematical concepts in order to utilise the new technology. He identifies probability theory, discrete variable systems and operational research as areas which will assume greater importance in the mathematics curriculum. At a seminar held by the European Society for Engineering Education (1985), discrete mathematics and probability were again noted as topics which should be included to take account of the significant input computers have made on engineering.

The first widely available computer algebra system, MACSYMA, was introduced in the USA in the 1960s. Since then it has been used to explore problems in fields as diverse as plasma physics, fluid mechanics and decision analysis in clinical medicine. MACSYMA, and other computer algebras such as REDUCE, MUMATH and DERIVE, which have been developed since, are large interactive computer software systems designed to assist scientists, engineers and mathematicians to solve mathematical problems. Since they can manipulate symbols rather than just numbers, they can perform complex mathematical operations analytically. Using exact arithmetic, the results are produced to any required precision. Until recently, computer algebra systems have been 'unfriendly' and difficult-to-use programs, requiring very expensive processors. In the near future, however, user-friendly multiple-representation systems incorporating numerical, graphical and symbolic facilities will be used routinely in industry and commerce.

The implications for mathematics teaching at tertiary level are enormous. Much of the current mathematics syllabus for service courses, with its emphasis on skills and techniques, will be of little use to the student in the workplace.

At present, the emphasis of many courses is on the development of competence in algebraic manipulation and the application of routine techniques and algorithms, most of which can readily be implemented as computer programs. For example, the solution of a linear programming problem with three or more variables, by the Simplex method, involves difficult and tedious matrix calculations which can now be carried out by a computer program. To tackle real problems, however, it is important for the user to acquire skills of formulating a problem in mathematical terms, interpreting the solution and analysing its sensitivity, all of which require a good understanding of the underlying concepts of the topic. Previously, undergraduate courses have concentrated on teaching the difficult computation, which had to be done by hand, leaving insufficient time to practise the wider problem-solving skills.

The ideas expressed in this section were reinforced by discussion groups at ICME6 in Budapest (1988). The consensus of opinion there was that routine application of techniques had been devalued whilst activities such as investigations and modelling were becoming revalued upwards.

1.2 Computer-assisted Learning in Mathematics

The computer can play an important role in fostering investigatory, problem-solving and communication skills but it has not always been used effectively. In their early days, computers enabled students to solve problems in areas which were too impractical otherwise and had thus not previously been tackled. Programs could be written, in a high level language such as Fortran, to perform numerical and statistical algorithms. Writing computer programs is difficult and time-consuming but, at that time (pre-1975), there was no alternative as there were few prepared software packages available for teaching purposes. Other early ventures into educational computing involved instructional systems designed to allow the student to learn at his or her own pace.

With the advent of the microcomputer, more software became available to assist the teaching of mathematics. For many years, however, the teaching approach and objectives of both the programs and the users remained the same as in pre-computer days. Most computer-assisted learning was limited to teaching the same things in much the same way but with the benefit of a computer to reduce the tedious arithmetic or control the pace of presentation of material. Whilst self-paced learning and the elimination of numerical drudgery are important features of computer use, it has many other potential advantages. In particular, the computer offers opportunities which did not exist previously for investigative work, realistic problem-solving and the use of graphics.

As interest in the use of microcomputers in schools and higher educational establishments has spread, new approaches to teaching and learning have begun to emerge. The Computer-Aided Teaching of Applied Mathematics project at Cambridge University (Harding, 1974) and, more recently, the Mathematics Department at Birmingham University (Beilby, 1987), both aim to develop an investigative approach towards problem solving. A supportive software environment enables students to write some of their own programs thus acquiring a deeper understanding of the mathematics of the problem. Within the school sector, several researchers have used the programming tool LOGO to develop mathematical problem-solving skills, encourage creative activities, and foster co-operative work and discussion (Pearson, 1986; Hoyles and Sutherland, 1987). With the aid of a package developed by Tall (1986), numerical and graphical methods can be used to introduce the fundamental concepts of calculus in a more meaningful way than the traditional symbolic approach.

There have been a number of studies describing mathematical packages and reporting on their use in the classroom or college environment (for example, Bajpai et al., 1984; Jacques and Judd, 1985) and some researchers have compared the effectiveness of a computer-aided learning (CAL) approach to traditional teaching methods (for example, Beevers, 1986 and 1988; Katsifli, 1986). There are, however, few published evaluations of changes in the nature of the learning which may take place when CAL is introduced. One study of mathematics learning in school pupils (Fraser, 1987), which did examine this aspect found that the use of computer packages encouraged problem-solving activities, investigations and better communication in the classroom. Improvements in mathematical discussion were also reported in a study involving the use of LOGO in schools (Hoyles and Sutherland, 1987) in which a pupil-centred, investigative approach was implicit. Although these studies demonstrate the wider potential of computers, the experimental nature of the research hinders its immediate acceptance by the tertiary sector. Long-lasting changes in higher education evolve as a result of gradual acceptance by lecturers, students and employers of the educational advantages of a new approach.

1.3 Development of Computer-assisted Learning of Mathematics at Napier

At Napier Polytechnic, Edinburgh, the Mathematics Department has been involved in the use of computers in teaching for many years (Leach, 1974). At first, teletype terminals to mainframe computers were used, but these were slow and unreliable. With the advent of microcomputers, members of staff believed that improved methods of using computers to enhance the learning of mathematics were possible. It was felt that an approach requiring students to write their own programs was not appropriate for the majority of the science and engineering courses being taught at Napier. Some microcomputer-based software was developed and a departmental laboratory was established. Awareness and interest in the potential of computers to enhance mathematical education grew steadily. More computers were purchased, more software was written or acquired, and more members of staff began to use packages with their classes. Much of the software has been developed within the department in response to needs identified by lecturers and has been designed in close consultation with them. The use of computer packages at Napier is integrated into the curriculum to illustrate, explore or extend a topic when it arises. Frequency of use of the laboratory varies widely depending on class size, availability of software and the suitability of the topics being studied. A few classes have weekly laboratory sessions, comprising perhaps one-quarter of their total mathematics time, whilst others use computers only to carry out an investigative assignment as part of their coursework.

1.4 Aims and Scope of the Research

For this project, two comprehensive computer-based mathematics learning packages were developed, tested and evaluated. The impact on the mathematics curriculum resulting from the use of the packages was investigated with a view to recording changes in the learning experiences of the students. The feasibility of transferring the packages developed to other establishments of higher education was also examined.

The packages share a common core of objectives, namely, to encourage investigative work, to facilitate problem-solving and to enhance student understanding of certain algorithms and methods. Unlike most other programs available for use in mathematics, the design of the software was determined by these basic objectives. The packages which were developed are:

LINPROG: This is a linear programming package with optional step by step progression through the Simplex method. The program features easy input of the data and menu-driven facilities that enable the original problem to be modified thus facilitating post-optimal analysis and the solution of integer programming problems by the branch and bound method.

NODES: This menu-driven package solves single or systems of ordinary, initial value differential equations numerically. Emphasis has been placed on graphical output and flexibility which allows problem parameters to be easily changed and the resultant effect on the solution observed. It is suitable for investigating both numerical methods and mathematical models.

Both packages are used in conjunction with appropriate worksheets, designed to encourage an investigative approach.

During the formative evaluation phase, modifications and improvements were incorporated and both packages were then marketed. Following this, a summative evaluation of the materials developed and their impact on the mathematics curriculum was carried out. In particular, changes in teaching approaches, learning outcomes, methods of assessment and in student attitudes towards mathematics were studied through observation, questionnaires and interviews.

Initially, two classes in which the use of computers was extensively integrated into the mathematics curriculum were selected for close monitoring. Questionnaires, supplemented by interviews with individual students, were used to gather evaluative data from the students in these classes. As the project progressed it became apparent that the use of computers in teaching at Napier was steadily growing, and more classes were using the laboratories for investigative work. Students from several other classes, who were using the NODES or the LINPROG package, were surveyed also. Additional data were gathered from lecturers in the Mathematics Department over a four-year period.

The study identifies a trend towards more investigative work and more testing and analysis of models when computer packages are used. Less time was spent practising routine manipulative skills and methods involving tedious arithmetic. Graphical output was used extensively to investigate the behaviour of functions, numerical methods and models. It was concluded that this work can lead to greater student understanding of the topic being studied. Student attitudes towards the use of computers as an aid to learning mathematics were found to be generally positive. Most of the students regard it as a useful and relevant aspect of the curriculum. A considerable number show enthusiasm for experimental and investigative work. Other learning outcomes deduced from the evaluation include a more student-centred approach to learning, improved communication skills and, in certain areas of mathematics, a better understanding of some mathematical concepts and algorithms. It is evident that a computer-based, investigative approach towards learning fosters the development of higher cognitive skills, thus leading to a higher quality of learning.

Recent and future developments, including implications of the introduction of computer algebra systems, are discussed. The desirability of regular laboratory work being included in the curriculum of all mathematics courses is noted.

CHAPTER 2

Learning Mathematics with Computers

2.1 The Role of Computers

The mathematical qualities required by future engineering, science and business graduates include the ability to think creatively, to apply their mathematical knowledge to unfamiliar problems and to communicate competently. The achievement of these aims will have important implications for the mathematics curriculum.

In 1977, Richard Hooper prophesied that computers would play an important role in education because of their inherent versatility. Unlike some previous educational innovations, the computer is not bound to any particular learning theory and so its survival does not depend on the prevailing ideology. A DES report (Ball et al., 1987) asserts that computers are:

> "...of particular significance for the teaching of mathematics, and have profound implications, both for the styles in which mathematics can be learned, and also for the content of the mathematics curriculum."

Thus, not only have microcomputers been the main influence behind calls for changes and improvements to the mathematics curriculum, they can also play a significant role in those changes. It is now widely accepted that, before embarking on a new teaching programme, it is necessary to consider the learning objectives. Unless such objectives are enumerated, the learning cannot properly be evaluated. This need not preclude the inclusion of additional objectives during the course of the teaching in response to observed outcomes.

Bloom's taxonomy (1956) classified educational objectives in the cognitive domain in ascending order of understanding as follows:

(i) knowledge
(ii) comprehension
(iii) application
(iv) analysis
(v) synthesis
(vi) evaluation.

Knowledge, comprehension and application imply the ability to learn techniques and apply them both routinely and to particular problems. Analysis involves identifying the different features affecting a solution and exploring the relationship between them. Synthesis is a creative activity in which relationships are developed. This is the level of understanding required to explain a concept to other people. The highest level is that of evaluation, where one gives judgement about the value of a method or solution.

The classification of affective objectives ranges from a basic level of awareness or willingness to learn at the lowest level through stages of increasing control over one's behaviour. Bloom's taxonomy has been criticised for suggesting that objectives in the cognitive and affective domains can be considered separately (Rowntree, 1982). Rowntree also considers that there is too much overlap between the cognitive levels specified. He proposes three broad objective domains for school education, namely, lifeskills, methodological and content which cut across the boundaries of the cognitive, affective and psychomotor spheres. At tertiary level, educators are concerned mainly with cognitive and affective skills. Bloom does recognise a close relationship between development in these two domains. His taxonomy relates closely to the objectives of mathematical teaching as outlined in the Cockcroft report (1982) and remains a useful guide towards curriculum development in higher education.

Recent reports on mathematics teaching in schools stress the need to develop flexible problem solving skills and the ability to communicate difficult concepts (Ball et al., 1987). The role of mathematics as a powerful and concise means of communication is cited by Cockcroft (1982). He also reports that evidence from professional bodies stressed the importance of being able to apply skills confidently in different ways and to think more deeply and critically.

Opportunities for discussion between teacher and students, and between students themselves, problem solving and investigational work are all listed by Cockcroft as teaching approaches which should be included in mathematics teaching at all levels. Problem-solving, investigative activities and effective communication quite clearly utilise cognitive skills at the upper end of Bloom's taxonomy. These are the pursuits which require the greatest depth of understanding of the mathematical concepts involved.

Skemp (1971, 1986) distinguishes between 'instrumental' understanding, the use of rules without knowing why, and 'relational' understanding in which new concepts are related to previous knowledge. He also introduces the idea of a 'schema' as a conceptual structure. Because of the hierarchical nature of mathematics, new learning must build on previous understanding. Real (i.e. relational) understanding results from assimilating new concepts into an appropriate schema. The process of assimilation involves a progression from generalising to abstraction, i.e. an awareness that the rule applies to a number of other structures (Plumpton, 1972). The great power of mathematics lies in its generality and abstraction. At the intuitive level of understanding, we do not reflect on the mental activities involved in a process, for example, when calculating 3 + 4. This is similar to the lowest levels of Bloom's taxonomy. Reflective thinking involves searching for the reasons why one does something. It is reflective activity that leads to generalisation and abstraction and, hence, relational understanding. Mason (1985) suggests that, if reflective thinking is followed by trying to explain the concept to another person, the understanding is further enhanced. This aspect can be exploited by including opportunities for small group activity in the teaching programme.

The distinction made by Cawley (1988) between holistic, or deep learning and surface learning relates to Skemp's two modes of understanding. Holistic learning, when the student thinks deeply about what is being studied and tries to relate it to previous learning leads to relational understanding. Surface learning restricts learning to memorising facts and methods and results only in instrumental understanding. Whilst there may be little difference in the short term between the performance of students who adopt deep and surface approaches, both long term retention and the comprehension of students using the deep approach is far superior (Saljo, 1975). Cawley concludes that open-ended investigative work, together with appropriate assessment, is crucial in encouraging deep learning.

Other authors support the use of open-ended investigations (Baron, 1972, Clements, 1984, and Grant, 1985). Baron asserts that such activity fosters creative thinking by promoting an environment in which making mistakes is accepted. The process of mathematical modelling is also an activity which encourages creative thinking due, mainly, to the open-ended nature of the problems encountered (Bajpai, 1975 and 1976, Lighthill, 1979, and Clements, 1984). In real life, problems are seldom precise or have a unique mathematical formulation or solution. By exposing undergraduate engineers to such problems they can develop the creative skills needed to find acceptable solutions (James, D.J.G., 1985). The process of modelling, normally a group activity, also stimulates the important skills of communication and critical judgement.

Very few realistic problems can be solved analytically without making many assumptions which greatly simplify the problem. A computer can be a valuable tool for modelling and investigational work. By using numerical methods, the realisation of a mathematical model can be implemented as a computer program. The ease with which a computer model can be tested, modified and re-tested greatly enhances the whole modelling process. Results displayed graphically give students qualitative information about a model and its behaviour. The student's time and energies can thus be directed towards the higher level tasks of formulating the model and analysing the significance of the results. A feature of any dialogue between computer and user is the impersonal nature of the computer so that the student is not embarrassed by his mistakes or when he repeats an action several times. Consequently, some students are more likely to persist in attempts to find a solution. Other students, particularly the weaker ones, may be deterred by this anonymity. The option to work in pairs or small groups can help to overcome this attitude whilst providing valuable opportunities for discussion. With a computer at his disposal, therefore, a student has a flexible piece of 'apparatus' with which he can conduct 'experiments' in mathematics. Students can also use computers to investigate simulations of theoretical models.

The ability to tackle realistic problems is an important motivational factor for engineers (Bajpai, 1976, and Cawley, 1988) and other non-specialist mathematicians (Gudgin, 1987). Problems which have no analytical solution or are too difficult to tackle analytically can frequently be solved using numerical techniques. Computer packages give the user access to numerical methods and thus facilitate problem solving. Further, the integration of numerical and analytical techniques can provide insight into the nature of a problem and hence improve understanding (Harding, 1976, Bajpai, 1975 and Rowe, 1985).

An advantage of the computational power of a computer is that it allows a large number of cases (or events) to be investigated very quickly. It can thus be used to illustrate or reveal relationships and generalisations. Such illustrations are invariably enhanced, where applicable, by graphical representation or dynamic display. Several authors have suggested that visual manipulation enhances the student's understanding of the underlying concepts (for example, Mills and Tall (1988), Driver and Scanlon (1988)). An appropriate computer program can direct the user's thinking towards specific mathematical concepts and lead to a restructuring of the student's knowledge (Tawney, 1979) and, hence, assimilation of the new theory into an existing schema. The nature of such conceptual changes in school pupils is one of the topic domains currently being studied by Driver and Scanlon.

As previously noted, it is not sufficient to concentrate only on cognitive skills which are desirable in the mathematics curriculum. In order to produce mature, highly motivated graduates who are mathematically confident, it is necessary to consider educational objectives in the affective domain also. A positive attitude towards mathematics, critical self-awareness and acumen and mathematical confidence could be considered affective objectives. Attitudinal development should include the desire for deeper understanding and the enjoyment of investigative and creative activities.

Several studies have shown that computer-aided learning (CAL) is a powerful motivating factor for students of many disciplines, for example, in mathematics (Harding, 1976), in economics (Gudgin, 1987), in operations research (Erikson and Turban, 1985) and in English (Johnston, 1985). The freedom to experiment and the flexibility of the response are two of the ways in which computers encourage a student's self-determination (O'Shea and Self, 1983). They allow information in various forms to be stored, manipulated and readily presented to the user. Students are also motivated to assume greater responsibility for their own learning because CAL allows them to learn whenever and whatever they wish. Whether the computer is being used in laboratory mode or for CAL, the student can work at his or her own pace.

Learning is more likely to take place when students are interested and involved in what they are doing. Computers help in the learning process when they arouse the user's curiosity, interest and involvement (Ball, 1987). Recent learning theories (Bruner, 1966 and Pask, 1976) stress that learning is most effective as an active process. Such learning means more than just 'learning by doing'. Rather than being a passive recipient of knowledge, the student must actively participate in the learning by discussion, written work or other means. Most computer programs require the user to participate to some degree although, in some cases, it may be little more than a "press space bar to continue" response. Genuine active learning can be stimulated by software which engages the user in dialogue that compels him to think about what is happening, allows the user to control his path through the program or to make decisions. Some programs allow users both to pose and solve their own problems (for example, function graph plotters). This flexible response to inputs is an important aspect of a computer's role.

This section has described many of the ways in which computers can help to make mathematical learning more effective and more appropriate to the needs of industry and commerce. These can be summarised as follows:

- it is an aid to problem solving
- it encourages investigations
- it facilitates simulations of models of physical systems
- it can display information graphically in a variety of ways.

Visualisation is a powerful aid to understanding.

- it provides a flexible response to students' inputs

- it promotes active student learning.

Past and current developments in computer-aided learning in mathematics have exploited these capabilities in various ways and with varying degrees of success. These will now be considered.

2.2 The Development of Computer-assisted Learning in Mathematics

In the early days of computers in education (pre-1975), very little educational software was available. Students had to learn a programming language such as Fortran and implement their own programs. Many statistical and numerical methods and algorithms involving tedious or complex arithmetic can be programmed relatively easily, thus enabling students to tackle problems which are impractical otherwise. Programming, however, is a very time-consuming activity. The pernickety nature of the prevailing compilers was a source of frustration, compounded by the slowness and unreliability of the mainframe computers and teletype terminals in use at the time. With obstacles such as these to overcome, there was a very real danger in seeing the program as an end in itself, rather than as a tool to be used to further the student's understanding of the topic being studied. Not surprisingly, these early ventures into computing were centred mainly on the universities, such as Cambridge, where their Atlas computer provided one of the first ever multi-access systems. At less fortunate institutions the students were hindered by batch-processing systems which must have dismayed all but the most persistent students!

Meanwhile, new developments were taking place, mainly in the USA, in the field of computer-assisted instruction (CAI), a successor to programmed learning. Tawney (1979) likens CAI to a dialogue between computer and student in which the computer questions, gives results, offers help, draws attention to particular features, etc., whilst the student responds, selects options, enters data and requests particular solutions. Proponents of CAI claim that it can individualise instruction and reduce labour costs. Tawney suggests that many tutorial systems are more concerned with efficiency of instruction than the quality of learning. Many teachers dislike the inflexibility of tutorial programs, and the fact that the approach has been predetermined by the author. Another major drawback is the initial high cost of producing the material.

Although some early CAI projects, for example, PLATO and TICCIT (Alderman, 1978, and Brerland, Amarel and Swinton, 1974), claimed potential educational benefits, there has been no widespread adoption of the systems by other institutions.

An effective way of reducing development costs is to use an authoring language to program the CAI units. One set of teaching units programmed in this way is the MALT system (Hunter, Rosenberg and Webber, 1985) which began development at the University of Glasgow under the National Development Programme in the 1970s. The authors stress that by far the most time-consuming stage of development is planning and structuring the educational content and presentation of each topic. The MALT interactive 'tutorial mode' teaching units and 'progress tests' are available for topics in 'A' level and first year undergraduate (Scottish) Mathematics, Statistics and Physics. The CAI units do not replace the normal lectures. They are intended to supplement the lecture material and broaden the student's experience. The advantage of such a system is that it encourages students to take responsibility for their own learning in terms of both content and pace.

A more recent development is the microcomputer-based CALM project (Computer Aided Learning of Mathematics) at Heriot-Watt University, although their software has been written in PASCAL to aid portability (Beevers, 1986 and 1988). The CALM units have been developed to assist the teaching of calculus to large classes of first year Engineering and Science undergraduates by replacing conventional tutorials with the computer-based system. A feature of the software is the effective way in which it exploits the graphical capabilities of the microcomputer to produce visually stimulating materials. Evaluation has shown that the students like the self-paced nature of the computer-based tutorials and that the weekly self-assessment built into the units is a powerful motivating factor.

Beevers reports a comparison of examination results at the end of first year between the computer group (49 Mechanical Engineers) and a group of 37 Civil Engineers matched for mathematics entry qualifications. In the calculus paper the overall average mark obtained is higher for the computer group, although the average of the algebra paper is lower by a similar amount. However, the superior marks are most pronounced for students who gained a Higher grade 'A' pass in Mathematics (23.4% higher), less striking for the largest group with a grade 'B' pass (10.2%) and, for 'C' pass students, the average mark of the computer group is actually 8.1% lower. It would appear, therefore, that the impersonal rigidly structured approach of the computer-based tutorials is more suited to high ability students, and that the very students who most need help receive least benefit. It seems likely that the computer tutorials give valuable reinforcement and practice to able students who readily assimilate new concepts, but fail to assist the understanding of the less able students.

A live tutorial is effective because the teacher has within him not only the knowledge of the subject matter but also of the learner or learners (Hooper, 1976). O'Shea and Self (1983) argue that, for CAI systems to be effective, they must incorporate artificial intelligence techniques to develop an understanding of the student's needs, thus treating the student as a thinking individual. The student model contained within the program should be adaptive, i.e. as the student learns, the model is modified so as to represent the current state of the student's knowledge. The program could then predict the student's future needs. Only then would learning become truly personalised.

Hartley (1987), too, stresses that future developments must be linked to the methodologies of cognitive science and artificial intelligence. He describes an on-line knowledge-based help system which works in conjunction with an application program to help develop a student's planning, investigative and problem-solving skills. The program contains an adaptive model of the student's knowledge, performance and mistakes, and gives advice accordingly.

The potential to develop CAL systems of this nature is limited by the sophistication of the programming languages used. Rushby (1986) argues that the authoring languages currently available have proved themselves inadequate as the developers of the majority of applications have reverted to general purpose languages for some sections of the software.

Until intelligent, adaptive computer systems are developed which do contain models of the knowledge outlined above, computer-aided tutorials cannot successfully replace the teacher. Production of such knowledge structures is costly and time-consuming, and progress to date has been slow.

2.3 The Computer as a Tool

The advantage of a human tutor over the computer is his ability to make decisions and adapt his teaching method and pace to suit the learner. The strength of the computer lies in its computational power, its graphics and its interactivity. The functions of the computer and teacher are therefore complementary.

When used as a tool, the computer does not contain any representation of the learner's knowledge. In this mode, the computer is primarily a computational aid, whether the user writes his own programs or uses prepared software. As computers have become cheaper, more reliable and available to larger numbers of students, more attention has been paid to developing software to assist the teaching of mathematics. For many years most of the software produced was badly written and difficult to use. With a lack of suitable support material, the use of such software is often limited and unimaginative.

The CALNAPS software, (Katsifli, 1986), written in FORTRAN for large mainframe computers, is designed to support the teaching of numerical analysis to undergraduates. In one of the few reported studies at higher education level of the effect on a student's learning of using computer packages, Katsifli found that the use of CALNAPS stimulated discussion between students and lecturer, and that the questions asked reflected more involvement with mathematical concepts than those asked by students using traditional methods. She concludes that the CALNAPS group achieved a deeper understanding of the numerical methods being studied. The number of students involved in the study is not given. Users of CALNAPS suffer from the usual drawbacks of mainframe computing, namely, a lack of colour to illustrate the output, uncertain response times, remote printing facilities and difficulty in trapping real time errors. Although the graphics are good, the method of inputting a function is cumbersome as FORTRAN is not usually an interpretative language. The screen output is verbose in order to assist first time users but, as a consequence, is irritating to the more experienced ones.

A dedicated mathematics microcomputer laboratory was introduced at

Paisley College of Technology in 1982 for first year students of the Mathematical Sciences degree course (Glen and Walker, 1985). During a weekly session in the laboratory the students work in small groups using prepared software to solve problems and experiment with the mathematical concepts and techniques of the course. Unreliability of the hardware proved to be a problem initially, but useful experience was gained and use of the laboratory has had a positive influence on the attitudes of both staff and students (SED, 1986).

One of several microcomputer-based packages available to support the teaching of mathematics is a suite of programs developed by Jacques and Judd (1985). Although intended to help students test the robustness and convergence of numerical methods, the lack of flexibility in the overall approach inhibits experimentation. After the initial selection of method, the order of progress through a program is usually fixed. The graphical output is limited and appears to have been added as an afterthought, rather than as an integral feature of the package which would enhance the student's understanding of the solution. For example, two solutions cannot be compared graphically on the screen. A more flexible program design is required to encourage an investigative approach by the students.

Inflexibility is also a drawback of the MIME software units, developed at Loughborough University of Technology for use in the teaching of mechanics and statistics to the 16-plus age group (Bajpai et al., 1987). Although robust and visually interesting, the programs are tutorial in nature and, thus, limit the user's control of the learning process. The units are difficult to use without frequently referring to the user documentation. Many of the topics are presented as animated sequences, which, though perhaps considered necessary to interest school pupils, are gimmicky and, at times, irritating.

A separate project concerned with software for undergraduate mathematics also adopts a highly graphical approach but with fewer gimmicks. The first unit to be completed, which deals with complex transformations, is a useful package but, again, is not easy to use without frequent reference to the user guide to identify the required input commands.

A set of simulation programs developed at the University of Sheffield to aid the teaching of mechanics at A-level (MAP, 1987) avoids the inflexibility of the MIME units whilst retaining an interesting presentation. The programs are designed to allow the teacher to fit them to his own teaching style, although the accompanying booklet does offer suggestions for their use and for investigational activities. Animation is used simply but effectively to illustrate, for example, the path of a projectile or the motion of a rotating body, but the user is actively involved at all times both in supplying input data and choosing between different forms of output. A good screen layout includes the animated display, graphical and numerical output and brief operational instructions. No user guide is required. In addition to simulation of general topics in mechanics, the package contains some more specific problems chosen to interest the user and to encourage him/her to apply his/her knowledge to everyday situations.

The quality of the software used can be crucial to the success of a project (Johnston, 1987). If a program fails to exploit the hardware or to motivate the students sufficiently, then its full educational potential will not be realised. It has been argued that developers should adopt a more consistent approach to screen layout, data input and the user interface (Bajpai <u>et al</u>., 1985). In particular, it is suggested that software must be interesting, robust, interactive and user-friendly. Of greater importance, however, is the way in which a program is used. When computers assist in changing what is being taught or how mathematics is being learned, then their impact is more fundamental.

2.4 New Teaching Approaches

Various specialist programs such as MINITAB and INSTAT have undoubtedly changed the face of statistics teaching in many higher education institutions within the past decade. In the Mathematics Department of Napier Polytechnic, all statistics learning is now aided by computer packages, supported by a comprehensive set of worksheet and handbook materials. Considerable emphasis is placed on an exploratory approach and more time is devoted to analysis and interpretation of results.

Stern (1987) argues that one of the main benefits of using computers in statistics is to enable the inclusion of topics that previously could not be taught or were under-emphasised. He cites experimental design and survey methods as examples. Large, realistic data sets can be used to give students experience of handling and presenting such data.

Programs used as computational tools focus the user's attention on to the output and thus encourage students to investigate the effect of changing initial values etc. However, a programming microworld such as LOGO provides a framework for exploring mathematical concepts (Hoyles, 1987). During a three-year longitudinal study of secondary school children using LOGO within the mathematics classroom (Hoyles and Sutherland, 1987), detailed data were collected from eight pupils. The children, working in pairs, were encouraged to experiment freely and to set their own goals. Further data were gathered from ten other classrooms into which the project was later extended. The findings suggest that a move towards a more pupil-centred approach, which was implicit in the project, encourages pupil initiative and stimulates collaboration and mathematical discussion between pupils. In addition to fulfilling an organisational role, it is important for the teacher to encourage the pupils to reflect on what is happening and to present new problems. Presenting the pupil with a carefully chosen investigation can direct the pupil's thinking towards particular concepts. The process of experimentation and of switching between graphical and symbolic representations helps to formalise the pupil's intuitive conceptions.

A similar pupil-centred approach has been followed by Pearson (1986) using LOGO in the mathematics classroom with 11- to 12-year-old children. As the pupils gradually adjusted to setting their own goals, she found that the pupil/teacher relationship changed. Instead of the teacher interrogating the pupils to test their knowledge, a conversation developed with each side genuinely asking the other for information, but also realising that there was not necessarily a unique, 'correct' answer to the question.

A LOGO or MINITAB environment represents a move away from the use of computers to reinforce existing teaching methods and curricula towards a more learner-oriented, investigative approach to mathematics. With the advent of cheap microcomputers, it has become possible to develop this new approach to learning with a wider range of applications.

A course in mathematical modelling, facilitated by a systems simulation package, is described by Clements (1986). This enables engineering students to model continuous systems which they are unable to solve analytically. A series of practical investigations assists the development of the skills necessary to tackle real-life problems. The integration of numerical and analytical techniques is seen by Clements as an important aspect of the course. Although familiar to engineering students, the block-oriented input required by the programs may not appeal to other users.

An investigation of a more meaningful way to introduce the calculus to school pupils resulted in the development of the Graphics Calculus I package (Tall 1986). A traditional symbolic approach emphasises the manipulative techniques used to obtain derivatives, integrals, etc. The computer program incorporates imaginative interactive graphics techniques which allow calculus to be introduced from an integrated numerical and graphical approach. Fundamental mathematical concepts, such as limiting processes and rates of change, can be explored dynamically, encouraging deeper understanding by the pupils. Tall (1987) believes that appropriate computer packages are useful both when introducing new mathematical concepts to pupils and for extended investigations. He suggests that simple programming also has a part to play, however, "in helping pupils to gain insight into the nature of mathematical algorithms", a role also advocated by Francis (1988). Programming promotes individual initiative, and pupils derive considerable satisfaction from solving the problems posed, but it is costly in terms of time and effort.

Harding (1984) claims that the close involvement which a student develops when writing and de-bugging a computer program to solve a mathematical problem can lead to a deeper understanding of the problem. This view led the developers of the CATAM (Computer-aided Teaching of Applied Mathematics) project at the University of Cambridge (Harding, 1974) to make students write their own programs but, in order to direct their energies towards investigating the mathematics of a problem rather than programming as an end in itself, a supportive software environment has been developed. This takes the form of a library of graphical and numerical routines which the student can incorporate into his program.

Harding points out that continuous formative evaluation of a project such as CATAM is necessary for it to achieve a cost-effective improvement in the student's learning. In the twenty years of its existence, CATAM has adapted and matured, in the light of changing availability of hardware, the needs of the students and the experience of the project staff. The fundamental aim of the courses, however, is still to develop an investigative approach to solving a wide range of problems numerically. A survey (Harding, 1976) indicated that the students appreciated the relevance of the course to real problems and the insight computer-generated graphical solutions gave to theoretical results.

A philosophy similar to CATAM has been adopted at the University of Birmingham where all first year mathematics undergraduates follow an introductory course of computer-based investigations (Beilby, 1987). During laboratory sessions the students use some prepared programs, usually of the demonstration type, but they gradually learn to write and modify their own programs, supported by a library of procedures and program constructions similar to that used by DAMTP at Cambridge. The computer investigations fulfil a dual role of reinforcing lecture material and introducing students to a wider base of mathematics than had previously been covered in first year.

Student response to the course is varied as a significant proportion of the students are attracted only by the mathematics and have no enthusiasm for computing. There is evidence that many students do enjoy the course and show a willingness to experiment. Staff-student relationships have improved through contact in the laboratory. In general, the students show a preference for writing their own programs rather than using prepared software, the former apparently allowing them to explore the mathematics in their own terms.

It is interesting to note that, both at Birmingham and at Cambridge, it was found that over 90% of the students entering the mathematics course at university had experience of using computers in schools. A considerable number, in fact, owned a computer. The situation regarding entrants to science and engineering disciplines may be different. At Napier Polytechnic, an average of only 50% of all students entering degree courses in science and engineering have had previous experience of computing (1987 survey).

2.5 Evaluation studies

The shortage of published evaluations of computer-aided learning has been criticised by several authors (Johnston, 1987, Ridgeway, 1986, Roblyer, 1985, Harding, 1976). This lack is particularly noticeable in the tertiary education sector. Those which have been reported tend to concentrate on a description of the software, the way in which it has been used and how the class was organised rather than an analysis of the learning experiences taking place or the impact on the curriculum.

Hartley and Bostram (1982) examine teachers' attitudes to computer-based teaching packages available at that time. Many programs were rejected because they were not considered relevant to the current classroom curricula. Qualities which teachers considered important in selecting a program for use in the classroom included:

(a) ease of use;

(b) flexibility in mode of use so that a program could be used either for a classroom demonstration or by small groups;

(c) presentation of help information on the screen rather than in a separate user guide;

(d) availability of various degrees of help/prompting to cater for a range of level of experience of the user;

(e) graphical output wherever possible.

An experiment to compare the effectiveness of CAL software with traditional methods to teach a topic area in mathematics showed a small but not statistically significant difference in favour of the CAL group in the post-test results. The authors point out that this result was not altogether unexpected since the CAL packages used closely followed the established style and content of teaching. Differences could only emerge, therefore:

> "not through differing cognitive demands made on the groups (confirmed by observation schedules), but through the interest and feedback provided by the program, and through additional opportunities for the teacher to instruct small groups of pupils".

It may be more effective to approach a topic in a new way in order to exploit the potential of the computer, particularly where conventional methods are not very successful.

Fraser (1987) asserts that the computer is a powerful aid to better communication in the classroom. The visual images and dynamic nature of the material presented by computer encourage the expression of ideas. Observation studies of 170 mathematics lessons to 13/14-year-old pupils found that, when using computer software, children:

> "moved into investigating and problem-solving activities more frequently than had been observed in the lessons without computer support."

She suggests that the children were imitating roles normally associated with the teacher or computer.

Software produced by the Shell Centre for Mathematical Education for GCSE pupils has been designed to encourage pupils to adopt 'teacher' roles of 'explainer', 'manager' and 'task-setter'. The software is supported by a comprehensive set of teaching materials including worksheets and lesson plans. Fraser deduces that such packages, when used as an integral part of the curriculum, promote an environment which encourages discussion, problem-solving and investigations.

By contrast, Ernest (1988) found no significant gain in achievement in an experiment designed to test the effectiveness of a microcomputer in teaching problem-solving skills to 13-year-old children. The experiment, which was conducted on a class of 21 children, consisted of just two and one-half hours of instructional activity plus two tests (pre-test and post-test), spread over a 10-day period. During this time the children worked through a sequence of problems, one half of the class working at computers in pairs and the others remaining as a group, whilst the teacher led a joint exploration of the problem and demonstrated the program. Over such a short time-scale it is hardly surprising that neither group showed any significant gain in achievement. It could also be argued that the written post-test was an inappropriate way to measure the students' achievements. Neither the skills used when solving problems with the aid of a computer nor the types of problem tackled are necessarily similar to those employed for a written test.

The results of the experiment reinforce the view that this is not an appropriate style of evaluation of the learning experience, since it fails to take into account the different type of learning which may take place when CAL is used. Qualitative evaluation of CAL, using observational techniques and the opinions of the participants obtained by interview or questionnaire, produce a more illuminating picture of the learning taking place.

2.6 Summary

Computer-aided instruction is basically directed towards teaching the same material to students but in a new, more user-paced, way. To this extent it has achieved some success, particularly where large classes are involved. It does little, however, to encourage creative and analytical skills which students need to develop to enable them to apply their knowledge confidently to problems of the real world.

There is evidence that the scope of problem-solving can be greatly widened by use of computer packages, and that such packages can also stimulate investigative activity. Experience at Cambridge and Birmingham suggests that, for mathematically able students at least, programming can enhance the effectiveness of investigations in the computer laboratory. Other authors believe that programming promotes deeper understanding of concepts. Unfortunately, few courses allow sufficient time for such an approach to be adopted, even with the back-up of a software library. Many of the benefits of programming can still be achieved, however, by careful design of prepared software. Flexibility, good graphics, ease of use and interactivity are essential. Worksheets used to direct the student's investigations must also be carefully constructed. Finally, any teaching approach requiring use of computer packages will be more effective if it is an integral part of the curriculum.

As it seems likely that not only the teaching approach but also the curriculum itself changes when computer-aided learning is introduced, traditional quantitative evaluation procedures are inappropriate. Illuminative techniques, based on observation, interviews and questionnaires, will provide a more detailed understanding of the changes that take place.

CHAPTER 3

The Learning Environment at Napier Polytechnic

3.1 Introduction

Napier Polytechnic is located on three main sites. Due to the wide diversity of courses serviced, mathematics staff are based at all the sites and teaching takes place at them all. There is a natural divide between the Professional Studies Faculty courses and the remaining courses which are science- or engineering-based. The nature and teaching emphasis of the mathematics for the business studies courses is understandably different from that for the more scientific disciplines. This study concentrates on the mathematics education of the science and engineering students.

The courses based within the Mathematics Department are a B.Sc. in Mathematics with Engineering Technology at honours and ordinary degree level and a SCOTVEC Higher Diploma course in Mathematics, Operational Research and Statistics. The Department has a major role in the B.Sc. in Science with Management Studies (honours and ordinary degree) and also teaches mathematics and statistics as service subjects to students on a wide variety of science and engineering degree and higher diploma courses.

A typical mathematics service course for an engineering degree involves 3-4 hours per week for 2 or 3 years of the course. Fifty per cent of the time would be devoted to lectures. The remaining 50% is occupied by tutorial sessions during which the students work through sheets of exercises and problems, with lecturers on hand to assist when necessary. These classes also provide an opportunity for students to seek assistance with any topic not understood in class. Class sizes range from 10 to 60 for lectures, but the ratio of students to staff is 20 to 1 or less for tutorials.

The majority of students at Napier are Scottish and enter the first year of their course directly from school at the age of 17 or 18. They will have completed either 5 or 6 years of secondary school education leading to Scottish Certificate of Education examination passes at Higher, Ordinary and Standard grade. Those students who stayed at school for six years may also have studied one or two subjects to the more advanced Sixth Year Studies level.

Scottish 'Highers' are sat at age sixteen/seventeen, a year earlier than English A-levels, and the knowledge content is consequently less. A Scottish school education is less specialised and pupils study a broader range of subjects up to Higher grade. The starting point of mathematics teaching in first year higher education courses is about halfway through the A-level syllabus.

A wide variety of teaching resources is available to lecturers either from the Department or through the Polytechnic's Learning Resources Unit. These include overhead projectors, Keller Plan units, a comprehensive set of tutorial sheets, books, video programmes, audio-tapes, open-learning packages and computer programs. A central computing facility, comprising PRIME computers, has terminal rooms at all the main sites. Large commercial software packages such as the NAG Library, MINITAB and ECSL (simulation package) are available on this system.

Careful selection of the most appropriate teaching method is an essential prerequisite for effective learning. It is important for the lecturer to devote time to selection of the most appropriate teaching medium for each part of the course and to the planning of the total learning package (Searl, 1985).

3.2 Early Use of Computers

Early experience of using computers to teach mathematics at Napier Polytechnic was gained during the National Development Programme in

Computer-assisted Learning (NDPCAL), between 1974 and 1977.

Implemented in FORTRAN on a multi-access computer, MATLAB (mathematical laboratory) was developed as an interactive problem-solving tool, which removes the arithmetical drudgery from mathematical, numerical and statistical techniques (Leach et al., 1977). Its mathematically-oriented input language does not dictate any particular teaching style but enables lecturers to adapt its use to their own needs. Back-up material provides examples of how to use MATLAB. During the 1976-77 session MATLAB was used, at Napier, by over 300 students from 16 courses for a total of 47 sessions involving 9 members of staff. It was also successfully transferred to two other Scottish colleges of higher education.

Serious problems with computer reliabilty, both software and hardware, hindered expansion of the use of MATLAB at Napier. In particular, the CTL modular one computer could only cope adequately when dedicated to MATLAB. Any attempt to run a non-FORTRAN program simultaneously increased the response time to unacceptable levels.

Although MATLAB has been superseded by more user-friendly, microcomputer-based software, the project introduced staff at Napier to the potential of computers at an early stage. Curriculum innovation is, inevitably, a slow process but the enthusiasm of a few lecturers had been fired. New ideas for using computers to assist the learning of mathematics and new ways of presenting the subject material gradually emerged.

By 1982 the Mathematics Department had acquired several PET and APPLE II microcomputers and a few lecturers had begun to use them in the classroom. Two members of staff had written some short programs in BASIC to illustrate statistical or numerical methods. In September 1982 I was appointed as a programmer in the Department and began to develop programs on an APPLE microcomputer to the specification of individual lecturers.

Gradually two important methods of using the computer as a tool to assist the learning of mathematics evolved (Mackie and Scott, 1988a).

The first of these involves using computer-based demonstrations within lectures, and has been successfully introduced over a range of courses with a wide variety of topics. A successful demonstration requires a microcomputer linked to monitors in such a way that the interaction is clearly visible to all. Using this arrangement the lecturer can illustrate important concepts, and solve problems in a manner which time and conventional facilities would not permit. For example, a good graph plotting program is ideal for illustrating the convergence of a Fourier series.

The second method followed naturally from the first, as the interest generated by this use of the computer made it evident that students would benefit from using the packages themselves. The Department considered that this could best be accomplished by acquiring appropriate computer equipment and establishing a dedicated mathematical sciences laboratory.

3.3 Mathematics Laboratories

Having investigated the equipment available, it was decided that ACORN BBC microcomputers offered the best value for money at that time. The speed, memory, graphics and networking capability of these computers would allow a wide range of mathematics software to be implemented in an efficient and stimulating manner.

Computers were duly purchased and a mathematical sciences laboratory was established at the Merchiston site in September 1984. It consisted of 10 BBC microcomputers linked to a file server and a 30 megabyte Winchester disc by an E-net network. The network also included a line printer and a graph plotter. The machines were arranged round the perimeter of the room, facing the wall, due to restrictions of the cabling and available power points. However, students could easily turn their chairs to face a lecturer at the front if necessary. The room also contained a mobile whiteboard, an overhead projector and a cupboard for the laboratory worksheets and user guides.

The first program to be implemented in the laboratory was a graph plotting package. Students quickly gained confidence when using this program and, being of such a general nature, it proved useful in a wide variety of applications. The library of software has gradually been extended to include packages covering several numerical methods, the solution of differential equations, graph plotting in 3D, Fourier analysis, linear programming, queue simulation and statistical packages. The majority of the software has been developed within the Department.

A second mathematics laboratory was established at the Craiglockhart site in February 1987 to cope with multi-site working and increased demand. This facility consists of 12 BBC Masters arranged in four rows of three, together with two line printers and a Winchester disc linked by an Econet network (Figure 3.1). The following year the Merchiston laboratory was moved to an adjacent room, extended to 17 machines with three line printers and converted to the Econet network system.

One of the priorities in setting up the laboratories was that the programs should be easily accessible to staff and students who had no previous experience with the system. They should be able to browse through the available software and feel free to experiment. Accordingly, all the teaching software is stored on the hard discs and accessed from any machine on the network by simple menus (Figure 3.2), introduced in June, 1987. The menus can easily be extended or modified as new

Figure 3.1: Mathematical Sciences Laboratory

Figure 3.2: Menu of Mathematics Programs

Computer-enhanced
mathematics

Help	NODES	NEWTON	LINPROG
GRAPH	DEPLOT	NUMINT	LPROG2D
SURF	TRIG	GAUSS	TRANS
MEI	FOURIER	EIGEN	QUEUE
COMPLEX	FTRS	EXPRND	

Use arrows to select option
then press RETURN

programs are added to the system. The menus and the software in the two laboratories are nearly identical, the difference being some additional facilities in the laboratory equipped with BBC Master computers. In general, the user does not require to be allocated a personal user number in order to access the menus and programs. Security units were purchased for all the computers so that the laboratories could be made available to students on an open-access basis when they are not time-tabled for class use and during the evenings.

3.4 Software and Worksheets

From the outset all the teaching software has been developed in response to requests and initial ideas from lecturers in the Mathematics Department. Close collaboration with the lecturer is maintained to ensure that the detailed specification and program design match his or her requirements. A team approach of this nature is considered by Nicolson and Scott (1986) as "the only viable method for producing quality software". They suggest that "the weakest link in the process is the specification stage where a classroom teacher must convey the crucial teaching ideas to a computer programmer". My experience as a mathematician, programmer and part-time lecturer undoubtedly helps me to bridge this potential gap in communication and, frequently, to assist with the detailed specification.

To some extent, a consensus has emerged amongst developers of CAL, mainly through demonstrations and workshops at conferences, as to what are 'good' program design features and approaches (see, for example, Bajpai et al., 1985, and Harding, 1986). For example, highlighting can be used for menu selection, a space at the foot of the screen for messages, default values for program parameters, etc. Many such features have been adopted for the mathematics software development at Napier.

The computer programs are not self-contained learning packages.

They are designed as tools to assist and enhance the learning of mathematics and to encourage investigations and modelling.

Of course, not many students will progress sufficiently if left to experiment completely freely. Just as in any other science laboratory, direction must be provided via a worksheet (Tawney, 1979). Such worksheets must be carefully designed and well prepared. Their design should reflect the aims and objectives of the laboratory session. Depending on the lecturer's perception of the exercise, the written material may take the form of a set of questions similar to the ones encountered in a traditional tutorial/problem class. However, it is more likely that a worksheet would include some open-ended investigative exercises. Some of the work undertaken might be of an experimental nature, leading to written reports for submission to the lecturer concerned.

Worksheets have been prepared by different lecturers, usually for use, in the first place, with one of their own classes. Gradually, a variety of worksheets has been accumulated and made available as a common resource.

3.5 The Continuing Development

Concurrent with developments in the use of microcomputers to support the teaching of mathematics, there has been a similar trend towards the use of mainframe software. In particular, use of a computer package such as MINITAB is now accepted as a necessary part of any statistics course taught by the Department.

The decision to use computers as a teaching resource for a particular class is usually at the discretion of the lecturer concerned. In general, use of computers is not an optional extra to a mathematics course, as in some institutions, but is an integral part of the curriculum. Occasional workshops/seminars are organised to familiarise staff in the department with new facilities and software in the mathematical science laboratories. A laboratory session takes the place of a traditional tutorial or lecture.

As with any innovative teaching, the use of computers entails much preparation and often involves more complex planning than a normal class, especially if access to computers is only required occasionally. The laboratory may not be free at the most convenient time. The lecturer must familiarise himself with the operation of the software before exposing his students to it, existing worksheets must be studied to determine their suitability for the class and, if necessary, new worksheets must be prepared. Worksheets are frequently designed by a lecturer for a particular course and so are highly relevant to that course. Where the class size is too large to permit supervised sessions to be accommodated in the existing timetable, assignments can be issued for students to complete in their own time after an introductory session. Some computer-based work is assessed.

The laboratory environment is not a static one. There is a continuing process of modification and refinement of the software and worksheets, as a result of input from several different lecturers.

Teacher-led innovation of this nature is usually the most successful way of introducing curriculum change and new teaching approaches (Candy, 1988). There has frequently been too much emphasis on developing educational software for its own sake without reference to the needs of teachers and students. The experiences of the teacher using a computer package in the classroom can make a valuable contribution towards subsequent modifications and improvements.

As a part-time lecturer and tutorial assistant I have been able both to observe students using the computers and to participate in the teaching. I have gained first-hand experience of difficulties encountered by students and staff when using the computers or software and also of the type of learning taking place. Informal evaluation of the software originally developed for the Apple computer enabled much improved versions to be written second time round for the BBC computers. In addition, a wide study of work taking place at other higher educational establishments has been undertaken. This includes several visits and attendance at a considerable number of seminars and conferences concerned with CAL. Familiarity with developments elsewhere has made it easier to exchange software with other institutions and, thus, to concentrate on producing programs which are either not available from other sources or whose approach is radically different.

3.6 Aims and Scope of the Investigation

As the use of computers to assist the teaching of mathematics at Napier has evolved, changing patterns of teaching and learning have become apparent. An increasing awareness of the potential of CAL, improved availability of micros in sufficient numbers to allow adequate student access and familiarity with projects at other institutions have all contributed towards this adaptation of the learning environment. In particular, with the introduction of laboratories, opportunities opened up for investigative work in many areas of the curriculum in which it was not previously possible. Not only is the laboratory environment conducive to experimentation, it also lends itself to co-operative working between pairs or small groups of students.

Ideas for program presentation and accompanying worksheets have developed in a similar fashion. Some software packages prove to be more effective learning tools than others. Many so-called educational programs are black-box calculators which simply provide the answer to a problem; others are difficult to use or inflexible.

Can the use of well-designed packages enhance the learning of mathematics by, for example, aiding the student's understanding of concepts or algorithms? What impact does the use of such packages have on the mathematics curriculum? Does the introduction of computer-based work result in more emphasis on some areas of the curriculum and less on others? The need to answer such questions led to this research project.

The success of the CATAM project at Cambridge (Harding, 1984) and the mathematics laboratory at Birmingham (Beilby, 1987) is, without doubt, partly due to the role of the computer having been determined by close examination of their own teaching/learning situation. Experience at Napier suggests that our students would benefit from the use of computer-based packages which are specifically designed to:

(i) enhance the students' understanding of particular concepts or algorithms,

(ii) facilitate problem solving, and

(iii) encourage investigative work.

The effective use of such packages will have a significant impact on the mathematics curriculum.

This project aims to test this hypothesis by:

(i) the development and evaluation of two packages designed to meet the criteria outlined above;

(ii) investigating the impact which the use of the packages has on the mathematics curriculum, in particular, with respect to teaching approaches and student attitudes towards mathematics;

(iii) examining the feasibility of transfer of the materials produced to other higher educational establishments.

CHAPTER 4

The Learning Packages

4.1 Design Criteria

Selection of topics for the learning packages was influenced by the perceived needs of the Mathematics Department at Napier and the criteria listed in section 3.6. The two contrasting areas of the curriculum chosen were linear and integer programming using the Simplex method and the numerical solution of initial-value differential equations. The packages would not be self-learning units but computer-based tools designed to enhance the students' learning.

It has already been shown that the design of educational software is important (section 2.6). Computer programs should be easy to use and, in general, not require a user manual although they must be accompanied by appropriate back-up material to direct and/or stimulate the learning process. They should be menu-based and flexible in operation, making full use of the computer's interactive and graphic capabilities. A program must allow the user to try any apparently reasonable experiment and, if the algorithm fails or the program produces an error message, it should tell him why. Experiments by Anderson et al. (1971) in computer-aided instruction provided some evidence that feedback from incorrect responses was more beneficial to the learner than acknowledgement given for correct answers. Kulhavy et al. (1976) further demonstrated that students who were confident in their response spent much longer considering the mistake when their answer turned out to be wrong.

4.2 The Linear Programming Package

Use of the Simplex method for linear programming involves matrix transformations which students find difficult and tedious. Traditional teaching enables them to progress, at best, to solving three or four variable standard problems. The complexity of the arithmetic tends to inhibit understanding of the process being carried out. Virtually no post-optimal analysis is tackled. Although the branch and bound method for integer programming is taught, students are normally given a data sheet (Figure 4.1) from which to work. This contains intermediate solutions to the problem for all combinations of constraints which are required to construct the branch and bound solution tree.

The aims of the teaching package, LINPROG, are twofold. The first of these is to assist students to understand the Simplex algorithm and the concept of duality. Secondly, having mastered the algorithm, an alternative mode of use facilitates post-optimal analysis or the construction of a branch and bound tree to enable the solution of integer programming problems. In this way, opportunities are created for problem solving and investigative work which were not previously possible. Research revealed no software available from other sources that would fulfil these aims.

Description

LINPROG copes with three main classes of problem:

(a) The standard class of linear programming problems where one is required to

either Maximise $f(\underline{x})$

subject to $g_i(\underline{x}) \leqslant b_i$

or Minimise $f(\underline{x})$

subject to $g_i(\underline{x}) \geqslant b_i$

There are many realistic problems of this nature. For these problems the user can follow a tableau by tableau display through the Simplex method. The user is asked to select the pivot element, to decide when the solution is optimal and to extract the solution from the final tableau.

Figure 4.1: Integer Programming Datasheet

Problem No.	Additional Constraints	x_1	x_2	x_3	x_4	I
1	ORIGINAL PROBLEM	0	22.9	18.1	1.43	24380.95
2	$x_1 \geq 4$ $x_2 \leq 20$	5	20	18.3	0	24333.33
3	$x_1 \geq 2$, $x_1 \leq 3$ $x_2 \leq 21$ $x_3 \geq 19$	2	20.5	19	0.25	24200.00
4	$x_1 \geq 2$ $x_2 \leq 22$	2	21.7	18.2	0.857	24361.91
5	$x_1 \geq 4$ $x_2 \leq 20$ $x_3 \geq 19$	4	19	19	0	24000.00
6	$x_1 \geq 2$ $x_2 = 22$	I N F E A S I B L E				
7	$x_2 \leq 22$	1.5	22	18.2	1	24366.67
8	$x_1 \leq 1$ $x_2 \leq 22$ $x_3 \leq 18$	1	22	18	1.5	24200.00
9	$x_1 \geq 2$, $x_1 \leq 3$ $x_2 \leq 21$	3	21	18.3	0.5	24333.33
10	$x_2 \geq 23$	0	23	18	1	24300.00
11	$x_1 \geq 2$, $x_1 \leq 3$ $x_2 \leq 21$ $x_3 \leq 18$	3	21	18	1	24200.00
12	$x_1 \geq 4$ $x_2 = 21$	I N F E A S I B L E				
13	$x_1 \geq 2$ $x_2 \leq 21$	3.25	21	18.3	0.5	24350.00
14	$x_1 \leq 1$ $x_2 \leq 22$	1	22	18.3	1	24333.33
15	$x_1 \geq 4$ $x_2 \leq 20$ $x_3 \leq 18$	4.67	20	18	0.667	24133.33
16	$x_1 \leq 1$ $x_2 \leq 22$ $x_3 \geq 19$	0	21.5	19	0.75	24200.00
17	$x_1 \geq 4$ $x_2 \leq 21$	4	20.6	18.3	0.286	24342.86

Minimisation problems are solved by first forming the dual problem. The algorithm used by the program follows that taught to the students at Napier.

(b) General linear programming problems where any combination of constraints is allowed:

For these problems, the program uses a BIG-M method (Zionts, 1974) but only the solution is given to the user.

(c) Integer and mixed integer programming problems:

The initial problem is solved as for a general linear programming problem. The user can then add further constraints and solve the new problem thus formed or return to the initial set of constraints.

Thus the branch and bound method can be implemented with ease.

Input to the program is straightforward. The user is led through a series of questions to establish the type of problem, number of variables and constraints. The program assumes that all the variables are $\geqslant 0$. The user chooses between a tableau by tableau solution using the Simplex method (Figure 4.2), in which he makes decisions at each stage, and a solution only option. The former option is available for the standard problem described above. If "solution only" is selected, the solution is followed by the comprehensive menu of options (Figure 4.3) which enables the user to carry out sensitivity analysis, parametric programming or the branch and bound method.

Prerequisites

The student is assumed to have been introduced to linear programming before using the package. In particular, the student would normally have been taught the Simplex method. Before attempting integer programming problems, the student should be familiar with the branch and bound method.

Figure 4.2: LINPROG: Tableau by Tableau Output

```
This is the initial tableau of the standardised problem

    X1       X2       X3
    3.000    6.000    3.000    1.000    0.000    0.000    22.000
    1.000    2.000    3.000    0.000    1.000    0.000    14.000
    3.000    2.000    0.000    0.000    0.000    1.000    14.000
-----------------------------------------------------------------
   -1.000   -4.000   -5.000    0.000    0.000    0.000     0.000

Is this an optimal solution (Y/N) ? n      Correct
Select next pivot column  3
Correct
Select pivot row  2                        Correct

    X1       X2       X3
    2.000    4.000    0.000    1.000   -1.000    0.000     8.000
    0.333    0.667    1.000    0.000    0.333    0.000     4.667
    3.000    2.000    0.000    0.000    0.000    1.000    14.000
-----------------------------------------------------------------
    0.667   -0.667    0.000    0.000    1.667    0.000    23.333

Is this an optimal solution (Y/N) ? n      Correct
Select next pivot column  2
Correct
Select pivot row  1                        Correct
```

Figure 4.3: LINPROG: Menu of Options

1	Display current problem
2	Solve current problem
3	Add a constraint
4	Change objective function
5	Alter coefficients of a constraint
6	Change r.h.s. of a constraint
7	Return to original problem
8	Terminate

The teaching objectives of LINPROG are:

(i) to enhance student understanding of linear programming and, in particular, the Simplex method;

(ii) to facilitate problem solving which involves the use of linear programming;

(iii) to facilitate the solving of integer programming problems by the branch and bound method;

(iv) to encourage investigative work, in particular, post-optimal analysis.

Implementation and use

Written in BBC-BASIC, LINPROG has been implemented on a stand-alone Acorn BBC microcomputer and on the ECONET network in the mathematics laboratory. A preliminary version was made available in February 1985 for field testing with students. Since version 3, described in this report, was released in December 1985, minor modifications have been incorporated as a result of formative evaluation. The package has been in regular use with several different classes at Napier each session since 1985.

LINPROG has been marketed by Napier Polytechnic since April 1986 (Mackie and Scott, 1986) and has been sold to over 20 educational establishments throughout the United Kingdom. It has been demonstrated at six conferences (Mackie, 1985; Scott, 1986), including the international ICME-6 conference in Budapest in August 1988. A PC-based version of LINPROG has now also been implemented.

Back-up material

A program which enables two-variable linear programming problems to be solved graphically is available in the mathematics laboraories with an accompanying worksheet. This program can be used during the initial stages of a linear programming course before introducing the Simplex method.

Three worksheets have been written to cope with topics covered by the LINPROG package (Appendix 1). These are:

(i) Introductory linear programming

Included in this worksheet are exercises to formulate a linear programming problem mathematically, examples which demonstrate the operation of the Simplex method and compare it to the graphical method of solving 2- and 3-variable problems and exercises which explore the concept of duality.

(ii) Post-optimal analysis

The exercises on this sheet enable the student to investigate the effect of changes in the objective function or constraints of a linear programming problem. Both sensitivity analysis and parametric programming are included.

(iii) Integer linear programming

Integer and mixed integer programming problems are solved graphically and by using LINPROG to implement the branch and bound method. For the latter the student constructs the branch and bound diagram on paper using the program menu options to form and solve the new problem at each node of the solution tree.

All three worksheets have been designed to include realistic problem solving and (ii) and (iii) to encourage investigative work.

4.3 The Differential Equations Package

One of the early programs written for the mathematics laboratory enabled the user to solve first order differential equations by a variety of Runge-Kutta methods. Although this program proved useful for a variety of teaching purposes, many ideas for additional features had been expressed by members of staff using it. The following features were identified that would be of particular benefit:

(i) graphical output of the solution;

(ii) the ability to compare the analytical solution, if known, with the numerical solution;

(iii) the ability to solve a system of differential equations;

(iv) the ability to obtain phase portraits;

(v) the addition of predictor-corrector methods.

Since several programs already existed for the numerical solution of differential equations, these were studied carefully to determine whether an existing package would fulfil our requirements (Jacques and Judd, 1985; Katsifli and Fyffe, 1984; Harding, 1974). None of the programs or approaches studied offered the facilities we required. Consequently, after discussion with several members of staff, a specification for a new package, NODES, was prepared.

Description

NODES solves a single or a system of ordinary differential equations subject to given initial conditions. The wide range of methods provided (Figure 4.4) enables the package to be used for comparative studies in numerical analysis and to investigate stiff equations. The equation(s), analytical solution, if known, initial condition(s), range and step size are input at run-time.

Figure 4.4: Numerical methods (enhanced version)

	Single Equation	System of Equations
Fixed-step Runge-Kutta methods	Euler Heun Improved Euler 3rd-order Runge-Kutta 4th-order Runge-Kutta	Euler 4th-order Runge-Kutta
Variable-step Runge-Kutta methods	4th-order user-controlled 4th-order Runge-Kutta-Fehlberg	
Predictor-corrector methods	Modified Euler 4th-order Adams-Bashforth-Moulton	

The output may be a table of results, a graph (showing the analytical solution also, if known) (Figure 4.5), or an error analysis incorporating the absolute or relative error in both tabular and graphical form (Figure 4.6). Different outputs may be selected in any order for a particular input. The highly flexible and interactive nature of the menu-driven program allows problem parameters or the method to be easily modified and the resultant effect on the solution observed. Successive graphs may be superimposed allowing direct comparison of solutions using either different step lengths, methods or initial conditions. When a system of differential equations is solved, the phase path of any two of the variables can also be plotted. Hard copy of either tabular data or graphical solutions may be output to the printer.

The screen layout has been carefully designed. Information about the current problem is displayed at the top of the screen and a small window at the base is reserved for instructions and error messages, leaving the central area for input, the menu or display of results. Default values for all input parameters are displayed in parentheses. These are continually updated to contain the value most recently input by the user. This facility is particularly valuable for classroom demonstrations and the inexperienced user but also simplifies modifications as only those parameters which have changed need to be re-typed.

When a fixed-step Runge-Kutta or the modified Euler method is used, the error is calculated by comparison with the analytical solution, if given. With the fourth-order variable step Runge-Kutta and predictor-corrector methods, however, the error at each step of a particular solution is estimated by the method itself.

Many practical problems and idealised systems in science and engineering can be modelled by differential equations. NODES allows

Figure 4.5: NODES graphical solution

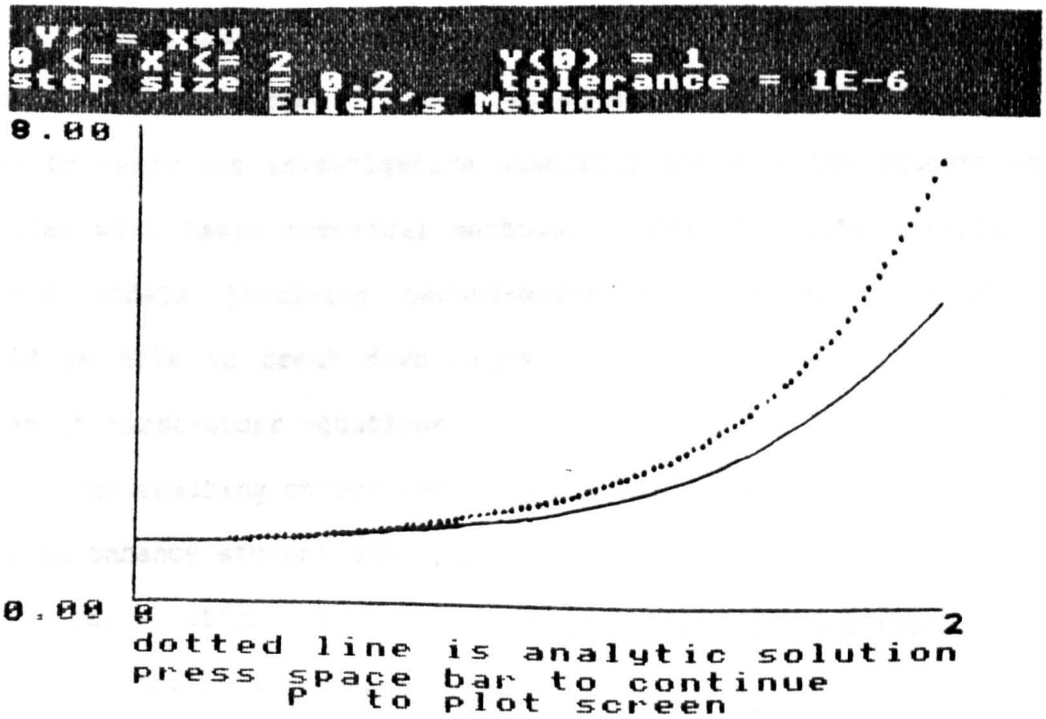
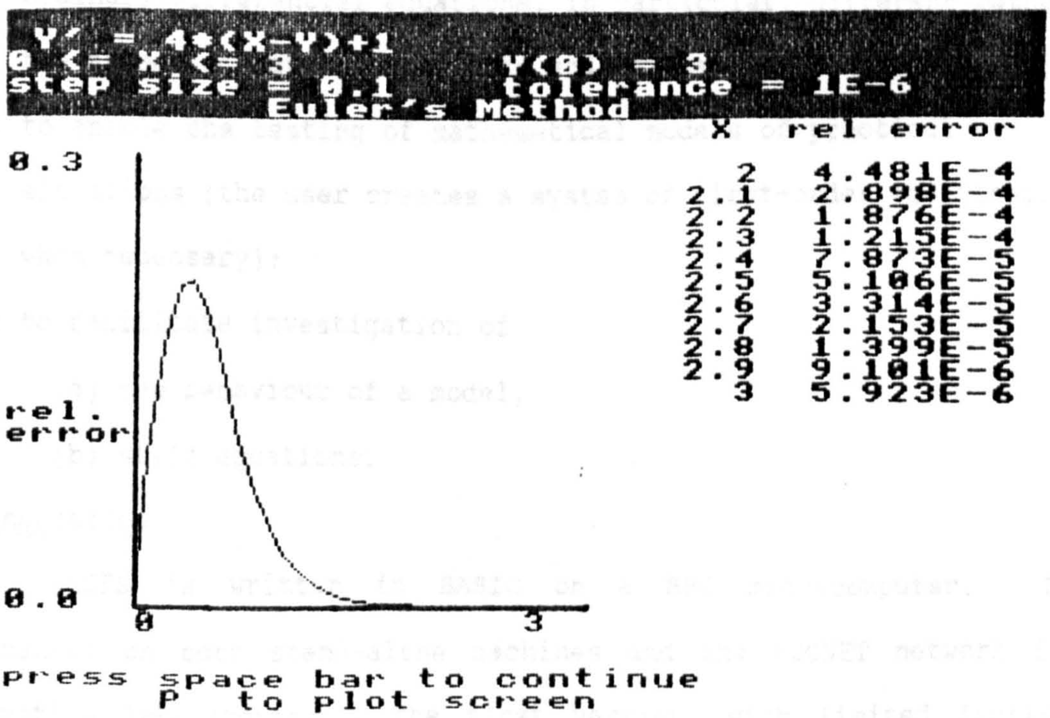


Figure 4.6: NODES error analysis



students to implement a model on the computer, to experiment with the model parameters and to observe the effect on particular solutions.

Before using this package, the student should have been introduced to first-order ordinary differential equations and simple modelling. In order to carry out investigative numerical analysis the student should be familiar with basic numerical methods. For stability analysis and for testing models involving second-order differential equations, he/she should be able to break down higher order equations and systems into a system of first-order equations.

The teaching objectives of the package are:

(i) to enhance student understanding of

   (a) solutions of ordinary differential equations (both numerical and analytical),

   (b) numerical methods,

   (c) stability of systems;

(ii) to facilitate investigation of numerical methods for solving ordinary differential equations, in particular, different methods, the effect of varying step size and errors;

(iii) to enable the testing of mathematical models of practical situations (the user creates a system of first-order equations, when necessary);

(iv) to facilitate investigation of

   (a) the behaviour of a model,

   (b) stiff equations.

Implementation

NODES is written in BASIC on a BBC microcomputer. It is implemented on both stand-alone machines and the ECONET network in the mathematics laboratories. The first version, with limited facilities, was installed on the network in November 1986 for student and staff use.

Throughout 1987, amendments and modifications were incorporated and further facilities added. As the program became too large to be accommodated on a BBC model B it was decided to implement two versions. The enhanced version, described in this report, is implemented on the BBC Master. A standard version which offers fewer numerical methods and error analysis for single equations only is available for the BBC model B. Version 3 was released in June 1988 (Mackie and Scott, 1988(b)) and demonstrated at the ICME-6 conference in Budapest in August that year. NODES has also been demonstrated at four other conferences (Mackie, 1986, 1988(a), 1988(b)) and is being marketed by Napier Polytechnic. The package has been used extensively at Napier since 1987.

Back-up material

A series of worksheets has been prepared to accompany this package, many of them specific to a particular application. Topics covered range from comparisons of different numerical methods and investigations of errors (or the stability of systems) to model building and analysis. A full list is included in Appendix 2. For example, one worksheet requires the student to construct and analyse a mathematical model of a simple vibrational system. The objectives are to study simple harmonic motion, damped vibrations and various forced vibrations, including the conditions leading to resonance. Many of the questions encourage the students to analyse the significance of their results. Where feasible, some open-ended questions have been included to foster creative thinking and further experimentation.

CHAPTER 5

The Process of Evaluation

5.1 Methodology

> "Evaluation is the means whereby we systematically collect and analyse information about the results of students' encounters with a learning experience."
>
> (Rowntree, 1982)

It is by identifying the outcomes, the strengths and the weaknesses of a teaching/learning system that it can be improved upon before using with another set of students. Such evaluation should also determine which objectives have been achieved and which have not. Scriven (1967) first introduced the distinction between formative and summative evaluation. The former occurs during the development phase of a learning system as a result of feedback obtained by continuous monitoring. Its primary purpose is to improve the system for the benefit of the present students. The appraisal of the entire system and its overall effectiveness constitutes the summative evaluation.

The distinction between these two phases in this project is relative rather than absolute. The development and use of CAL in the Mathematics Department at Napier, described in Chapter 3, has been an iterative process. Evaluation of small-scale innovations, with the close involvement of the lecturers concerned, enables modifications to be made to the new materials and ways of presenting them. Further CAL is introduced as a result of individual self-evaluation and in response to specific needs. Rowntree notes that self-appraisal at the end of a teaching unit is a summative evaluation of that unit but may appear formative to the evaluation of a system as a whole. The main distinction in this project is that the formative evaluation concentrates on the materials developed whereas the summative assessment considers the outcomes resulting from the use of those materials.

It used to be thought that, to be of value and to demonstrate validity, educational research had to follow the experimental methods of scientific research. As discussed in section 2.6, however, the quantitative pre- and post-test measurements typical of experimental evaluation fail to provide sufficient insight into all the changes in learning which may be taking place as a result of significant curriculum innovation.

Several recent studies in the field of computer-assisted learning (Hoyles and Sutherland, 1987; Johnston, 1985; Gudgin, 1987; Fraser, 1987) have relied on the qualitative techniques of illuminative evaluation (Hamilton et al., 1977). This approach is anthropological in nature, relying on observation, interviews and questionnaires for much of its data. Anecdotes may be used to reveal particular examples of attitude or change.

For this project illuminative technqiues have been embodied in a framework of action research (Cohen and Manion, 1980). In advocating the teacher as researcher, Candy (1988) argues:

> "The close investigation into the learning and teaching processes at work when a computer is introduced, can be carried out only by considering the impact and effectiveness in the school and classroom context. The changes observable by the teacher, reflecting on a number of different factors, can be related to previous experience and assessed with a view to subsequent action."

Continual appraisal allows subsequent teaching to be modified and new goals and strategies to be set in the light of experience. The advantage of the dynamic nature of this approach is its ability to recognise that objectives and priorities are continually evolving. Thus, this methodology not only leads to a clearer understanding of the processes involved but is more likely to assist in the identification of all the significant outcomes of the learning process, not just those it was designed to produce.

The views of participating lecturers are a crucial aspect of the evaluation. These have been determined by informal conversation, by interview and by questionnaire. Most researchers agree that student evaluation can be valuable also (Arubayi, 1986), although variables such as sex, experience, grade, class size and time of day course is taught can influence students' ratings of instruction. Interviews and questionnaires have been used to gather student evaluation data.

The validation of the learning packages developed is the main function of the formative evaluation. There are two main objectives:

(i) to assess the performance of each program, and whether it is a convenient and effective learning tool;

(ii) to determine the extent to which the teaching objectives of the package have been achieved.

Attributes such as the reliability, "user-friendliness" and usefulness of the programs need to be assessed. What difficulties or deficiencies have been encountered in use? Does the program have any unexpected advantages? Are the accompanying worksheets clear, relevant and useful?

Both student and staff ratings of the programs have been measured but, by analysing these separately, any significant variations will be noted. One problem in judging the reliability of a program is the user's inability to distinguish between hardware and software faults. Deficiencies in the network operating system may lead an inexperienced user to rate a package as unreliable. By encouraging users to report all faults either verbally or in a book provided for the purpose in the laboratories, hardware faults should be quickly rectified and the software reliability should improve as the project progresses.

Organisational and operational aspects of the mathematical sciences laboratories were also reviewed as part of the formative evaluation.

The summative evaluation

(i) investigates all the learning outcomes;

(ii) examines changes in (a) teaching approach,

(b) the mathematics curriculum,

(c) student attitudes;

(iii) determines the feasibility of the transfer of the packages to other higher educational establishments.

The primary teaching objectives are the promotion of problem-solving and investigative work (sections 4.2 and 4.3). The role of computers in learning mathematics was discussed in section 2.1 and the following desirable student outcomes of problem solving and investigative work in the laboratory were identified:

assimilation/reinforcement of concepts

mathematical discussion

involvement of both weak and able students

students' knowledge is broadened

encouragement towards becoming more independent learners

increased motivation

improvement of problem solving skills

experience of mathematical modelling.

Since outcomes in both the cognitive and affective domains are expected, a variety of techniques have been used to identify them. The students' learning experiences are assessed informally but frequently by both the teacher and the students themselves. Their judgments have been polled by questionnaires. Written data gathered from the questionnaires can be verified and elaborated by interviews with staff and students. Student motivation can be inferred from scaled ratings in surveys. Observation of overt behaviour in the mathematical sciences laboratories can be used to validate the written responses.

5.2 Classes selected for Monitoring

Initially two classes were selected for close monitoring during the 1985/86 academic session:

(i) B.Sc. in Science with Industrial Studies, 4th year

(ii) B.Sc. in Communications and Electronic Engineering, 4th year (Honours).

The lecturer concerned, who teaches both classes, felt that both would benefit from the introduction of computer-assisted learning, utilising microcomputers in the mathematical sciences laboratory, available for the first time the previous session. As both groups study aspects of linear programming as part of their course, opportunities exist for the use of the LINPROG package. Obviously the willingness of the lecturer to participate in the research was also a requirement. Prior to this session both classes had occasional microcomputer-based classroom demonstrations of mathematical programs but neither had hands-on experience of such software.

B.Sc. in Science with Industrial Studies, 4th year (SIS4)

This is an interdisciplinary science-based sandwich degree with two six-month placements at the end of Years 2 and 3. The course leads to an ordinary degree after four years or an honours degree after five years.

In their first year, students study Physics, Chemistry, Biology, Mathematics, Computing and Industrial Studies. One science subject, but not Mathematics, is dropped at the end of Year 1 and another the following year. They thus continue to study two science subjects, one of which may be Mathematics, for the remaining two or three years of the course. The course structure and mathematics syllabuses are detailed in Appendix 3. (See also Leach, 1978).

As part of the computing course, students learn at least one high level programming language, FORTRAN, in Year 2. Those who choose the Mathematics option from Year 3 onwards also continue to study computing. The content of the mathematics syllabus is almost 50% statistics. Students use MINITAB and other computer packages regularly from the first year of their course.

There were 12 students in SIS4 in 1985/86. The three hours of mathematics per week shown on the scheme of work in Appendix 3 forms half of the total mathematics curriculum for the fourth year of this course. The balance between lectures and tutorials is flexible but, in general, laboratory sessions replaced lectures. They had one classroom demonstration and three laboratory sessions using packages, but were encouraged to make further use of the packages in their own time.

Students on this course study linear programming and the use of the Simplex method in their third year. In the fourth year the topic is extended to include integer and mixed integer programming problems and their solutions by the branch and bound method. LINPROG was used to reinforce the teaching of this method. The student's use of LINPROG was not assessed directly.

As no non-linear programming software was available, the students, working in pairs, wrote their own programs in BBC BASIC based upon routines given in their textbook. Each group developed a program for one of the methods, then presented their work to the rest of the class including an explanation of the algorithm itself. This exercise involved 5 laboratory sessions for the group presentations which were assessed. The programming was done in the students' own time.

B.Sc. in Communications and Electronic Engineering, 4th year (Honours) (CEE4)

This 4/4½ year sandwich course, with bifurcation at the start of Year 3, leads to an ordinary or honours degree in Communication and Electronic Engineering. The principal subjects studied are Electrical and Electronic Engineering, Engineering Science, Mathematics, Computing, Information Technology, Communication Systems and Industrial Studies. Details of the course structure and mathematics syllabuses are given in Appendix 3. There are two 18-week periods of supervised work experience, at the end of Years 2 and 3, during which students work in a variety of industrial environments.

Throughout their course, the students are exposed to a wide range of computer hardware, including microcomputers, microprocessors and a high frequency laboratory. They learn both a high level programming language, PASCAL, and microprocessor programming techniques during the first three years of their course.

In 1987, in response to the Finniston Report, this course was converted to a B.Eng. degree, with a greater emphasis being placed on engineering applications.

The provision of a relevant programme of engineering applications aims to stimulate creative thought and self-awareness in the students. The first phase of the programme, EA1, equips the students with the practical machining and electronic skills they require. During the second phase, EA2, individual creative skills are developed by the application of the theoretical work to practical solutions of real problems. This includes a computer-based mathematics assignment of an experimental or investigative nature, in each of the first four years of the course.

In Year 4, the students have three hours of mathematics per week, in which they study probability and statistics, numerical analysis, optimisation and queueing systems. The three hours comprise one hour each from two different lecturers plus a shared tutorial hour. The scheme of work followed by one lecturer for the 1985/86 session, given in Appendix 3, shows the balance between lecturing, classroom demonstrations and students' hands-on experience in the mathematics laboratory. It also illustrates how the use of appropriate computer packages is integrated into the mathematics curriculum. Laboratory sessions with this class replaced lecture hours. The students were encouraged to explore the use of the computer packages further in the laboratory in their own time. There were 16 students in this class.

The linear programming studied in the fourth year of the course consists of the graphical solution of two-variable problems, the Simplex method, the concept of duality, and an introduction to post-optimal analysis. The solution of integer programming problems by the branch and bound method is also covered. The LINPROG package was used to enhance the teaching of all aspects of this topic.

Computer packages were also used to assist the study of queueing theory. The software required to solve boundary value problems by the finite difference method was not available in time, but one of the students wrote a program on his own computer which was then demonstrated to and used by the rest of the class. This class also made use of the programs for non-linear programming methods developed by SIS4.

The students had a total of 10 laboratory-based classes, five of them using LINPROG. They also had one classroom demonstration. None of the computer-based work was assessed directly. Their EA2 assignment was a joint exercise in mathematics and communication systems to simulate random flow in networks.

One of a number of other classes which used the mathematics laboratory regularly during this session was the first year B.Sc. Applied

Chemistry class (AC1), comprising 24 students. In the capacity of tutorial assistant, I participated in laboratory-based classes with this group and was, therefore, able to observe their experiences. A variety of programs for function plotting, Newton-Raphson iteration and numerical integration were used. By gathering data from this first year class also, their response to the use of computer-based materials can be compared to that of the more mature and mathematically-experienced fourth year classes.

5.3 Observation of Laboratory Sessions

During the 1985/86 academic year I attended twelve laboratory sessions, three each with SIS4 and CEE4 and six with AC1. Although with the latter class I participated as a tutorial assistant, whereas my involvement with the other classes was simply as an observer, the distinction was blurred. In all three classes I gave advice on operation of the software, when requested, and discussed difficulties and results with the students.

The LINPROG package was in use during three of the observed sessions. Being at a formative stage of development, several minor 'bugs' and operational problems were discovered during these sessions.

At each laboratory class the students were given a worksheet to direct their efforts. In most cases this had been specially prepared for the package being used but, in a few cases with AC1, an existing tutorial sheet was used. In all instances, when a package was being used for the first time, the lecturer gave the students a short introduction to its operation.

5.4 Student Surveys

All the students in the three classes listed in section 5.2 were asked to complete questionnaires in October 1985 and May 1986 (Appendix

4). The first of these, SQ0, was brief, designed to assess the students' previous experience of and attitude to mathematics and computing. The second questionnaire, SQ1, had two main aims:

(i) to appraise student attitudes towards the use of computers as a learning tool for mathematics, and

(ii) to determine which packages had been used and to rate these packages.

The students were also asked for their opinions on assessment of computer-based work, group working and unsupervised working in the laboratory. These, and subsequent questionnaires, were handed out to students in class either by me or the lecturer. The students were asked to complete them and hand them back as soon as possible.

In addition to the questionnaires, I interviewed ten students individually, three from CEE4, four from SIS4 and three from AC1. These students were selected by the lecturer to represent a cross-section of views, sex and abilities within each class. The interviews were informal and wide-ranging, lasting about twenty minutes each. The students were told that the interviews were for research purposes only, but not the exact nature of the research. They could not, therefore, guess the 'correct' responses to the questions asked and were encouraged to talk frankly. The aim was to gain a deeper insight into the views and attitudes of these students towards mathematics and the use of computers than could be ascertained from a questionnaire. A further aim was to identify unforeseen points of significance which could be followed up in later questionnaires.

During the following session, 1986-87, the SIS4 and CEE4 classes followed the same scheme of work. They were again surveyed at the start and end of the session using questionnaires SQ0 and SQ3, respectively. The second questionnaire, SQ3 (Appendix 4), was similar to SQ1 used the previous year but modified to reflect the wider range of computer packages available and the additional laboratory.

At this time a new mode of use of the laboratories emerged. As explained previously (secton 5.2), the Communications and Electronic Engineering degree, now converted to a B.Eng. degree, required the mathematics course to include an engineering applications (EA2) component. Students on this course are taught methods for solving first- and second-order differential equations analytically in first year. Methods for second- and higher order differential equations and simultaneous equations are developed further in second year and numerical methods are introduced. It was felt that an investigation of the mathematical model of a physical system using the NODES package would be an appropriate and useful engineering applications assignment. The first and second year classes, CEE1 and CEE2, a total of 116 students, used an early release of the NODES package for this purpose early in 1987. The worksheet DE7 was used as the CEE1 assignment whilst CEE2 used DE11 (Appendix 2).

Students in these classes had little previous experience of using computer packages in mathematics. At the start of the session, the first year class had been introduced to the facilities of the laboratory and, in particular, use of the graph plotting program, and, thereafter, encouraged to use it in their own time to follow up topics encountered in class. NODES was used in class with the second year group prior to the EA2 assignment to demonstrate various numerical methods for the solution of ordinary differential equations. The students' preliminary instruction for the EA assignment included an introductory lesson using NODES in the laboratory and being shown how to break a second- or higher order differential equation into a system of first-order equations. They were then required to complete the assignment in their own time.

Both classes were asked to complete questionnaires SQ0 in October 1986 and SQ2 (Appendix 4) after completion of the EA2 assignment. The objectives of the SQ2 survey were:

(i) to rate NODES for its ease of use, reliability, usefulness and flexibility;

(ii) to pinpoint particular features of the program which were liked or disliked;

(iii) to find out student attitudes towards use of the NODES package;

(iv) to assess students' perceptions of ways in which use of a computer could help their learning.

An improved version of NODES was available for the 1987-88 session and was used by CEE2 in the same way as in the previous session. As this class had completed the SQ2 survey the previous year, an amended questionnaire, SQN2 (Appendix 4) was prepared and issued to the class in March 1988. Many questions remained the same for comparison with the previous year's findings. A four-point scale for ratings replaced the three-point scale because of the tendency of participants to 'sit on the fence'. I also interviewed three students from CEE2, selected to represent a cross-section of abilities and attitudes within the class, to obtain a more detailed and subjective reaction to this assignment. The interviews were recorded on audio tape.

In March 1988, the SIS4 and CEE4 classes were again asked to complete a general questionnaire, SQ4 (Appendix 4). This questionnaire was based on SQ3 but modified to use a four-point rating scale. Previous questions for which sufficient data had already been gathered were omitted but ratings for the packages used were again requested in order to complete the formative evaluation. New questions were included to ascertain ways in which students thought that the use of computer packages had assisted their study of mathematics.

Data from the interviews conducted with CEE2 students and previous questionnaires contributed towards two new questionnaires, one each for LINPROG and NODES, which were developed for the final year of the study, 1988-89 (LPAQ and NAQ, Appendix 4). In these attitude surveys students were asked to respond to a series of statements and to state the degree of agreement with each statement on a scale of 1 to 5 (Likert, 1932) as follows:

5	4	3	2	1
Strongly agree	Agree	Not sure/ Doesn't apply	Disagree	Strongly disagree

For example:

"Using LINPROG helped me to understand the branch and bound method for integer programming."

"It was interesting to investigate the sensitivity of solutions to differential equations."

"I couldn't understand what the program was doing."

These surveys were designed to assess:

(i) the student's enjoyment and interest in the topic being studied and the use of the program,

(ii) the student's assessment of the package as a learning tool, and

(iii) the types of learning taking place.

The statements were mixed with respect to positive and negative attitudes to avoid a set pattern of responses and to enable a check to be made on the internal reliability of the survey. Each survey was given to two classes who had made use of the appropriate package that session. The classes asked to complete the NODES survey were B.Sc. Mathematics with Engineering Technology, Year 2 (MET2) and the B.Sc. Physics, Year 2 (PHYS2), a total of 26 students. The LINPROG survey was given to the SIS4 and CEE4 classes, totalling 35 students. The student surveys carried out are summarised in Table 5.1.

Table 5.1: Student Surveys

Session	Class	No.of students	Questionnaires		No.of Completed Questionnaires	Number Interviewed
1985/86	SIS4	12	SQ0	SQ1	8	4
	CEE4	16	SQ0	SQ1	13	3
	AC1	24	SQ0	SQ1	7	3
1986/87	SIS4	20	SQ0	SQ3	16	
	CEE4	23	SQ0	SQ3	23	
	CEE1	65	SQ0	SQ2	51	
	CEE2	58	SQ0	SQ2	42	
1987/88	SIS4	11		SQ4	11	
	CEE4	20		SQ4	17	
	CEE2	64		SQN2	50	3
1988/89	SIS4	13		LPAQ	10	
	CEE4	26		LPAQ	25	
	MET2	16		NAQ	16	
	PHYS2	10		NAQ	9	

5.5 Staff Surveys

A microcomputer laboratory was first established in the Mathematics Department in 1984. Resultant changes in teaching approach by the staff were monitored by a series of questionnaires issued in June 1985 and the three successive years 1986 to 1988 (Appendix 5). Information requested included:

(i) amount and frequency of time spent using computers,

(ii) packages used,

(iii) reasons for use, and

(iv) ratings for packages used.

All permanent, full-time members of staff of the Mathematics Department based at Merchiston or Craiglockhart were asked to complete the questionnaires. Science and engineering courses are taught at these two sites.

In 1989, four lecturers were asked to complete a more detailed survey, two each for LINPROG and NODES (LQL2 and LQN2, Appendix 5). The emphasis of these was on his/her objectives when using the package and the extent to which these were achieved, the teaching approach adopted and the effectiveness of the package as a learning tool.

Further detailed data were gathered by extended, informal interviews in June 1989 with five lecturers who have gained experience of using computers to assist their teaching over several years. The lecturer responsible for teaching the SIS4 and CEE4 classes was among those interviewed. These interviews were recorded on tape so that details and implications could be summarised later on paper. By using a checklist of points to be covered, it was possible to obtain comprehensive data on individual changes in teaching emphasis and approach as a result of the use of computer packages.

5.6 Use in Other Institutions

Contacts were established with other Scottish higher education institutions early in the project with a view to transferring the packages developed. Lecturers at Paisley College of Technology and Dundee College of Technology expressed an interest in LINPROG and were duly supplied with copies of the program, handbook and worksheets in May 1986, with later updates as these became available. Similarly, the NODES program, with accompanying user guide and worksheets, was given to Dundee College in 1987 and to Robert Gordon's Institute of Technology in Aberdeen in 1988. Of the four packages supplied, only one has actually been used with students.

At Dundee College a lecturer used LINPROG with a class in both 1987 and 1988. In February 1988 I visited the College and observed the class using the program. The observed lesson was part of the mathematics course of a third year B.Sc. Science degree, during which they study linear programming using the Simplex method. The course includes a two-hour practical session each week, usually held in the BBC laboratory, but covering different aspects of the course and often using the BBCs as terminals to a mainframe computer. The use of LINPROG represented their first use of a microcomputer-based CAL package.

I left questionnaires for both students and the lecturer (SQL1 and LQL1, Appendix 6) to be completed and returned to me.

The Dundee lecturer explained that lack of class time had prevented him from using the NODES package since differential equations were not covered until late in the session, by which time the students were feeling the pressure of examinations. Robert Gordon's Institute of Technology expressed interest but, soon after receiving NODES, they acquired a new IBM-PC compatible computer laboratory and no longer had ready access to BBC computers.

In April 1988 a questionnaire, LQL2 (Appendix 5) was sent to the 20 customers to whom a copy of LINPROG had been sold, with an accompanying letter requesting completion by a lecturer who had used the program. Since purchase orders for software are handled by the central administration in many colleges and universities, it was not possible to identify the department from which the order originated in all cases. It is not known, therefore, how many questionnaires did in fact reach the intended user. Six completed questionnaires were returned.

CHAPTER 6

Formative Evaluation of the Learning Packages

6.1 LINPROG

From 1986 onwards, LINPROG has been used with students in CEE4 (B.Sc., Communications and Electronic Engineering, 4th year Honours class) and SIS4 (B.Sc., Science with Industrial Studies, 4th year class). For three successive years, the students were requested, by questionnaire, to rate the computer packages they had used for ease of use, reliability and usefulness of output on a scale ranging from 'Very highly' (4) to 'Poor' (1). The results are shown in Figure 6.1. There are many factors which may influence students' ratings. These include the time of day, week or session that the questionnaire is answered, their most recent experience with computer packages and their peer group. A different format of questionnaire in 1987/88 may also have affected ratings for that year. These results, therefore, can only provide a broad indication of student opinion, but they do demonstrate a consistent rating of 'High' or 'Very high' by at least 46% of respondents for all aspects of the program under consideration.

The data for 'Reliability' shows a sustained improvement over the period of study. Table 6.1 contains student comments relating to their use of LINPROG. Two aspects of the program which proved advantageous were the step by step solution of problems by the Simplex method and being able to implement the branch and bound method.

Figure 6.1: LINPROG - Students' Ratings (percentages)

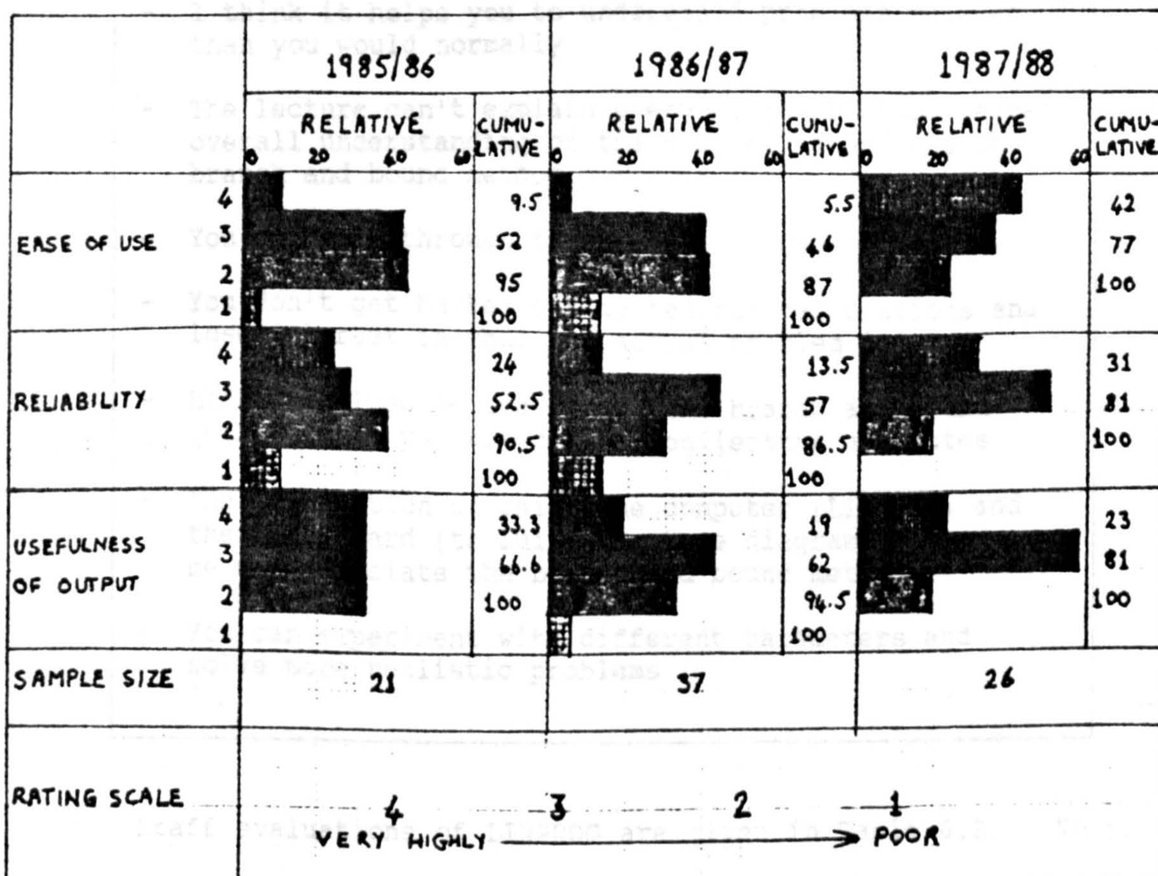


Table 6.1: Student comments relating to LINPROG

- Allows you to concentrate on the problem and therefore understand the method more easily
- Use of the program helped my understanding. I got lost when reading through my notes. The screen layout and prompts aid understanding
- I think it helps you to understand problems quicker than you would normally
- The lecture can't explain everything - LINPROG helped overall understanding of the Simplex method and the branch and bound method
- You can work through the method at your own pace
- You don't get hacked off by tedious calculations and lose interest in what the actual problem is
- LINPROG helped me understand the branch and bound method. It was not clear from lecture and notes
- The combination of using the computer (LINPROG) and the whiteboard (to build the tree diagram) helped me to appreciate the branch and bound method
- You can experiment with different parameters and solve more realistic problems

Staff evaluations of LINPROG are given in Table 6.2. No program ratings were gathered in 1988. As the number of ratings obtained is small, no generalisations can be made. Staff may be less critical and rate programs higher than the students do because they are more aware than students of the amount of arithmetical drudgery eliminated by the programs and the wider educational opportunities opened up. LINPROG was rated particularly highly for the usefulness of its output. It scored less well, however, for 'Flexibility/range of options offered'.

Table 6.2: Staff LINPROG ratings 1986-87

Program attribute		Number of staff 1986	Number of staff 1987
Ease of use: very highly	4	2	1
	3	0	2
	2	0	1
poor	1	0	0
Reliability: very highly	4	2	1
	3	0	3
	2	0	0
poor	1	0	0
Usefulness: very highly	4	2	3
	3	0	1
	2	0	0
poor	1	0	0
Flexibility/options: very highly	4	no data available	0
	3		3
	2		1
poor	1		0
Sample size		2	4

Further data were obtained from questionnaire LQL2 in 1989 from lecturers who had, by then, made considerable use of the package. The two lecturers who evaluated LINPROG rated it on a scale of 4 (very highly) down to 1 (poor) as follows:

	Lecturer 1	Lecturer 2
Ease of use	4	4
Reliability	4	3
Usefulness of output	4	4
Flexibility/options offered	3	3

The features of the program most liked by the reviewers were the ease with which students became familiar with the program and the tableau by tableau option which aided understanding of the Simplex method. These results reinforce the findings of the students' evaluations. The ease of use and the labour-saving aspects were two reasons why they considered that the students reacted favourably to using the package. One lecturer would like the program to allow more variables and constraints in order to solve more realistic problems. Both respondents suggested that an option to delete a constraint should be added to the menu. Access to the menu options from the tableau by tableau mode for post-optimal analysis would also be welcomed. The results in Table 6.3 indicate that LINPROG had been used for all its intended teaching objectives (Section 4.2) and was judged to have been very successful for all purposes except enhancing student understanding of the concept of duality.

Table 6.3: LINPROG Evaluation 1989

Teaching Objective	Degree of success	
	Lecturer 1	Lecturer 2
To enhance understanding of the Simplex method	Very successful	Very successful
To enhance understanding of duality	Fairly successful	Not sure
To solve realistic L.P. problems	Very successful	*
To carry out post-optimal analysis	Very successful	*
To solve integer programming problems	Very successful	Very successful
Support material used	LINPROG worksheets	LINPROG worksheets
Used for part of overall assessment?	No	Yes

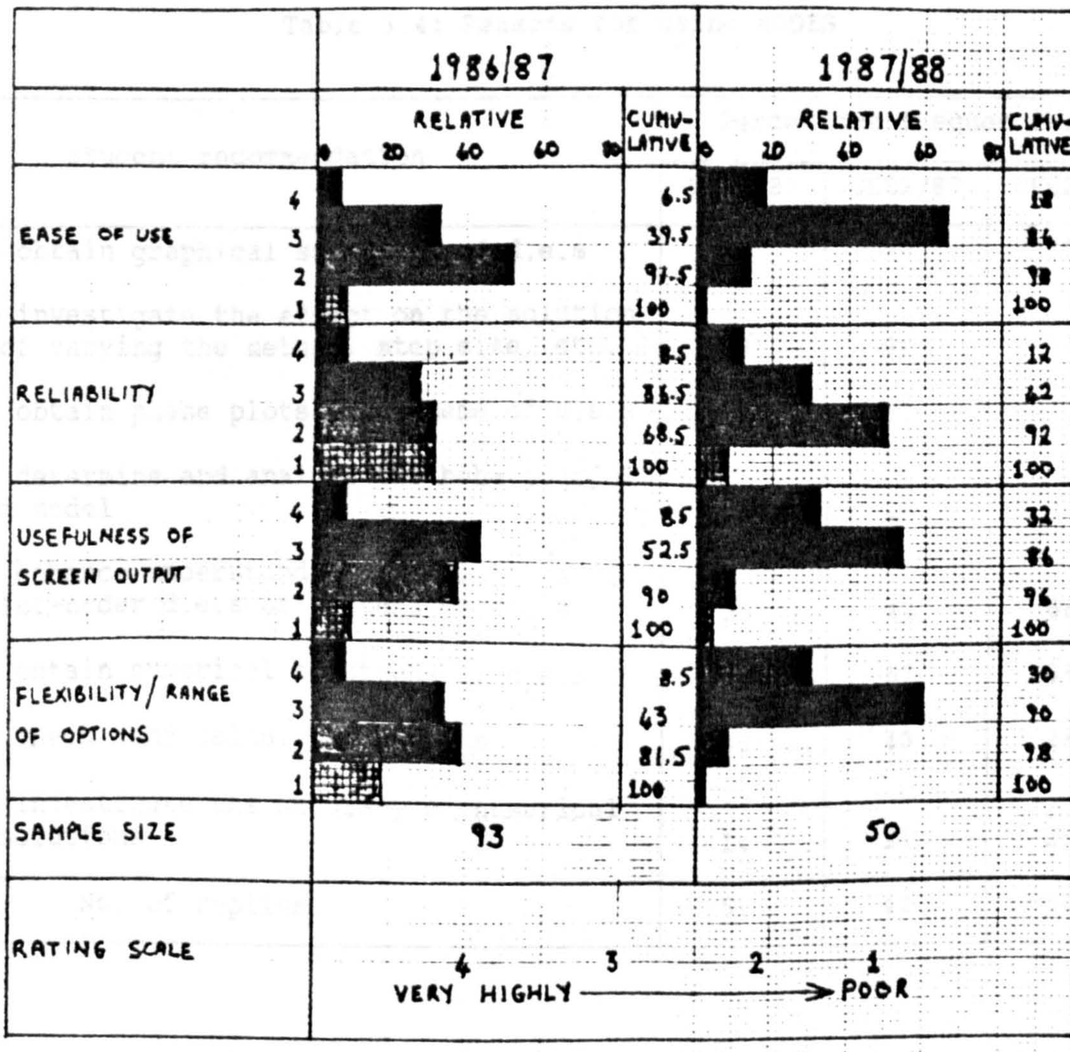
Note: * program not used for this purpose

6.2 NODES

The NODES package was first used in the 1986/87 session with classes CEE1 and CEE2. Results from questionnaires SQ0 and SQ2, completed by both classes, are shown in Appendix 9. Although only 7% of the students responded positively to the question "Have you enjoyed using NODES?", a further 75% rated their enjoyment as 'OK'. Features of NODES which were particularly liked by students in 1987 included the program's ease of use, the clarity of the graphical output and the ability to superimpose graphs. Features identified as being 'liked least' were difficulties in editing input data, the tendency for the program to 'crash' easily and problems encountered when outputting graphs to the printer.

In 1988, the CEE2 class used an updated version of NODES and subsequently completed questionnaire SQN2, the reuslts of which are given in Appendix 9. Student ratings for NODES, summarised in Figure 6.2, clearly reflect the substantial improvements implemented for the 1988 session, notably an easier method of editing equations, default values for parameters and a screen dump facility to print graphs. The program's ease of use and graphical output were again rated as its most liked features, whilst the lack of a proper queueing system for the printers and occasional run-time errors were the least liked. These would account for the relatively low rating for reliability in both years of the survey. The run-time errors, which were caused by 'printer busy' messages, proved difficult to overcome on the Econet network but were finally eliminated in a later version.

Figure 6.2: NODES - Students' Ratings (percentages)



The students were asked to select reasons for which they would recommend using NODES. Table 6.4 shows the percentage of respondents who selected each reason in approximate descending order of frequency. It is interesting that, whereas, in 1987, only 25% of CEE1 (and 43% of CEE2) felt that use of NODES could enhance their understanding of the solution of differential equations and systems of equations, in 1988, that percentage of the same group of students, now in CEE2, had risen to 68%, suggesting that further use of the package and, possibly, deeper reflection on the previous year's use, had caused them to change their minds on this issue.

Table 6.4: Reasons for using NODES

Student recommendation	Percentage frequencies		
	CEE1/87	CEE2/87	CEE2/88
To obtain graphical solutions of d.e.s	88	90	96
To investigate the effect on the solution of varying the method, step size, etc.	69	90	78
To obtain phase plots of systems of d.e.s	29	83	80
To determine and analyse the behaviour of a model	63	45	74
To enhance understanding of solutions of 1st-order d.e.s or systems of d.e.s	25	43	68
To obtain numerical solutions of d.e.s	45	57	38
To check hand calculated results	49	40	28
To investigate the accuracy of numerical solutions	14	14	28
No. of replies	51	42	50

Several students suggested the addition of a facility enabling the user to specify the x- and y-axis limits (over-riding the default values). This would overcome a problem in the CEE2 assignment where the default range of y-values did not allow the graphs of various different solutions to be superimposed. This facility has since been implemented.

Some of the students' comments relating to their use of NODES are given in Table 6.5. They illustrate the importance of the graphical output as an aid to understanding the solutions of the differential equations.

Staff ratings of NODES in 1987 are given in Table 6.6. No program ratings were gathered in 1988. In 1989, two lecturers who had been using NODES for two or more sessions completed questionnaire LQN2. NODES was rated very highly by the two lecturers who evaluated it as follows:

	Lecturer 1	Lecturer 2
Ease of use	4	4
Reliability	4	4
Usefulness of output	4	4
Flexibility/options offered	4	3

Table 6.5: Student comments relating to NODES

- You can see the problem more clearly when visually displayed
- Easier to see how a function behaves
- Brought to life the abstract ideas of Runge-Kutta
- You can see what is happening
- Shows how maths is related to real-life applications
- The package helped me understand what (the solution to) a differential equation did and how it worked with respect to the initial values
- Seeing the graphical solutions of equations helps to understand the equations
- Shows how mathematical models behave under different conditions in a graphical format thus showing what the model does instead of just finding a solution

Table 6.6: Staff NODES ratings 1987

	Ease of use	Reliability	Usefulness of output	Flexibility/ options
Very highly 4	3	1	3	1
3	0	1	0	2
2	0	1	0	0
Poor 1	0	0	0	0
Sample size	3	3	3	3

The most liked features were the graphical solutions, the speed and the visual, self-explanatory nature of the program. Other results from this survey, summarised in Table 6.7, show that the lecturers considered their use of NODES to have been very successful for investigative work and analysis of the behaviour of models. They both included computer-based work using NODES as part of the overall assessment for the class.

The teaching objectives of NODES (Section 4.3) are:

(i) to enhance student understanding of

(a) solutions of ordinary differential equations,

(b) numerical methods,

(c) stability of systems;

(ii) to facilitate investigation of numerical methods for solving ordinary differential equations, in particular, different methods, the effect of varying step size and errors;

(iii) to enable the testing of mathematical models of practical situations;

(iv) to facilitate investigation of

(a) the behaviour of a model,

(b) stiff equations.

Clearly, (ii), (iii) and (iv)(a) had been achieved whilst the outcome of (i) was still uncertain. As previously mentioned, however, 68% of CEE2 recommended using NODES as an aid to understanding the solution of differential equations in 1988 (Table 6.4) and this finding is strengthened by student comments (Table 6.5). This suggests that teaching objective (i)(a) had been achieved also.

Table 6.7: NODES Evaluation 1989

Teaching Objective	Degree of success	
	Lecturer 1	Lecturer 2
To obtain graphical solutions	Very successful	*
To obtain numerical solutions	Very successful	*
To enhance understanding of the solution of 1st-order o.d.e.s	Not sure	*
To determine and analyse the behaviour of models	Very successful	*
To investigate the accuracy of numerical solutions	*	Very successful
To investigate the effect on the solution of varying method, step size, etc.	*	Very successful
Frequency of use supervised sessions student assignments classroom demonstrations	 12 hours 1 2 hours	 5 hours
Support material used	NODES worksheets	NODES worksheets
Used for part of overall assessment?	Yes	Yes

Note: * program not used for this purpose

6.3 Mathematical Sciences Laboratories

At each of the 12 laboratory sessions which I attended the students were working through a set of exercises at their own rate, either alone or in pairs. The lecturer introduced them to the program to be used and, in some cases, led them through an introductory example. The topics being studied ranged from trigonometric graphs to integer programming using the branch and bound method, but, in all cases, the relevant theory had been covered in a recent lecture.

In general, the students seemed to enjoy using the computers and were quick to gain confidence in using the various programs. In contrast to traditional tutorial classes they frequently entered the laboratory and started working before the lecturer arrived. This was particularly noticeable with the first year class, AC1. The 'settling down' period was short and, throughout the session, there was less casual conversation than usual. The students worked hard and made good progress through the worksheets.

When working in pairs, there was usually some meaningful discussion about results. Results which were obviously incorrect generated most discussion as the students tried to discover what had gone wrong. Most students did not record their results unless told to do so. If discussion was lacking or a student was working alone, the lecturer could stimulate it by posing questions at an appropriate moment. Queries such as "Why does the graph have that shape?" or "What happens when θ is much larger?" can force the student to draw conclusions or generalise.

One laboratory session with AC1 was unsatisfactory because the worksheet provided was a standard set of exercises, some of which could not be handled by the program. No experimental or investigative questions were included.

On several occasions students suggested amendments or new facilities which might be incorporated into programs. The lack of printer or plotter output caused frustration at times.

During interviews in 1986 with a cross-section of students from classes AC1, SIS4 and CEE4 (Appendix 7), various suggestions were given for improving worksheets:

(a) to include more explanation of the mathematics and perhaps an example of the use of the program, and

(b) to include more investigative work. (This suggestion came from students who had used packages for which no specially prepared worksheets were available.)

Some students felt that worksheets should be given out in advance of the computer session as, on some occasions, most of the time in the laboratory had been spent reading and understanding the worksheet.

Another suggestion was that an introductory session in the mathematical sciences laboratory be held at the start of the year to explain the facilities and packages available. Having sampled some of the facilities, they felt that there might be others which could also be of use to them and were frustrated by lack of knowledge but insufficiently experienced to find out for themselves.

In the 1986 and 1987 questionnaires, SQ1 and SQ3, students were asked to suggest ways in which the organisation of the mathematical sciences laboratories could be improved. Replies are given in Table 6.8. Improvements implemented since this survey include:

(i) March 1987: printing facilities improved

(ii) December 1987: graph printing from GRAPH and NODES implemented

(iii) January 1988: laboratory open-access time extended to include some evening time

(iv) October 1989: full-time technician appointed.

Table 6.8: Suggested improvements to the laboratories

	AC1	SIS4	CEE4
More help available	6	4	4
More tutorial time	2	4	6
Extend hours of opening	0	7	2
Better printing facilities	0	8	9
More desk space	0	2	8
Better worksheets	6	0	1
Graph-printing facility	0	1	6
More computers	1	0	5
Sample size	19	34	38

In annual surveys between 1985 and 1988, mathematics lecturers were asked to give details of their use of computer-based packages and to evaluate the laboratory facilities. In the 1985 and 1986 surveys, lecturers' recommended improvements to the laboratory included more computers, a better level of fittings (i.e. workbenches, chairs, etc.) in the laboratory, more software and worksheets, better printing facilities, in particular, hard-copy graphical output and more user guides. In addition, many amendments to specific programs were suggested. This high level of constructive comments in the early years of the laboratory suggested a keen interest by many members of staff and a willingness to try out the facilities. One lecturer commented that the use of a package:

> "...allows students to do more investigations on their own allowing a change in emphasis in assessments from set exercises."

However, a common complaint was summed up by:

> "With large classes of 30-40 it is difficult to organise a laboratory session."

The survey in 1987 followed the opening of the second laboratory and the addition of many more packages and worksheets. The most commonly suggested improvements in this survey were for permanent technical support, general instruction sheets for the use of packages and printers, hard-copy graphical output and a standard procedure for accessing and running programs. With the implementation of a menu-access program, graphical output to the printers and on-line help facilities later that year, good progress was made towards satisfying these requests.

Table 6.9 shows that the GRAPH package was consistently the most frequently used program over the four years of surveys. The statistical program, MICROTAB, which, like GRAPH, is of a general nature and thus

Table 6.9: Computer programs used

Computer Programs	Number of users 1985	1986	1987	1988
GRAPH	8	9	11	9
MICROTAB	N/A	N/A	9	7
SURF	N/A	1	6	5
NEWTON	8	6	5	6
LINPROG	N/A	2	4	4
QUEUE	2	3	4	1
NODES	N/A	N/A	3	6
NUMINT	3	4	3	5

Note: N/A program not available

suitable for a wide range of purposes, was also well used though not so well liked. Use of most programs remained fairly constant or rose slightly. Use of both LINPROG and NODES increased each year after their introduction in 1986 and 1987 respectively.

6.4 Student attitudes

A general impression of the students' reactions to the introduction of computer-based learning packages in mathematics in 1986 was gained from questionnaires SQ0 and SQ1 (Appendix 8). These reactions were examined more closely during interviews with ten of the students. A resume of the interviews is contained in Appendix 7.

The first year students to whom I talked obviously found it difficult to articulate their feelings; the fourth year students were able to express their attitudes towards their course and towards learning mathematics much more clearly.

Of the ten students interviewed, eight felt that the use of computers had contributed positively towards their course to varying degrees. Three students had made use of the laboratory in their own time. Most of those students who enjoyed using computers would welcome more investigative work in the laboratory. All the first year students wanted more time spent both on suprvised sessions and on assignments to be completed in their own time.

Students' comments relating to the use of computers in assisting the learning of mathematics, which are given in Table 6.10, highlight the importance of graphical output and the ease with which experiments can be carried out. Three fourth year students emphasised that they found writing their own programs on the BBC micro very helpful as it "forces you to understand" the problem.

Two female students in SIS4, who both enjoy mathematics, did not perceive computers as being useful towards their learning; they both gain their satisfaction in mathematics from working through a problem and getting the correct answer, like solving a puzzle. Neither mentioned

Table 6.10: Student comments relating to computer-aided learning

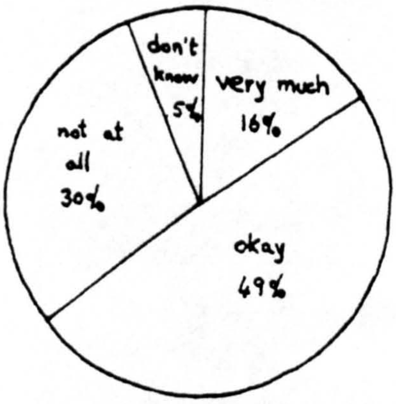
- I learn by doing and like experimenting.
- It helps understanding and makes the notes clearer. Graphics output helps. I learn by doing. Computers are fascinating.
- They allow 'what if' questions.
- I don't like mathematics but enjoy using computers in mathematics.
- You can work at your own pace.
- Useful for solving problems relevant to the work being done.
- Packages allow you to try out things and experiment to see what happens if things are varied.
- Experimenting with different parameters helps as it would take a long time to perform enough examples by hand to reach conclusions.
- They present the data in logical steps, present data graphically and inform on mistakes.
- As long as methods are understood, then mundane calculations are removed. Graphical outputs are useful.
- Solving hard problems in the laboratory helps me to solve easier ones by hand.
- Computers are good for guiding you through a problem.

experimenting or investigations. One objected that the computer doesn't show how it gets its answers: "I must know what is happening". The other student, who was older than average, had no confidence in computers or the output from programs. Initial experience of computing in first year was discouraging and she had never really recovered.

The first questionnaire, SQ0, showed that most of AC1 considered themselves of average mathematical ability and had a moderate enjoyment of the subject. Very few had much previous experience of using computers though most thought that they would be a very useful aid towards learning mathematics. The SIS4 and CEE4 students showed more enthusiasm for mathematics, and had all used computers at college previously and written programs of their own. Enjoyment of computing was mixed, but about two-thirds considered that computers can be a useful aid towards the learning of mathematics.

There was a poor response to the second questionnaire in 1986. It was given out too late in the session, after the end of formal classes, and there was no further opportunity to see the students collectively to receive completed forms or issue reminders. It cannot be assumed that those students who did respond were either a representative or a random sample, therefore data from this questionnaire referring to enjoyment of the use of computers has been ignored. Results from questionnaires SQ3 and SQ4 which students in SIS4 and CEE4 were asked to complete in 1987 and 1988 are contained in Appendix 8. Figure 6.3 illustrates that, in 1987, the percentage of students who enjoyed using computers in mathematics 'very much' at the end of the academic year was greater than those who enjoyed using computers 'very much' at the start of the year. Further, only 8% did not enjoy using computers in mathematics at all, compared with 30% at the start of the session who did not like using computers.

Figure 6.3: Students' enjoyment of use of computers



Start of session

End of session

Students surveyed in 1986 and 1987 were asked: "In what ways, if any, do you think computer packages help you in mathematics?" The most frequent responses from these surveys are shown in Table 6.11. The results in this table were used to compile a list included in the 1988 questionnaire to SIS4 and CEE4 (SQ4). The students were asked to name particular packages which had helped in any of the ways shown. The most frequently mentioned packages were LINPROG, SURF (3-D function plotting), GRAPH (2-D function plotting), EIGEN (numerical approximations of eigenvalues, used by SIS4), and QUEUE (queue simulation, used by CEE4). The responses, summarised in Table 6.12, are given as percentages of the number of students in the class or classes using the particular package. LINPROG achieves the highest score in all but one category. Its score is significantly higher than that of the other programs for "Better understanding of a topic", "Solving problems", and "Checking results", and equal to the score of the two function plotting programs for helping the user "To visualise a solution more clearly". In the same survey, 6 out of 28 students thought that the use of computer packages was a 'very useful' aid to learning mathematics, and a further 14 felt they were a useful aid.

Table 6.11: Ways in which computer packages help

	AC1	SIS4	CEE4
Less arithmetic	2	11	12
Visual display aids understanding	2	6	6
Experimenting with different parameters	2	6	14
.... leading to greater understanding	0	0	3
Quick solutions allow more problems to be tackled	0	7	9
Reinforcement of theory	0	2	3
Working at own pace	0	4	2
Solving realistic problems	1	9	5
Working step-by-step through a problem with mistakes corrected	0	0	6
Checking tutorial answers	0	0	6
Sample size	19	34	38

Table 6.12: Computer packages as an aid to learning (percentages)

How package helps	LINPROG	SURF	GRAPH	EIGEN	QUEUE
Better understanding of a topic/ algorithm/method	43	21	18	27	18
Able to visualise solution more clearly	54	43	54	36	24
Solving problems	61	36	36	45	29
Experimenting/investigating	36	32	29	18	35
Working at own pace	39	29	29	9	41
Checking results	64	25	29	9	35
Sample size	28	28	28	11	17

6.5 Summary of the formative evaluation

LINPROG

LINPROG was used with a class by four different lecturers during the survey period. It was rated highly or very highly for its ease of use, reliability and usefulness of output by all the lecturers who had used it. The package was used for all its intended teaching objectives, namely:

(a) to enhance student understanding of linear programming and, in particular, the Simplex method;

(b) to facilitate problem solving which involves the use of linear programming;

(c) to encourage investigative work, in particular, post-optimal analysis.

Students in two classes were asked to rate LINPROG for the same attributes as the lecturers. It was rated highly or very highly by about 50% of the students in 1986 and 1987, and by over 75% in 1988. The usefulness of its output was rated higher than the other characteristics. Student comments highlighted the role of LINPROG as an aid to understanding both the Simplex method and the branch and bound method.

One disadvantage mentioned by both staff and students was the lack of a facility to delete a constraint. It would appear that some way must be found to include this facility in a future version of the program.

NODES

The three lecturers who used NODES in 1987 rated it very favourably, particularly for its ease of use and the usefulness of its output. Graphical solutions were pinpointed as the most liked feature. Evaluation by two lecturers in 1989 revealed that NODES had been successfully used in the following ways:

(a) to obtain graphical solutions;

(b) to obtain numerical solutions;

(c) to investigate the accuracy of numerical solutions;

(d) to investigate the effect on the solution of varying the method, step-size, etc.;

(e) to determine and analyse the behaviour of models.

Student ratings of NODES were less favourable in 1987 but improved greatly, with the exception of reliability, in 1988. More than two-thirds of the students would recommend using NODES for (a) and (d) above, and one half for (e). In 1988, 68% recommended its use as an aid to understanding solutions of differential equations. Student comments emphasise the important part the graphical output plays in assisting their understanding of the solution and of the behaviour of a model. Criticisms related mainly to difficulties in using the printers to obtain hard-copy graphical output. This facility and the general reliability have been improved in later versions.

All the teaching objectives have been successively achieved with the following exceptions which have yet to be tested:

(i) to enhance student understanding of numerical methods and the stability of systems;

(ii) to facilitate investigation of stiff equations.

Both LINPROG and NODES have been used for computer-based student assessments.

Mathematical sciences laboratories

Most students enjoy using computers in the mathematical sciences laboratories because there is less tedious arithmetic and they like investigative work. They find graphical output easier to intepret than numerical output. A few students dislike computers, in some cases because of bad experiences early in their college course.

In response to student demands, open-access hours to the laboratories have been extended. Many students were critical of the printing facilities and the limited help available in the laboratories. In addition to these complaints, lecturers requested more software and worksheets. The range of software and worksheets has steadily been expanded, more technician help is now available and printing facilities have been improved. Many modifications and extensions suggested by staff and students have been incorporated into programs and worksheets. Staff were also concerned about difficulties in accommodating large classes. One solution to this problem has been to set more computer-based assessments to be completed by the student during open-access time.

Findings presented in this chapter from questionnaires and interviews provided a base from which to develop a summative evaluation of the computer packages. The main aims were to examine the overall impact on the mathematics curriculum and to evaluate the learning experiences of the students.

In general, students react favourably to new approaches to learning but, once the novelty value has worn off, will they still be enthusiastic and judge it worthwhile? This issue, too, was investigated further. The results are presented in the next chapter.

CHAPTER 7

Results of the Summative Evaluation

7.1 Introduction

The study of the impact on the mathematics curriculum resulting from the use of computer-based packages examined changes from three viewpoints:

(i) the teaching approach,

(ii) the emphasis and content of the curriculum, and

(iii) methods of assessment.

From the students' perspective, the effect on their learning experiences and attitudes was investigated. Finally, the feasibility of transfer of the learning packages developed to other higher educational establishments was studied.

7.2 Teaching approach

Changing approaches to teaching mathematics were monitored by annual questionnaires to lecturers in the Department. The number of lecturers using the laboratories for teaching purposes increased steadily over the survey years (Table 7.1). The number of staff based at the two campuses covered by the surveys, Merchiston and Craiglockhart, increased by three over this period. The opening of a second laboratory at Craiglockhart early in 1987, in response to demands for additional computers, accounts for large increases in use in 1987 and 1988. It is very likely that those lecturers who did not return their questionnaires had made no use of the laboratories. This small number have continued to make no use of the computer-based facilities despite several departmental workshops and encouragement by the Head of Department.

Table 7.1: Number of staff using laboratories

Type of use	Number of lecturers 1985	1986	1987	1988
(a) For a formal class meeting	7	11	13	14
(b) By setting student assignments	3	6	12	10
Own use	9	10	9	12
Not at all	1	1	1	1
Number of questionnaires issued	15	17	18	18
Number of replies	12	16	14	15

The frequency of use of the laboratories for supervised class sessions is summarised in Table 7.2. By estimating the average number of hours of use per month for each frequency category, as shown in the table, the total number of hours use can be approximated. The monthly total rose in the second and third years. The slight decrease in the final year is not significant, given the crude system of measurement. More detailed analysis of the figures reveals a gradual shift towards more frequent use over the first few years, as would be expected. A new user is likely to approach the laboratory cautiously at first to experiment with new ideas by himself before introducing them to his students, perhaps to one class in the first year, and to additional classes in subsequent years. Of course, not all computer-based learning/teaching will be judged to have been successful or worthwhile. Thus a certain amount of consolidation takes place which could account for the levelling-off in 1988.

Table 7.2: Use of laboratories for formal class meetings

Frequency of use	Average hrs/month	Number of lecturers 1985	1986	1987	1988
More than 2 hours per week	10.0	1	1	2	1
1-2 hours per week	6.0	2	4	3	3
1-4 hours per month	2.5	2	2	4	7
Less than 1 hour per month	0.5	2	4	4	3
Estimated total hrs/month		28	41	50	47

Of even more interest, perhaps, are the reasons why lecturers choose to use the laboratories (Table 7.3). The three most frequently named reasons were:

to reinforce/enhance understanding of an algorithm/method

as a means of solving more realistic problems

to carry out investigations and/or experiments.

Of those lecturers who used computers for any of their teaching, the percentage selecting each of these reasons increased between the years 1986 and 1988. This could indicate a growing awareness among these lecturers of the potential of computer-based packages to improve these aspects of their teaching. Other reasons mentioned were for student projects, to motivate non-mathematically inclined students, for preparation of classwork and to provide 'hands-on' experience.

Table 7.3: Lecturers' reasons for using the laboratories

Reason	Number of lecturers 1985	1986	1987	1988
An improved teaching method	6	-	-	-
To reinforce/enhance understanding	-	8	10	13
To solve more realistic problems	5	5	8	10
Investigative work	6	8	12	13
To set student assignments using packages	2	7	9	5
Student programming assignments	0	1	1	2
Total no.of lecturers using labs.	7	11	13	14

Reasons for not using the laboratories as often as they would have liked are summarised in Table 7.4. The most frequently given reason was that the class size was too large. Some lecturers were able to overcome this problem by setting computer-based assignments to be completed in the student's own time. Generally, the class would be split into manageable groups who would be given an introductory session in the laboratory. The assignment would then be completed in the student's own time during open-access. It is interesting to note that the number of positive replies to the first five reasons listed increased annually. This suggests that the desire to use the laboratory facilities was growing at an even greater rate than the actual usage. Physical and practical considerations limited the potential use of the laboratories, with large class sizes and insufficient class time being the most difficult obstacles to overcome.

Table 7.4: Reasons for not using the laboratories

Reasons	Number of lecturers 1985	1986	1987	1988
Class size too large	1	5	7	8
Appropriate software not available	2	4	4	5
Worksheets not available	1	2	3	3
Insufficient class time	0	2	5	9
Laboratory already booked	0	3	0	4
Laboratory out of order	0	0	6	4
Teaching at another site	4	9	2	1

Further detailed information was obtained from extended informal interviews with five lecturers from the Department of Mathematics in 1989 (Appendix 10). All those interviewed had been using computers to assist their teaching for several years. They had, therefore, had time to develop and evaluate (informally) their new teaching approaches and to reflect upon the value and effectiveness of computer-based learning.

All the staff interviewed stressed that the use of computers is integrated into their lecture programme. Laboratory sessions most often replace conventional tutorial classes but, in some cases, they take the form of laboratory lectures. One lecturer explained that all his time with some classes was timetabled in the laboratory. There was then no distinction between lectures and tutorials, and computers could be used whenever appropriate. All the lecturers relate work being done in class to packages available in the laboratories. If there is insufficient time to use a package during class time, the students are encouraged to go and try it out for themselves. There is evidence from both students (student interviews, 1986, Appendix 7) and lecturers that some students did use the packages in their own time. Referring to his first year physics class, lecturer B said he had "broken the barrier". The students were "quite familiar with the lab. and happy to use it" both for mathematics and to assist their physics assignments. Several of the staff felt that lectures were becoming less formal with more participation from the students. This was achieved, in one case, by the use of courseware booklets based on MINITAB and, in another case, by using 'interactive handouts' which the students completed as the lesson proceeded. Lecturer C used a computer-illustrated textbook (Bowman and Robinson, 1987) as the basis for a course in statistics. At appropriate points in the text, the student is invited to use the accompanying software to illustrate the topic being studied. Spells of working through the textbook as a whole class were interspersed with periods when the students worked individually, or in pairs, at the computer. Most lecturers also used other resources such as videos to assist their teaching when appropriate.

The interviews confirmed the reasons given in questionnaires for using computer packages (Table 7.3). Lecturer A commented that, with recent developments in hardware and software, it was difficult to see how one could avoid using computers. He uses packages to reinforce ideas and to explore them further, for example, with numerical integration. More use is made of graphical output and more time is spent on experimental and investigative work.

Students spend their time in the laboratory doing more interesting things according to lecturer C. Investigations are presented in an open-ended way to encourage students to take them as far as they like. In this way, there should be a greater challenge for more able students, but only two lecturers felt that this was being achieved.

Three interviewees stressed the advantages of graphical output as an aid to understanding. For example, when finding stationary values of a function of 2 or more variables, students can use a 3D graphical package to draw the surface and thus obtain a visual check of the minimum or saddle point previously located using calculus.

> "This boosts their confidence .... and makes it more interesting .... It is easy for students to follow the recipe for obtaining and classifying stationary points - but they forget what they are really doing."

Supervised laboratory sessions entail harder work for the teacher as they involve continual interaction with students, usually one or two at a time. The lecturer asks more questions of the students than he or she would do in a conventional classroom setting and students are more responsive.

> "The awareness of the facilities leads one to .... pose more questions", for example, "as to what the graph of a function looks like."

Questions asked by the lecturer in the laboratory may also be more complex and take longer to respond to. Lecturer B indicated that the questions asked are more demanding conceptually. He uses questions to probe understanding. For example, the idea of steady state is difficult to get across analytically, but is obvious when seen graphically on a computer.

> "That is one (concept) I would always query them about .... particularly when you get a steady state being an oscillation instead of a horizontal line. What has it got in common with the previous steady state? What has it got different? These are questions I wouldn't really ask analytically. They would be too tied up with the maths to think about it .... I rely heavily on the

graphical output."

Lecturer F pointed out that there can be more support for weaker students as the lecturer can keep prompting the student to interpret his/her results. Another claimed:

> "There is not as big a difference between the weaker student and the stronger student in the laboratory .... I have also noticed that some of the weaker students analytically often do better in the laboratory because they realise that there is something here that they can come to grips with and they take it very seriously."

Investigative work, the use of graphical output and appropriate questioning by the lecturer can, therefore, all contribute towards improved understanding of mathematical concepts by the students, thus enhancing the quality of their learning.

All the staff interviewed enjoy teaching more as a result of using computers. There is less drudgery and it is more interesting. It can also generate interesting discussions with students on points which would not have arisen otherwise. Some explained how using computers had affected aspects of their own attitude towards mathematics. One felt that her understanding of the practical side of handling data had benefitted, whilst another has found that certain topics, such as the phase solutions of differential equations, have become much more alive and meaningful as a result of the new graphical approach. Several teachers mentioned that they sometimes feel hampered if a computer is not available when they are teaching.

Computer-based learning passes more responsibility to the students who can work through an algorithm or method at their own pace and return to the computer as often as they wish. They can explore topics further, experiment with parameters and choose from a variety of outputs. In general, co-operation between students in the laboratory is seen as a good thing. Three lecturers actively encourage students to work in pairs on the computers, whilst the others prefer them to have a machine each but to confer freely.

7.3 Emphasis and content of the curriculum

The use of computers in the teaching of mathematics affects not only how some topics are taught but also what is taught. In the past, numerical analysis consisted solely of learning techniques. Two lecturers now use NODES to investigate numerical methods for solving single and systems of differential equations. For example, students no longer work through many examples using the 4th-order Runge-Kutta method by hand, but do spend more time on error analysis which is more easily understood using graphical output from NODES. Students now also study the stability of numerical methods, using NODES to investigate the solution as the step length varies.

Techniques are played down and there is more emphasis on applications which can be explored with the help of a computer package. NODES has been used very effectively to investigate the behaviour of given mathematical models which would not be possible without computers. The SIS5 syllabus has changed significantly. Differential equations now form almost two-thirds of the course whereas, formerly, they occupied one-third. Far more time is spent investigating models and applications. More discrete mathematics such as non-linear difference equations (leading to chaos) is also covered. Leslie matrices, which are a related idea, have been reintroduced to SIS5 because the difficult mathematics can be done by computer. These two topics now make up the remaining third of the course and stochastic processes have been dropped. With other classes lecturer B has initiated the use of computer algebra packages to enable students to do mathematics that they could not otherwise do, thus showing the power of such packages.

Lecturer C uses time saved on computation within a topic to extend the topic. The students

"are still learning all the things they learned before ....

but are not endlessly applying algorithms and techniques."

For example, in linear programming, whereas students used to spend a lot of time solving a few examples by hand, they now solve more problems using LINPROG, devote more time to the formulation of the problems and are introduced to post-optimal analysis. The use of computer packages allows students to solve problems which would take far too long to do otherwise, such as integer programming problems using the branch and bound method. Lecturer C claims that this is a particularly successful use of computers which combines the use of a package (LINPROG) to solve all the intermediate linear programming problems with pen and paperwork to construct a tree diagram. Both SIS4 and CEE4 classes have benefitted from using LINPROG in this way.

The use of computer packages in the teaching of statistics is well established at Napier. Two of the staff interviewed are primarily involved in teaching statistics, and lecturer F has developed courseware booklets based on the use of MINITAB. Both lecturers indicated that about 50% of tutorial time is spent in the computer laboratory. Lecturer D disclosed that a few years ago he tended to use computers too much and too soon but he feels he has achieved a better balance now. Use of packages such as MINITAB enables realistic data sets to be accessed, eliminates tedious arithmetic and allows the student to do more.

The use of computer packages has led to some topics being approached in a completely new way, starting from data rather than starting from the theory. The balance of importance between topics has changed with more emphasis being placed on regression and hypothesis testing. Within topics the balance has shifted from calculation to understanding and interpretation of results. Students now use real data for project work, often gathered from their own discipline (e.g. biology, chemistry, business studies) and then analysed using statistical packages.

The use of computer packages has enabled more of the educational objectives at the upper end of Bloom's taxonomy to be achieved. It is evident that, by involving students in more investigative work and analysis of models, NODES and LINPROG, in particular, have been used to enhance and extend the quality of their learning experiences.

7.4 Assessment

Interview data revealed a trend towards the use of more computer-based assignments as part or all of the assessed coursework replacing class tests. This seems to be the most appropriate way of assessing computer-based work at present. The engineering applications component of the mathematics course for the Communications and Electronic Engineering degree now comprises a computer-based investigation of a mathematical model, as described in section 5.4. For many courses changes in methods of assessment require the approval of a multi-disciplinary course team. Lecturers from other departments have sometimes been reluctant to relinquish formal mathematics tests in favour of computer-based assignments to be completed in the student's own time but such opposition is gradually diminishing. An innovation tried in the 1989/90 session, for such a course, was a formal computer-based assessment in the laboratory during a timetabled session.

Lecturer B has introduced a system with some classes whereby all laboratory tutorials are assessed and contribute towards an overall coursework mark. His worksheets contain blank spaces for answers to specific questions which the students complete and hand in at the end of the session.

> "The students always work in pairs. They prefer it and I encourage it because they learn more. It is more difficult to assess but it is primarily a learning situation rather than an examination situation and I want them to learn as much as possible."

Occasionally he sets a more extended assignment which may be completed over two or three sessions, but he tries to avoid asking them to write lengthy reports as he feels many classes are already overburdened with coursework. Other lecturers see report writing as an important part of an assignment, encouraging communication skills and reflection upon results.

Two lecturers have also used computer-generated data or tests for assessed coursework. According to lecturer D, this reduces "coursework by committee. Students can discuss the work with each other but not copy results."

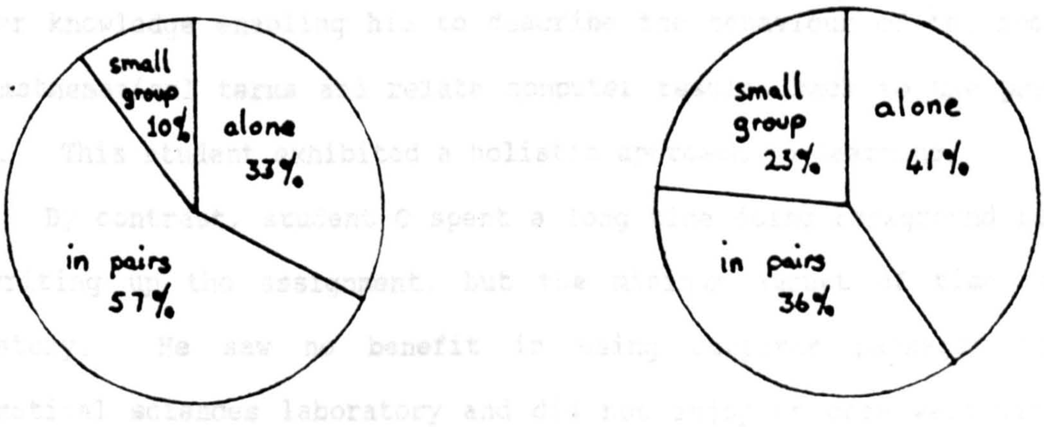
The content of examination questions has been slowly changing as a result of the wider use of computers. This trend is most marked in statistics, where a paper can now include several pages of MINITAB output. The student might be required to analyse data or interpret results by extracting information from the computer print-out. In linear programming problems a question might include several tableaux of the solution by the Simplex method and the student be asked to pick out the solution and interpret it. Students are no longer asked to perform a numerical method such as 4th-order Runge-Kutta by hand in an examination if they have used NODES to investigate the method in the laboratory. Instead, more conceptual questions, concerned with the behaviour and accuracy of the method, would be asked based on their laboratory work.

7.5 Student attitudes

All the lecturers interviewed agreed that a few students do not like using computers but the majority enjoy it and, as a result, are more interested and attentive. "Their ability to communicate improves dramatically", and they become more mathematically confident. Some students show more desire to experiment; NODES is particularly good for this. Their investigative skills improve, probably because they spend more time talking about the problem and solutions. They are not in so much of a hurry to get to the computation, which used to be the time-consuming stage, since the computer now takes care of that, and thus spend more time thinking. If their formulation is wrong, less time has been wasted and it doesn't take long to get a new solution. The mathematics students tend to ask more questions and discuss more with each other. At first, the questions are mainly concerned with the operation of the computer and the package but, later, they are interested in the meaning of the results.

Whereas students in classes SIS4 and CEE4 used the mathematical sciences laboratory mainly for supervised sessions and had little or no computer-based work assessed in the years 1986-88, the CEE1 and CEE2 classes who were surveyed in 1987 and 1988 used the laboratory primarily in their own time for an assessed computer-based assignment. Results from student questionnaires reveal that 67% of the latter group, but only 52% of the former group, thought that some computer work should be assessed. The higher percentage of the group who were assessed being in favour of such assessment may be due to the amount of effort required to complete their assignment or may reflect satisfaction with this form of assessment. Figure 7.1 shows that the majority of students preferred to work co-operatively at the computer, but students in the assessed group were more likely to work alone.

Figure 7.1: Preferred mode of working at computer



(i) non-assessed group (ii) assessed group

Interviews with three students from CEE2 in March 1988 provided more insight into these students' learning experiences (Appendix 11). This class used NODES to carry out an engineering applications assignment to investigate the Van der Pol oscillator, including the study of limit cycles. The students exhibited different approaches towards the computer-based exercise.

Student A enjoyed investigating the mathematical model and diligently produced a great many hard-copy graphs, many of which he did not use for his written report. The graphical output was an important factor in his understanding of the model but he found it difficult to write the report. He did not consider the subject matter relevant to his course.

Student B was very enthusiastic about the investigative aspects of the assignment and the use of NODES. He also appreciated the wider relevance of the exercise and would not expect the model chosen for investigation necessarily to have been covered in classwork. There is "no point in knowing all the theory and how to solve all the equations if you've never come across practical applications". He considered the report writing stage important as it forced him to draw conclusions about his laboratory work. He believed that the standard of his report writing had improved since last year. This was due both to more practice and greater knowledge enabling him to describe the behaviour of the model in more mathematical terms and relate computer results back to the physical model. This student exhibited a holistic approach to learning.

By contrast, student C spent a long time doing background reading and writing up the assignment, but the minimum amount of time in the laboratory. He saw no benefit in using computer packages in the mathematical sciences laboratory and did not enjoy or cope well with any computer-based work. He did not consider it to be as important as 'examinable' topics and therefore did not devote much time to it. His priority was to attain a good overall assessment. He had no interest in the assignment and did not consider it a useful or relevant exercise. He believed that this view was shared by the majority of his class.

Data from the 1988 questionnaire completed by 50 students in this class (SQN2, Appendix 9) do not corroborate this opinion. The questionnaire findings show that:

73% of the class found 'investigating a model' interesting,

39% found it enjoyable, and

60% found it useful towards the rest of their course.

Only 12% found report writing interesting, and even less found it enjoyable, but 40% considered it to be useful towards the rest of their course.

64% of the class (all male) enjoyed mathematics (rated 1 or 2). Only 26% said that the use of computer packages contributes towards their enjoyment of mathematics and 16% said it didn't contribute at all. However, 61% thought that the use of packages enhanced their understanding of aspects of mathematics, the most commonly cited aspects being the behaviour of mathematical models and being able to study the solutions of differential equations graphically.

Student attitude surveys

Information for the 1989 LINPROG and NODES attitude surveys (Appendix 4) was coded as follows. Statements with which agreement implies a generally positive attitude were coded:

Strongly agree	5
Agree	4
Not sure or doesn't apply	3
Disagree	2
Strongly disagree	1

For the remaining statements, agreement with which signifies a negative attitude, the scoring was reversed so that 5 represented 'Strong disagreement' and 1 'Strong agreement'. In either case, a score of 3 indicates 'Not sure or doesn't apply'. Another lecturer was asked to categorise the statements as positive or negative independently to confirm the groupings shown below:

Program	Positive statements	Negative statements (coding reversed)	Neutral
LINPROG	1,3,4,6,8,9,11,12, 14,16,18	2,5,7,10,13,15	17
NODES	1,3,4,5,7,9,10,12, 13,14,17,18,20	2,6,8,11,15,16	19

The surveys were then analysed using the MINITAB statistical package. For the LINPROG survey, scores for the 18 statements were read into 18 data columns, each row representing one student's score. Similarly, the NODES data were read into 20 columns, one for each statement in the survey. Each column thus contained all the scores for a single question, whilst each row held the data for a particular student.

One drawback to using a Likert scale for attitude measurement is that identical total row scores may be obtained in different ways and may, therefore, have different meanings (Oppenheim, 1966). It is important, therefore, to look at the pattern of responses also.

Results for LINPROG

The coded survey data for LINPROG is given in Appendix 12. The following statements had the highest column sums, thus showing the strongest agreement or disagreement:

Strong agreement

16 Similar computer programs should be used to enhance the learning of other topics

1 I enjoyed using LINPROG

12 Using LINPROG made the topic more interesting

14 Knowledge of linear programming techniques would be useful when working in industry or commerce

Strong diaagreement

2 I couldn't understand what the program was doing

10 I had difficulty using LINPROG

Row sums were calculated omitting the neutral column 17. The stem and leaf diagram (Figure 7.2) shows that 73% of those surveyed have a generally positive attitude towards the use of LINPROG. Only one student showed a slightly negative attitude.

Figure 7.2: Student attitudes towards use of LINPROG

Attitude	Stem	Leaf
Minimum 17		
Negative		
	4	9
--- Neutral 51 ------	-- 5 ---	- 1 1 1 2 2 2 2 ----
	5	5 7 7 7 7 8 8 8 8
Positive	6	0 0 1 2 2 3
	6	5 5 6 7 8
Strongly positive	7	1 2
Maximum 85		

Notes: (i) *N = 30; leaf unit = 1*
(ii) *Rows with one item missing have row mean added to row sum*
(iii) *Rows with more than one item missing have been omitted*

A principal component analysis of the responses of the 28 students (omitting those with data items missing) to the 18 survey statements was used to identify possible relationships between statements (Appendix 12). The first principal component of the analysis shows a 23% variation between students, attributable mainly to statements:

3 Using LINPROG helped me to understand the Simplex method

4 Using LINPROG helped me to understand the concept of duality

7 I learn more by working it out by myself on paper (reversed)

9 LINPROG reinforced my understanding of linear programming

11 Using LINPROG helped me to understand the branch and bound method for integer programming

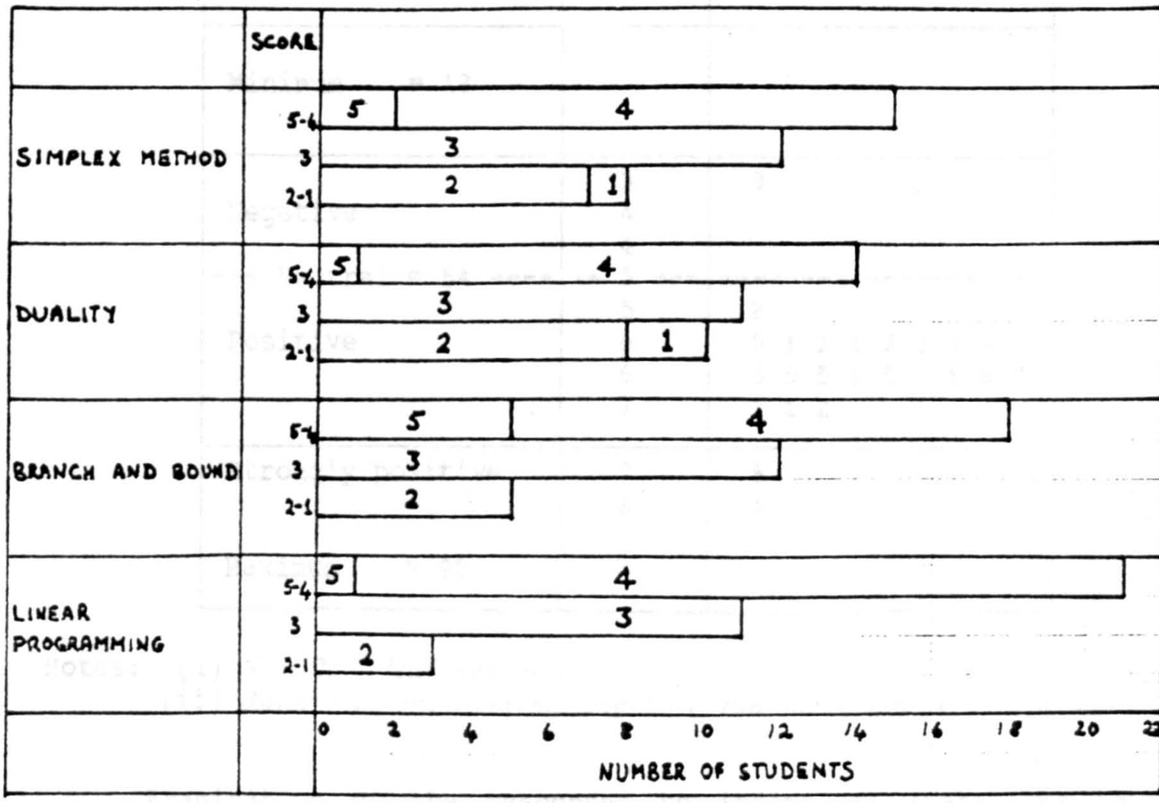
12 Using LINPROG made the topic more interesting

15 I am only really interested in passing my exams (reversed)

16 Similar programs should be used to enhance the learning of other topics.

This group contains all the statements which relate to understanding aspects of linear programming and the algorithms used and also statements which imply an active interest in the subject and course. This suggests that those students who felt that their understanding of aspects of linear programming was improved by the use of LINPROG were also well motivated generally. An analysis of the scores for the questions relating to understanding is given in Figure 7.3.

Figure 7.3: LINPROG as an aid to understanding



Results for NODES

The coded survey data for NODES is given in Appendix 12. Row sum totals, indicating overall attitude, have been calculated by summing all columns except 4 and 19. Statement 19 concerns preference for single or group working. Twenty out of twenty-five respondents scored 3 ('Not sure or doesn't apply') for statement 4, and it was subsequently confirmed that neither class had studied stiff equations. Since 18 statements have thus been used to estimate overall attitude, a score of 18 × 5 = 90 is the maximum possible score, 18 × 1 = 18 is the minimum and 18 × 3 = 54 indicates a neutral attitude. The stem and leaf diagram (Figure 7.4) indicates that, with one exception, all the students showed an overall positive attitude towards the use of NODES in their mathematics course, with 16% expressing a strongly positive attitude.

Figure 7.4: Student attitude towards use of NODES

Attitude	Stem	Leaf
Minimum = 18		
	3	9
Negative	4	
	4	
--- Neutral = 54 ----	-- 5 ---	--------------------
	5	8
Positive	6	0 1 1 1 2 2 3 4
	6	5 5 5 5 5 7 8 8 8
	7	1 1 2
Strongly positive	7	6 7
	8	4
Maximum = 90		

Notes: (i) *N = 25; leaf unit = 1*
(ii) *Rows with one item missing have row mean added to row sum*

Examination of the responses to individual statements revealed that the strongest agreement or disagreement was with the following:

<u>Strong agreement</u>

10 Graphical solutions of differential equations are easier to understand than numerical values

14 Investigating a model using NODES is a useful exercise

18 Similar programs should be used to enhance the learning of other topics

17 Experience of investigating mathematical models would be useful in industry

1 I enjoyed using NODES

9 I like to experiment with different parameters in the program and see how they affect the solution

13 Using NODES made the subject more interesting.

<u>Strong disagreement</u>

11 I had difficulty using NODES

2 I couldn't understand what the program was doing.

A principal component analysis of the 20 questions in the NODES survey was carried out to explore possible relationships between groups of statements (Appendix 12). A summary of the information gained is given below.

The first principal component of the analysis accounts for 32% of the variation between students. The statements contributing most to this variation were:

14 Investigating a model using NODES is a useful exercise

17 Experience of investigating mathematical models would be useful in industry

12 NODES reinforced my understanding of the behaviour of solutions to differential equations

9 I like to experiment with different parameters in the program and see how they affect the solution

13 Using NODES made the subject more interesting,

and to a lesser extent:

16 I am only really interested in passing my exams (reversed)

15 I did not learn anything new about solving differential equations (reversed)

3 Using NODES helped me understand numerical methods for solving differential equations

18 The program was user friendly.

These nine statements can be classed as:

(i) concern with the usefulness or relevance of using NODES or similar packages (14,15,17);

(ii) showing interest in the subject being studied and the course (9,13,16);

(iii) acknowledgement of its role in reinforcing understanding of aspects of differential equations (3,12).

Students who scored highly on this set of statements would be exhibiting an overall positive attitude towards their course in general, and the use of NODES in particular. Since most of these statements received high overall scores in the survey, this supports the finding that a large proportion of the students surveyed exhibited a positive attitude. For this group of students the main benefits accruing from their use of NODES could be summed up as follows:

> Investigating a model using NODES and experimenting with different parameters to see how they affect the solution is an interesting and useful experience.

7.6 Transfer to other institutions

Dundee College of Technology

Evaluative data for LINPROG were gathered from students and their lecturer at Dundee College of Technology in 1988, as described in section 5.6. This third year B.Sc. science class used LINPROG to reinforce their study of the Simplex method for solving linear programming problems.

At the observed session in February, 1988, nine students were present, five female and four male. Two students shared a terminal, whilst the others had one each, but all were seen to be collaborating roughly in pairs throughout the two-hour session. The class had a hand-out explaining the Simplex method, but had not previously worked through any examples by hand. They were, in effect, using the program to learn the method, although they did understand terms such as slack variables, basic feasible solution, etc. After the lecturer had guided them through the procedure to set up the initial tableau, the students worked through examples on a given sheet (prepared by the lecturer) using the 'tableau by tableau' option.

The students had few difficulties with the operation of the program but asked frequent questions about the problems they were trying to solve. They all had previous experience of using computer packages for mathematics, but LINPROG was their first involvement with a microcomputer-based package. One student neatly summed up her reaction to using LINPROG as follows:

"Use of the program helps you to become familiar with the use of the correct terminology and layout. When you are working on your own on paper, you don't always use the correct terminology and thus don't always understand results at a later stage. You get through lots more problems, and thus become familiar with these things and the method more quickly."

The students completed questionnaire SQL1 after their third linear programming laboratory session. The above explanation was supported by comments in a questionnaire:

"It helps you to understand the Simplex method and duality. It is a good back-up to lectures. You can get through more examples than you would by hand calculating. It helps you to become accustomed to the terms used and it is quick. You need to do hand calculated examples, too, to get a good understanding. You can't depend on LINPROG alone. It doesn't explain slack variables."

These students' ratings for LINPROG are shown in Table 7.5, with those given by Napier students in 1988 (Figure 6.3) in parenthesis for comparison. The percentage cumulative frequencies are similar for the three aspects common to the two surveys. The Dundee students rated reliability more highly, but, as explained in section 5.1, hardware factors may have influenced these ratings as students do not necessarily distinguish between hardware and software reliability. Flexibility/options offered scored a lower rating than the other aspects, thus reinforcing Napier staff appraisals.

Table 7.5: LINPROG student ratings, Dundee, 1988

	Ease of use		Reliability		Usefulness of output		Flexibility/ options	
	No.	Cumulative %	No.	Cumulative %	No.	Cumulative %	No.	Cumulative %
Very highly 4	4	36 (42)	5	45 (31)	2	18 (23)	1	9
3	6	90 (77)	6	100 (81)	7	81 (81)	6	63
2	1	100(100)	0	(100)	2	100(100)	4	100
Poor 1	0		0		0		0	
Sample size	11	(26)	11	(26)	11		11	

Note: *Figures in parenthesis are Napier student ratings, 1988*

Other questionnaire results, detailed in Appendix 13, convey that the students were more enthusiastic about their use of LINPROG than about the use of mathematical computer packages generally, but were not in favour of having computer-based work included in their overall assessment. Although one student suggested that the package "could be more user friendly", four commended it for its "ease of use" and one for being "easy to understand". Two others liked the speed of calculation.

The lecturer at Dundee also completed a questionnaire in which he rated LINPROG 4 (very highly) for 'Ease of use' and 3 for the other aspects listed above. He adjudged his use of LINPROG to reinforce understanding of the Simplex method to have been very successful, due to its "interactive capability in the learning environment". He disliked the fact that the program forces maximisation to be the standard problem and also found the tableau by tableau output difficult to read for some larger problems. Overall he felt that the students' reaction had been very favourable:

> "They took to it easily and picked up the Simplex procedure from hands-on experience (with no prior class tuition)."

Other higher educational establishments

Six completed questionnaires were received from customers who had purchased LINPROG between 1986 and April 1988. Five of these customers had used the program with students and their evaluations of the resulting learning experience are shown in Table 7.6. One user observed:

> "It certainly improved their facility in the mechanics of the method but true understanding is unlikely to come from such work."

The program ratings of the six purchasers are shown in Table 7.7.

Table 7.6: Customers' reasons for using LINPROG

Reason for use	No.of users	Degree of success: Very successful ----------> Not successful 4	3	2	1
To enhance understanding of the Simplex method	4	1	2	1	0
To enhance understanding of duality	1		1		
To solve realistic LP problems	2		2		

Table 7.7: Purchasers' ratings

	Ease of use	Reliability	Usefulness of output	Flexibility/ options
Very highly 4	2	0	1	0
3	2	3	3	2
2	1	1	2	0
Poor 1	1	1	0	2
Don't know	0	1	0	2

The most critical comments came from a purchaser who was using the package with business studies students. He described the program as poorly designed and rated no aspect higher than 2. Comments and ratings from other reviewers, at least three of whom were teaching mathematics students, were more favourable. The detailed step by step approach to the Simplex method was commended as was the ease of entering and amending input data. Criticisms included the automatic conversion of minimisation problems to the dual problem, a program error, and being unable to add or subtract constraints when using the Simplex method.

The back-up material used with the program consisted of sheets of straightforward linear programming problems. No one had used the package for post-optimal analysis, or mentioned doing any investigative exercises. Several customers commented that it was still too early to assess fully the usefulness of the package, but they clearly intended to use it again.

CHAPTER 8

Discussion and Developments

8.1 The Learning Packages developed

Problem solving and investigative work should form an important part of the mathematics curriculum of science and engineering students. This research has confirmed that these activities can be made more effective with the aid of computer-based packages. Unlike some software, labelled as educational, LINPROG and NODES were specifically designed as teaching tools to satisfy needs which emanated from real teaching situations at Napier Polytechnic. The design and content of the programs resulted from consultation with lecturers, thus ensuring their suitability for the teaching objectives for which they were intended. Acceptance of the packages as effective teaching tools is evident from the increase in their use over the survey period to a level which has subsequently been maintained.

Both packages were rated favourably by staff and students for their ease of use, reliability and usefulness of the output. Student comments emphasised LINPROG's role as an aid to understanding both the Simplex method for linear programming and the branch and bound method for integer programming. The graphical output from NODES was highlighted as an important feature, which assisted student understanding of the behaviour of solutions of differential equations.

In his final report of the National Development Programme, Hooper (1977) asks that evidence of institutionalisation should be the primary gauge of success of a project rather than generalisations derived from experimental classroom use. At Napier the crucial stage of institutionalisation has been accomplished and both packages are now firmly established in the teaching repertoire of several lecturers.

The successful transfer of a package to other institutions not only helps to justify the high development cost of the materials but is also a useful indicator of its educational value. Complete transfer of a package is unusual. It is more likely that a lecturer would wish to use selected parts of a package or to adapt accompanying notes or coursework to fit his student's needs more closely. LINPROG has been successfully transferred to Dundee College of Technology for some of its teaching objectives. The computing laboratory facilities and the mathematics curriculum of the course for which LINPROG was used at Dundee were similar to those at Napier. Student and lecturer reaction was very favourable. The lecturer prepared his own worksheets, the content of which was restricted to enhancing the existing teaching approach for the Simplex method. No attempt was made during the period of evaluation to use LINPROG to introduce investigative work such as post-optimal analysis.

The response of purchasers of LINPROG was more varied. The most successful use was achieved by lecturers using the package with mathematics students. Reaction from a lecturer of business studies students was more critical. No mention was made by any of the respondents of using LINPROG to facilitate investigative work. Many teachers are reluctant to make significant changes in their teaching strategies even when computers become available (Hartley and Bostram, 1982). This study has demonstrated that a more investigative approach to teaching and learning at Napier evolved over a number of years after computers were introduced. Depending upon previous experience, it may take several years of gradual advancement before lecturers at other establishments are willing fully to exploit the potential of this computer package.

Although the use of NODES within Napier has been judged highly successful, it was not transferred to any other institution during the research period. A number of copies of the program have been sold but no data has yet been gathered from purchasers regarding its use. The booklets accompanying NODES and LINPROG contain examples of use of the package, but experience indicates that it might be beneficial to add suggested laboratory exercises in order to encourage lecturers at other institutions to explore a wider range of educational possibilities.

8.2 A Changing Role for the Teacher

Use of the mathematical sciences laboratories at Napier Polytechnic for timetabled classes increased by about 70% over the four-year period 1985-88. The majority of lecturers in the Mathematics Department now use computer-based packages as a tool to support their teaching. Many lecturers, therefore, have had to reconsider their teaching objectives and adapt their teaching approach to accommodate new styles of learning and the wider opportunities offered by computers. Computer packages have enabled teachers to arrange learning situations which were not previously possible, for example, using graphics to explore the behaviour of a function, to analyse the solution of a mathematical model or to investigate the stability of a numerical method.

Co-operative learning has flourished in the laboratory as most students prefer to work in pairs or small groups. Opinions vary amongst the lecturers as to whether the students should work alone or in pairs on the computer, but all lecturers encourage students to discuss problems and results both with each other and with the teacher. As a result, the laboratory environment is less formal than a conventional classroom where one lecturer faces rows of students. Control is shifted away from the lecturer towards the students who, working autonomously, manage their own learning. The teacher's role, however, has not diminished in importance and class contact time has not been reduced. On the contrary, it was found that class sessions in the laboratory can be harder work for the lecturer because the students ask more questions, some of which involve concepts that would not have been encountered otherwise. The role of the teacher and the computer are complementary and both contribute towards the learning process. The computer is used as a tool to remove tedious calculation, to present information graphically and to increase a student's involvement in his own learning. The teacher's skills lie in recognising and seeking to rectify a student's particular weaknesses or misunderstandings and exploiting the learning situations which arise. All five lecturers interviewed were enthusiastic about their use of computers and enjoy teaching more as a result of using them. By reflecting on the educational advantages of their new approach, these lecturers have responded positively to their changing role and will remain receptive to further developments and opportunities.

A few lecturers failed to make any use of computers in their teaching during the survey period. Large classes and lack of class time were mentioned by one lecturer, whilst another claimed insufficient time for preparation due to other commitments. Some teachers may be apprehensive of the changes which the introduction of computers would bring to their established teaching practices. They may even feel that others are using the new technology as a substitute for teaching skills. Further research is required to investigate fully the reasons for this reluctance.

8.3 <u>Learning Outcomes</u>

The learning outcomes which have resulted from using computer-based packages as a tool for mathematics represent a broader range and higher quality of cognitive and attitudinal outcomes than previously present in the mathematics curriculum at Napier Polytechnic.

<u>Student understanding of some mathematical concepts and algorithms has been improved or reinforced</u> as a result of using computer

packages. More than 40% of the students in two classes (SIS4 and CEE4) considered that use of LINPROG helped them to understand the Simplex method, duality, the branch and bound method for integer programming problems, or some other aspect of linear programming. Use of the NODES package for engineering applications assignments resulted in increased understanding of the solutions of differential equations by 68% of respondents in a second year class. In particular, many students found that being able to study the solution of equations graphically using NODES improved their understanding of the behaviour of the mathematical model.

Investigative and experimental work has increased and more emphasis is placed on this type of work. The use of computer-based packages has been successfully integrated into the curriculum of several courses at Napier. In such courses there is now less time than previously spent teaching techniques and more time spent solving problems, formulating and testing mathematical models and doing investigative work. In linear programming, for instance, more time is available for post-optimal analysis, which is introduced in a meaningful way with the aid of the LINPROG package. Almost all the lecturers using the laboratories cited "To carry out investigative and/or experimental work" as one of the reasons for choosing to do so. Extensive use is made of graphical output to examine the behaviour of functions, to analyse models and to investigate the errors in, or the stability of, a numerical method, using one of the packages available in the laboratories. Students' work in the laboratory is directed by worksheets which have been designed to accompany the computer programs and which contain some open-ended investigations. Results from student surveys show that the majority of students consider investigative work to be both interesting and useful.

Students solve more realistic problems. There is much wider scope for problem solving, both in statistics, where 'real' data is used and in areas such as linear programming. As the time spent on computation has been greatly reduced, more time is spent formulating problems and there is more emphasis on the analysis and interpretation of results.

Students spend more time testing and analysing mathematical models. Computer-based packages enable a much wider range of problems to be tackled. In the first four years of the Communications and Electronic Engineering degree, the students carry out a computer-based assignment as part of the engineering applications element of the course. This involves investigating the mathematical model of a physical system using a computer package. NODES was found to be an excellent tool for this purpose.

Mathematical discussion has increased. The importance of being able to communicate mathematical concepts confidently was highlighted in Chapter 2. The use of computer-based packages stimulates mathematical discussion in the classroom both between teacher and student and between the students themselves, confirming findings from a previous study (Katsifli, 1986). Graphical output offers the lecturer many unrivalled opportunities to probe the students' understanding by appropriate questions. Evidence from lecturers also suggests that students' competence in discussing mathematical ideas improves as a result of using computer packages in the laboratory. Working through an algorithm step by step on the computer (for example, the Simplex method for linear programming), not only aids student understanding of the method but also helps him to become familiar with the mathematical terms being used. This boosts the student's ability to communicate clearly.

The above outcomes relate to the higher levels of Bloom's taxonomy of cognitive learning objectives, discussed in Chapter 2.1. In particular, the skills of analysis and synthesis are fostered by the mental processes involved in computer-based exercises. The ease with which the user can change the parameters of a model or problem encourages her to explore particular instances of mathematical behaviour. Systematic variation of parameters enables the student to progress towards the stage of generalisation where she can predict the change in behaviour when a parameter is changed. As this new learning is assimilated into existing mathematical schema, relational understanding develops (Skemp, 1976).

The natural curiosity of some students will lead them through the above stages with little assistance but, for the majority, the structure of the worksheet plays a crucial role in directing their mental activity towards relational understanding. Discussion with fellow students or a lecturer also contributes towards the generalisation and assimilation process.

The above results are broadly similar to those observed during experimental computer-based projects with schoolchildren (Fraser, 1987). Learning outcomes in the affective domain are also important and, in many cases, closely interrelated to cognitive outcomes. Results reported in Chapter 7 show that, for most students, the introduction of computer-assisted learning has had a positive influence on their attitude towards their mathematics course.

<u>Learning is more student-centred</u>. Computer-based work is usually more self-paced than traditional coursework and allows the student to control the level of explanatory detail required. It thus passes more responsibility to the student for his or her own learning. Explanatory feedback from errors enables a student to learn from his own mistakes. Many laboratory worksheets include open-ended exercises to encourage the students' creativity and collaboration. Some students return to the laboratory in their own time to use packages to reinforce ideas, explore them further or to assist with coursework for other subjects.

<u>Many students enjoy their mathematics course more</u> as a result of

using computer packages. Attitude surveys concluded that many of the students surveyed felt that use of the LINPROG and NODES packages had made the topic being studied enjoyable and more interesting. Increased motivation probably leads to an improvement in educational achievement. There was also strong agreement that similar programs should be used to enhance the learning of other topics. A minority of students dislike using computers. In some cases, early computer programming experience has left the student confused and lacking in confidence. Additional help should be made available to such students, perhaps in the form of a computer-based tutorial, to increase their basic computing skills. A few students felt that computer-based work was not so important as more directly examinable parts of the course.

Most students consider their computer-based work useful and relevant, both to their course and to their future career in industry or commerce. This outcome was evident from the results of the student attitude surveys and could be summed up by one student who commented that there is

> "no point in knowing all the theory and how to solve all the equations if you've never come across practical applications".

One of the three students interviewed after completing a computer-based engineering applications assignment showed evidence of a holistic approach to learning. He used the assignment to improve his overall understanding of the topic and to build upon existing knowledge. The report writing stage entailed reflective thinking as it required him to draw conclusions about his laboratory work. When a student is well-motivated, it seems likely that an investigative assignment of this nature encourages a holistic approach to learning, as asserted by Cawley (1988). By contrast, another student in the group had adopted a surface approach to learning which excluded anything not regarded as immediately relevant to achieving a good mark in the next examination. As computer-based work came into this category, he did not devote much time to the assignment and, consequently, derived little benefit from it. To increase the motivation of such students and encourage a deeper approach to learning it would be necessary to place greater emphasis on the assessment of computer-based work. This should not, however, be at the expense of allowing students sufficient time to progress, at their own rate, towards the goal of understanding.

The success of the Napier approach to computer-aided learning can be attributed to various factors. The computer programs and accompanying worksheets were developed in close consultation with lecturers in the department, but with the benefit of previous programming experience and cognisance of other concurrent developments in the field. The resulting programs are reliable, user-friendly, effective and appropriate to the needs of the Mathematics Department. They not only enable a wide range of problems to be tackled but also encourage users to vary problem parameters and to test the sensitivity of solutions. Graphical output helps students to interpret the results more effectively and to achieve a better understanding of the concept or model being studied.

The use of computers evolved gradually over a period of years, encouaged by the provision of mathematical science laboratories. Inspired by the enthusiasm and success of early users, more lecturers became involved and tried out the facilities for themselves. They displayed the determination required to master the operation of the software and hardware, to overcome the logistical problems of timetabling, class size etc. and to prepare suitable worksheets. As new software and worksheets were developed, the level of the awareness of the inherent potential of computer packages to enhance mathemtical education and the quality of learning increased.

It was deduced in Chapter 2 that teaching involving the use of computer packages is more effective if it is an integral part of the curriculum. Experience at Napier strengthens this opinion. The integration of computer-based work ensures that students appreciate its relevance and that its benefits feed directly into their course.

8.4 Implications for the Curriculum

Methods of assessment are slowly changing to reflect curriculum developments. Many courses now include a computer-based assignment as coursework replacing a class test. A majority of students surveyed were in favour of some computer-based work being assessed. In some examinations, students are asked to interpret computer output from a package they have used and they might also be asked questions of a conceptual nature based on work done in the laboratory.

The reported shift in teaching emphasis away from techniques towards understanding, applications and investigations is likely to continue, and, indeed, to accelerate, as the use of computer algebra systems increases. The new, broader range of learning outcomes described in the previous section calls for further thought to be given to the most appropriate means of assessing students' work. Tentative steps have been taken at Napier towards laboratory-based examinations. As computer facilities improve, these may become commonplace.

One difficulty which still exists is how to extend the benefits of laboratory work to students in larger classes. This has been partially accomplished by setting computer-based assignments which the students are required to complete during open-access time following an introductory laboratory session. It is usually possible to split a large class into manageable groups for one or two introductory sessions in the year. This approach gives students valuable experience of using computer packages as an investigative tool in mathematics and exposes them to a broader range of mathematical models than would be possible otherwise. Of course, it is only possible to explore a very few topics from the curriculum by means of computer-based assignments. In order to extend the advantages of using computer packages to other areas of the curriculum, additional access must be arranged.

Two other approaches have been adopted by some lecturers. Where large monitors or projection facilities are available, computer packages can be demonstrated during lectures at appropriate points in the course. Students can also be encouraged to try out packages in the laboratory in their own time. However, more lecturers would use the mathematical sciences laboratories if class sizes were smaller or laboratories larger. This may only become possible when mathematics is fully recognised as a laboratory-based subject and time-tables are organised such that large classes are sub-divided for laboratory work on a regular basis. An alternative solution is to provide much larger laboratories, but these would not only be difficult to supervise but expensive to establish and uneconomic unless the majority of classes were large.

Insufficient class time also deterred some lecturers from using the mathematical sciences laboratories as often as they would have liked. Again, the solution lies in recognising laboratory work as an essential element of a mathematics course. Only then will Boards of Studies or course committees be willing to amend curricula to include regular laboratory work in mathematics.

8.5 Developments

In 1989, twelve PC-compatible computers were purchased by the Mathematics Department and a third laboratory was opened. Versions of LINPROG and several other mathematics packages are now available on these computers, running under MS-DOS, and conversion of NODES will take place at an early date. Thus the substantial learning resources and expertise is being transferred to the new laboratory without disrupting lecturers' teaching programmes. Staff and students will, however, notice the improved running speed of the programs which the more powerful computers offer. The greatly increased memory has enabled LINPROG to be adapted to allow larger linear programming problems to be tackled. Further planned improvements include deletion of constraints, a default problem and the grahical solution of two variable problems.

As described in Chapter 3, there is a continuing process of modification and refinement of the software and worksheets in the mathematical sciences laboratories, as a result of input from lecturers and students. New worksheets are regularly added to the system and more open-learning material is being developed. During interviews, one lecturer reported his use of a Computer Illustrated Text (CIT) for a statistics course. CITs have since been used in other courses to supplement existing course material. A CIT combines the advantages of a textbook for browsing and reference with the flexibility and visual nature of a good computer package (Harding, 1988). The underlying philosophy of CITs, which integrate illustrative and investigative use of programs into the teaching material, closely reflects the approach which has emerged at Napier.

Students frequently have difficulty relating what is taught in their mathematics class to practical applications or even to what they learn in other subjects. The aim of the second stage of the engineering applications programme for B.Eng. courses is to develop the students' ability to apply their knowledge to the solution of physical problems. In an effort to improve the practical aspect of the EA2 mathematics assignment for CEE2 in 1989, the investigation of a mathematical model was linked to an experiment in the mechanical engineering laboratory. Apparatus was set up to enable the students, in small groups, to conduct experiments in mechanical vibrations, including simple harmonic motion, damped, undamped and forced vibrations. In the mathematics laboratory, the same experiments were performed on a mathematical model of the system using NODES and using some data obtained from the physical apparatus. The exercise has since been repeated in a modified form using electrical apparatus and an appropriate mathematical model. Despite logistical difficulties, initial student reaction has been favourable. It is intended to evaluate this innovative exercise.

With the addition of a PC-compatible laboratory, it is possible to run much larger packages, including computer algebra systems (CAS). One such system, DERIVE, has already been used in the laboratory with a class and there is considerable interest in its use both within the department and elsewhere. Much work has still to be done in developing and evaluating courseware for such packages. There is a need for evaluative studies to be conducted to assess the effectiveness of CAS as teaching tools for mathematics.

However, the use of CAS has wider implications for the curriculum. There has already been debate at ICME-6 (1988) and elsewhere about possible changes to mathematics syllabi for engineers and scientists. Is it still necessary for engineers to learn several different techniques of integration if a CAS is available? It seems likely that, in future, less time will be spent learning techniques. This study has reported a trend at Napier towards more emphasis on understanding and investigations when computer packages are used as mathematical tools. The introduction and wider recognition of the power of CAS may prove to be the catalyst required for course committees to reduce the techniques content of syllabi and to recognise fully mathematics as a laboratory-based subject. The advantages of using computer packages to enhance the learning of mathematics and to encourage investigative work could then be extended into other areas of the curriculum and across more courses than is presently possible.

8.6 Model Curricula

Much of the discussion in this Chapter can be summed up by presenting a model for mathematics curricula which embodies the findings of this research project. The underlying philosophy for such curricula recognises the computer as an educational tool which not only assists the teaching but, if used effectively, enhances the quality of the learning. Mathematics curricula should be designed to allow regular time to be devoted to laboratory work in addition to lectures and tutorials. The balance between these modes of learning should be flexible and left to the discretion of the lecturer to vary according to the topic being studied. At least one computer should be available during lectures and tutorials for demonstration purposes and/or checking results. The system must be flexible enough to allow each lecturer to develop the teaching style which he considers most appropriate to the course and with which he is most comfortable. A further development favoured by some lecturers envisages groups of students spending some of their course working on open-learning materials with staff present to help when required. Student-centred coursework of this nature would not resemble the instructional learning packages of the 1970s but might include computer illustrated textbooks and similar material.

Experience at Napier has shown that the effective use of computer packages enables an investigative approach to be incorporated into the curriculum and, further, that when such an approach is adopted, the students' learning experience is enhanced. If, as seems likely, the use of computer algebra systems beomes widespread, there will be less need for the majority of science and engineering students to spend a large proportion of their time mastering algebraic skills and techniques. The time thus saved should be spent investigating concepts and methods, solving realistic problems, formulating models and analysing and interpreting results, using computer packages when appropriate. The improved understanding and higher cognitive skills resulting from such work will better prepare the students for the current needs of industry and commerce.

BIBLIOGRAPHY

Alderman, D.L. (1978), "Evaluation of the TICCIT Computer-Assisted Instructional System in the Community College", Final Report, Educational Testing Service, Princeton, New Jersey 08541, U.S.A.

Anderson, R.C., Kulhavy, P.W. and Andre, T. (1971), "Feedback Procedures in Programmed Instruction", J. Educ. Psychol., 62, 148-156.

Arubayi, E., (1986), "Students' Evaluations of Instruction in Higher Education: A Review", Assess. and Eval. in Higher Educ., 11 (1).

Bajpai, A.C., Mustoe, L.R. and Walker, D., (1975), "Mathematical Education for Engineers. Part I", Int. J. Math. Educ. Sci. Technol., 6, 361.

Bajpai, A.C., Mustoe, L.R. and Walker, D., (1976), "Mathematical Education for Engineers. Part II: Towards Possible Solutions", Int. J. Math. Educ. Sci. Technol., 7 (3), 349-364.

Bajpai, A.C., Fairley, J.A., Harrison, M.C., Mustoe, L.R., Walker, D. and Whitfield, A.H., (1984), "The MIME Project at Loughborough - a first report", Int. J. Math. Educ. Sci. Technol., 15 (6), 781-810.

Bajpai, A.C., Fairley, J.A., Harrison, M.C., Mustoe, L.R., Walker, D. and Whitfield, A.H., (1985), "Mathematics and the Micro: Some Hints on Software Development", Int. J. Math. Educ. Sci. Technol., 16 (3), 407-412.

Bajpai, A.C. and Downend, I.J., (1987), "Development of Computer Enhanced Learning Units under the MIME Project", Inst. Math. and its Appl., 23, 122-127.

Ball, D. (1987), "Microcomputers in Mathematics Teaching", Brit. J. Educ. Technol., 18 (3), 247-255.

Ball, D., Higgo, J., Oldknow, A., Straker, A. and Wood, J. (eds.), (1987), "Will Mathematics Count?", Report of the National Mathematics and Statistics Computer-based Learning Review Panel, Dept. of Educ. and Sci., AUCBE, England.

Baron, M.E., (1972), "The Nature of Mathematics - Another View", Chapter 2 in "The Process of Learning Mathemstics", ed. L.R. Chapman, Pergamon Press, Oxford.

Beevers, C., (1986), "Having a CALMing Influence on Calculus", Conference Report, Mathematics Teaching 1986, University of Edinburgh.

Beevers, C., (1988), "The CALM Project - Evaluation and Development", Conference Proc., "Mathematics and Statistics Curricula in Higher Education for the 1990s", Napier Polytechnic, Edinburgh.

Beilby, M., (1987), "Experience of Computer-assisted Education in a University Department" in "Trends in Computer-assisted Education", eds. R. Lewis and E.D. Tagg.

Bloom, B.S. (ed.), (1956), "Taxonomy of Educational Objectives: Cognitive Domain", David McKay, New York.

Bowman, A.W. and Robinson, D.R., (1987), "Introduction to Statistics", Computer Illustrated Text Series, Adam Hilger Ltd., Bristol.

Brerland, H.M., Amarel, M. and Swinton, S., (1974), "The Evaluation of PLATO: Educational Analysis of the Elementary School Components", Project Report, Educational Testing Service, Princeton, New Jersey 08541, U.S.A.

Bruner, J.S., (1966), "Toward a Theory of Instruction", Harvard University Press, Cambridge, Mass.

Candy, L., (1988), "Computers and Curriculum Change: An Action Research Perspective", J. Computer Assisted Learning, 4 (4), 203-213.

Cawley, P., (1988), "The Effectiveness of Engineering Degree Courses", Assess. and Eval. in Higher Educ., 13 (3).

Clements, R.R., (1984), "The Use of the Simulation and Case Study Techniques in the Teaching of Mathematical Modelling", reprinted from "Teaching and Applying Mathematical Modelling", eds. J.S. Berry et al., Ellis Horwood.

Clements, R.R., (1986), "The Role of System Simulation Programs in Teaching Applicable Mathematics", Int. J. Math. Educ. Sci. Technol., 17 (5), 553-560.

CNAA, (1988), "CNAA Handbook", Council for National Academic Awards, London.

Cockcroft, W.H., (1982), "Mathematics Counts", Her Majesty's Stationery Office, London.

Cohen, L. and Manion, L., (1980), "Research Methods in Education", Croom Helm, London.

Driver, R. and Scanlon, E., (1988), "Conceptual Change in Science", J. Computer Assisted Learning, 5, 25-36.

Erikson, W.J. and Turban, E., (1985), "Teaching Operations Research on Microcomputers", OMEGA Int. J. Man. Sci., 13 (3), 191-200.

Ernest, P., (1988), "An Investigation into the Use of Microcomputers to Teach Mathematical Problem-solving Skills to 13-year-olds", Int. J. Math. Educ. Sci. Technol., 19 (6), 807-818.

European Society for Engineering Education, (1985), "Seminar Proceedings", Eur. J. Eng. Educ., 10 (3/4), 275-360.

Finniston, M., (1980), "Engineering our Future: Report of Committee of Inquiry into the Engineering Profession", Her Majesty's Stationery Office, London.

Francis, R., (1988), "Teaching Mathematics with the Inspiration of a Computer", Teaching Math. and its Appl., 7 (2).

Fraser, R., (1987), "Learning Activities and Classroom Roles: Communication", Inst. Math. and its Appl., 23, 114-117.

Freese, R., Lounesto, P. and Stegenga, D.A., (1986), "The Use of muMATH in the Calculus Classroom", J. Comp. Math. Sci. Teaching, 6 (1), 52-55.

Glen, J.A. and Walker, L.J., (1985), "Knowledge Engineering in an

Undergraduate Mathematical Sciences Laboratory", Int. J. Math. Educ. Sci. Technol., 16 (3), 355-364.

Grant, A., (1985), "The Formation of Mechanical Engineers", The Grant Report, Institution of Mechanical Engineers.

Gudgin, A.N., (1987), "Computer-assisted Learning of Introductory Economics", Brit. J. Educ. Technol., 18 (2), 84-93.

Hamilton, D. et al., (1977), "Beyond the Numbers Game: A Reader in Educational Evaluation", Macmillan, London.

Harding, R.D., (1974), "Computer-aided Teaching of Applied Mathematics", Int. J. Math. Educ. Sci. Technol., 5 (3), 447-455.

Harding, R.D., (1976), "Evaluative Development of a Computer-assisted Learning Project", Int. J. Math. Educ. Sci. Technol., 7 (4), 475-483.

Harding, R.D., (1984), "CATAM Revisited", Comp. Educ., 8 (1), 113-125.

Harding, R.D., (1986), "A Software Toolkit for Computer-aided Mathematics", Inst. Math. and its Appl., 22, 76-80.

Harding, R.D., (1988), "Computer Illustrated Texts and the CITEXT Group", Inst. Math. and its Appl., 24, .

Hartley, J.R., (1985), "Some Psychological Aspects of Computer-assisted Learning and Teaching", Program. Learn. Educ. Technol., 22 (2).

Hartley, J.R., (1987), "The Innovation of Computer-Assisted Learning", Brit. J. Educ. Technol., 18 (3).

Hartley, J.R. and Bostrom, K., (1982), "An Education of Micro-CAL in Schools", Int. J. Man. Machine Studies, 17, 127-141.

Hooper, R., (1976), "To CAI or not to CAI - That is the Question", Int. J. Math. Educ. Sci. Technol., 7 (4), 461-473.

Hooper, R., (1977), "National Development Programme in Computer-assisted Learning: Final Report of the Director", Council for Educ. Technol., London.

Hoyles, C., (1987), "Computer-based Micro Worlds: Can They Help Children Learn Mathematics?", Conference Proc., "Mathematics Teaching 1987", University of Edinburgh.

Hoyles, C. and Sutherland, R., (1987), "Ways of Learning in a Computer-based Environment: Some Findings of the LOGO Maths Project, J. Computer Assisted Learning, 3, 67-80.

Hunter, J., Rosenberg, P. and Webber, D.B., (1985), "MALT for Mathematics Undergraduates", Int. J. Math. Educ. Sci. Technol., 16 (3), 379-389.

ICME-6, (1988), "Proceedings of the Sixth International Congress on Mathematical Education", eds. K. and A. Hirst, Janos Bolyai Mathematical Society, Budapest, Hungary.

Jacques, I.B. and Judd, C.J., (1985), "Use of Microcomputers in Teaching Numerical Mathematics", Int. J. Math. Educ. Sci. Technol., 16 (3), 347-354.

James, D.J.G., (1985), "How Should the Mathematical Training of an Undergraduate be Conducted?", Int. J. Math. Educ. Sci. Technol., 16 (2), 163-171.

Johnston, V.M., (1985), "Introducing the Microcomputer into English", Brit. J. Educ. Technol., 16 (3).

Johnston, V.M., (1987), "The Evaluation of Microcomputer Programs: An Area of Debate", J. Computer Assisted Learning, 3, 40-50.

Katsifli, D., (1986), "A CAL Phenomenon in Numerical Analysis", Int. J. Math. Educ. Sci. Technol., 17 (2), 209-227.

Katsifli, D. and Fyffe, D., (1984), "CALNAPS", School of Mathematics, Kingston Polytechnic, Kingston upon Thames.

Kulhavy, R.W., Yehovich, F.R. and Dyer, J.W., (1976), "Feedback and Response Confidence", J. Educ. Psychol., 68, 522-528.

Leach, D.F., (1974), "A Computer-based Laboratory for Service Mathematics

Teaching", Proc. Int. Conference, "Frontiers in Education", 212-215.

Leach, D.F., (1978), "A Science-based Generalist Degree: Science with Industrial Studies", Studies in Higher Education, 3 (2).

Leach, D. et al., (1977), "Final Review of the Mathematics Laboratory Project (MATLAB) at Napier College", The Council for Educational Technology.

Lighthill, J., (1979), "The Mathematical Education of Engineers", Bulletin Inst. Math. and its Appl., 15.

Likert, R., (1932), "A Technique for the Measurement of Attitudes", Arch. Psychol., No. 140.

Mackie, D., (1985), "Exploring Linear Programming with a BBC Micro", Conference Proc., "Mathematics Teaching 1985", University of Edinburgh.

Mackie, D., (1986), "Computer-based Mathematics in Further Education", Conference Proc., "Mathematics Teaching 1986", University of Edinburgh.

Mackie, D., (1988(a)), "Computer-based Investigations in Mathematics", Conference Proc., "Mathematics and Statistics Curricula in Higher Education for the 1990s", Napier Polytechnic, Edinburgh.

Mackie, D.M., (1988(b)), "Finding the Unstable Orbit - a Tertiary Level Investigation", Conference Proc., "Mathematics Teaching 1988", University of Edinburgh.

Mackie, D. and Scott, T., (1986), "LINPROG, a Linear Programming Package for the BBC Micro", Napier Polytechnic, Edinburgh.

Mackie, D. and Scott, T., (1987), "Computer-based Investigations", Scottish Mathematics Council Journal No. 17.

Mackie, D. and Scott, T., (1988(a)), "Using Computers to Enhance the Learning of Mathematics", Int. J. Math. Educ. Sci. Technol., 19

(1).

Mackie, D. and Scott, T., (1988(b)), "NODES, a Differential Equations Package for the BBC Micro, Napier Polytechnic, Edinburgh.

Macsyma, (1983), "MACSYMA Reference Manual", The Mathlab Group, Laboratory for Computer Science, M.I.T.

MAP, (1987), "Mechanics in Action" software package, University of Sheffield.

Mason, J.H., (1985), "Fostering Mathematical Thinking from Cradle to Grave", Conference Proc., "Mathematics Teaching 1985", University of Edinburgh.

McQuade, E. (1985), "Future Technology and Mathematics", Int. J. Math. Educ. Sci. Technol., 16 (3), 431-436.

Mills, J. and Tall, D., (1988), "From the Visual to the Logical in Mathematics", Inst. Math. and its Appl., 24.

Nicolson, R.I. and Scott, P.J., (1986), "Computers and Education: The Software Production Problem", Brit. J. Educ. Technol., 17 (1).

Oppenheim, A.N., (1966), "Questionnaire Design and Attitude Measurement", Heinemann, London.

O'Shea, T. and Self, J., (1983), "Learning and Teaching with Computers", Harvester Press, Brighton.

Pask, G., (1976), "Conversational Techniques in the Study and Practice of Education", Brit. J. Educ. Psychol., 46, 12-25.

Pearson, J., (1986), "The Pupil-Teacher Relationship in a Logo Classroom", Conference Proc., "Mathematics Teaching 1986", University of Edinburgh.

Plumpton, C., (1972), "Generalisation and Structure", Chapter 5 in "The Process of Learning Mathematics", (ed. L.R. Chapman), Pergamon Press, Oxford.

Ridgway, J., (1986), "Research Needs in CAL", J. Computer Assisted

Learning, 2, 131-140.

Roblyer, M.D., (1985), "The Greening of Educational Computing: A Proposal for a More Research-based Approach to Computers in Instruction", Educ. Technol., 25 (1).

Rowe, G.W., (1985), "The Impact of Computers on the Teaching of Mathematics to Engineers", Int. J. Math. Ed. Sci. Tech., 16 (2), 181-185.

Rowntree, D., (1982), "Educational Technology in Curriculum Development", (2nd edn.), Harper and Row, London.

Rushby, N., (1986), "A Knowledge Engineering Approach to Instructional Design", Proc. ETIC '86, "Flexible Learning Systems", Napier Polytechnic, Edinburgh.

Saljo, R., (1975), "Qualitative Differences in Learning as a Function of the Learner's Conception of the Task", Acta Universitates, Gothenburgensis.

Scott, T.D., (1986), "Learning Linear and Integer Programming in a Mathematics Laboratory", Proc. ETIC '86, "Flexible Learning Systems", Napier Polytechnic, Edinburgh.

Scottish Education Department, (1986), "Microelectronics in Further and Higher Education", Report by HM Inspectors of Schools, Her Majesty's Stationery Office, Edinburgh.

Scriven, M., (1967), "The Methodology of Evaluation", in "Perspectives of Curriculum Evaluation", (ed. R.E. Stake), Amer. Educ. Res. Ass., Monograph Series on Curriculum Evaluation No. 1, Rand McNally, Chicago.

Searl, J.W., (1985), "Mathematics for Engineers and Scientists: Some Teaching Approaches", Int. J. Math. Educ. Sci. Technol., 16 (2), 275-283.

Skemp, R.R., (1976), "Relational Understanding and Instrumental

Understanding", Mathematics Teaching, No. 77.

Skemp, R.R., (1971 and 1986), "The Psychology of Learning Mathematics", (1st edn. and 2nd edn.), Penguin Books, Harmondsworth, Middlesex.

Skinner, M.E., (1988), "Attitudes of College Students toward Computer-assisted Instruction: An Essential Variable for Successful Implementation", Educ. Technol. 28.

Stern, R.D., (1987), "The Use of Computers in the Teaching of Statistics", Conference Proc., "Mathematics and Statistics Curricula in Higher Education for the 1990s", Napier Polytechnic, Edinburgh.

Tall, D., (1986), "Graphics Calculus I, II and III", Glentop Publishers, London.

Tall, D., (1987), "The Complementary Roles of Prepared Software and Programming in the Learning of Mathematics", Inst. Math. and its Appl., 23, 128-133.

Tawney, D.A., (1979, "Learning through Computers", Macmillan, London.

Zionts, S., (1974), "Linear and Integer Programming", Prentice Hall Inc., U.S.A.

APPENDIX 1

LINPROG worksheets

NAPIER COLLEGE

DEPARTMENT OF MATHEMATICS

MATHEMATICS LABORATORY

Subject Area : LINEAR PROGRAMMING

Author : T.D. Scott

Date : 23rd May 1985

TOPIC : **INTRODUCTORY LINEAR PROGRAMMING**

PROGRAM : LINPROG

OBJECTIVES :

(i) To formulate a L.P. problem mathematically.

(ii) To solve a 2-variable problem graphically.

(iii) To solve problems with 2 or more variables using the computer.

(iv) To demonstrate the operation of the Simplex Method for a 3-variable problem.

(v) To explore the concept of duality.

1. A firm manufactures two products A and B, the market for each being virtually unlimited. Each product is processed on each of the machines I, II and III. The processing times in hours per item of A or B on each machine are given in the table:

	I	II	III
A	0.5	0.4	0.2
B	0.25	0.3	0.4

The available production time of the machines I, II and III is 40 hours, 36 hours and 36 hours respectively each week. The profit per item of A and B is £5 and £3 respectively.
The firm wishes to determine the weekly production of items of A and B which will maximise its profit. Formulate this problem as a linear programming problem and solve it graphically.

2. Maximise $P = 5x_1 + 3x_2$

subject to $x_1 \geqslant 0,\ x_2 \geqslant 0$

$$2x_1 + x_2 \leqslant 160$$
$$4x_1 + 3x_2 \leqslant 360$$
$$x_1 + 2x_2 \leqslant 180$$

3. Verify that the graphical solution of Question 1 agrees with the answer obtained on the BBC for Question 2. Why is this the case?

4. Maximise $I = 3x_1 + 5x_2 + 2x_3$

subject to $x_1, x_2, x_3 \geq 0$

$$x_1 + 3x_2 \leq 12$$
$$x_1 + x_2 + 2x_3 \leq 6$$

comparing your solution, tableau by tableau, with the diagram shown in Fig. Q.4. What conclusions do you reach from this comparison?

5. (a) A manufacturer has available three inputs A, B and C, from which he manufactures 3 products X_1, X_2 and X_3. The quantity of each input used, the income from 1 unit of each product, and the total supply of the three inputs is summarised in the following table:

		A	B	C	Income/Unit (£)
Composition of the products	X_1	1	1	0	0.15
	X_2	2.5	0	0	0.10
	X_3	1	1	1	0.10
Total Supply ($\times 10^6$)		10	5	3	

Find the production programme which maximises the manufacturer's income.

(b) A dealer offers to purchase all of the available inputs, A, B and C, at prices which guarantee that the manufacturer receives at least as much for the contents of each product as he could obtain by making and selling the product. Assuming that the manufacturer agreed to sell his total supply, the dealer seeks to minimise his total outlay. Verify that this problem is the dual of the primal problem of part (a).

(c) State the solution to the dual problem of part (b).

(d) Verify the principle of complementary slackness for this example.

Mathematical Formulation

Find x_1, x_2 and x_3 which

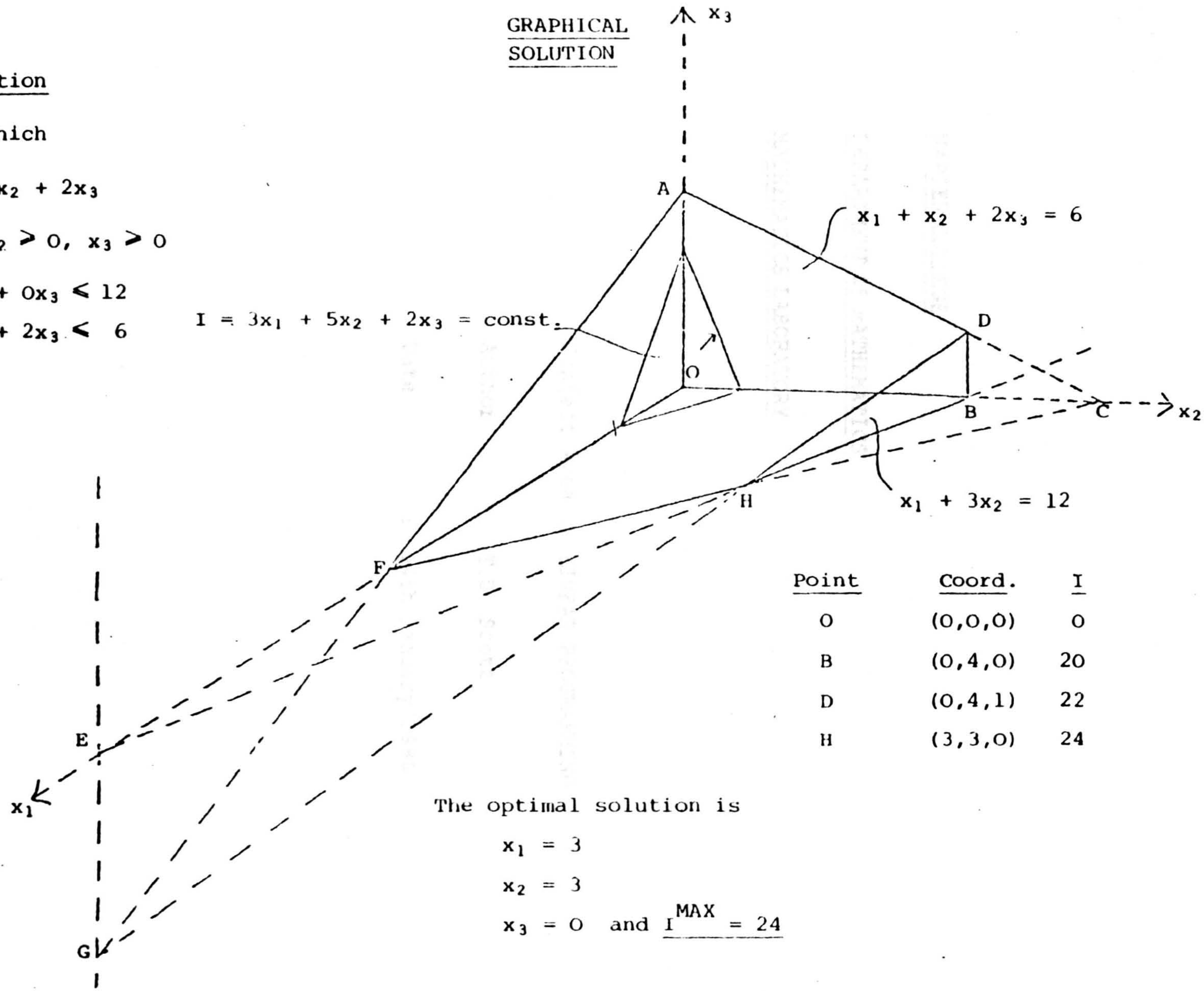
maximise $I = 3x_1 + 5x_2 + 2x_3$

subject to $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$

$$x_1 + 3x_2 + 0x_3 \leq 12$$

$$x_1 + x_2 + 2x_3 \leq 6$$

GRAPHICAL SOLUTION



Point	Coord.	I
O	(0,0,0)	0
B	(0,4,0)	20
D	(0,4,1)	22
H	(3,3,0)	24

The optimal solution is

$x_1 = 3$

$x_2 = 3$

$x_3 = 0$ and $I^{MAX} = 24$

Fig. Q.4

NAPIER COLLEGE

DEPARTMENT OF MATHEMATICS

MATHEMATICS LABORATORY

Subject Area : LINEAR PROGRAMMING

Author : T.D. Scott

Date : 24th January 1986

TOPIC : POST-OPTIMAL ANALYSIS

PROGRAM : LINPROG

OBJECTIVE: To investigate the effect of the following changes in a L.P. problem:

(i) Changes in the RHS values (of the constraints).

(ii) Changes in the coefficients of the objective function.

(iii) The inclusion of more variables.

(iv) The addition of extra constraints.

Problem 1

(a) Minimise $C = 12y_1 + 6y_2$

subject to $y_1 \geq 0,\ y_2 \geq 0$

$$y_1 + y_2 \geq 3$$

$$3y_1 + y_2 \geq 5$$

$$2y_2 \geq 2$$

(b) Investigate the solution to this problem with the altered objective function:

$$C = (12 - 2\theta)y_1 + 6y_2$$

and, in particular, obtain the range of values of θ for which the optimal solution remains as in part (a).

(c) Assume that, in the original problem, the R.H.S. become 3, 5 and 2 + θ respectively. Determine the range of values of θ which allow the solution to the problem to remain optimal.

(d) Suppose that the first constraint of the problem becomes $(1 - \theta)y_1 + (1 + \frac{\theta}{2})y_2 \geq 3$. Show that the solution to the problem is unaffected for values of $\theta \geq -\frac{1}{2}$.

Problem 2

(a) A firm produces self-assembly bookshelf kits in two models A and B. Production of kits is limited by the availability of raw material (high quality board) and machine processing time. Each unit of A requires 3 m^2 of board and each unit of B requires 4 m^2 of board. The firm can obtain up to 1700 m^2 of board each week from its suppliers. Each unit of A needs 12 minutes of machine time and each unit of B needs 30 minutes of machine time. Each week a total of 160 machine hours is available.

If the profit on each A unit is £2 and on each B unit is £4, determine the weekly production programme which maximises total profit.

(b) The production manager, who is not a mathematician, requires the solution to part (a) as well as answers to the following questions:

(i) Suppose we can buy extra board from a second timber merchant. How much per square metre are we prepared to pay for it?

(ii) Suppose we can obtain extra machine time by working overtime. If this costs £7 per hour extra, is it worth it?

(iii) Suppose that the profit on each A unit is £P_1 and on each B unit is £P_2. For what possible values of the ratio $P_1:P_2$ is the solution we have obtained optimal?

(iv) Suppose a third type of kit, C, can be made. Suppose it uses 4 m^2 of board and needs 20 minutes of machine time. If the profit on each unit is £P, should we make it?

(v) Suppose that, during a period of economic recession, the sales team report that the market will not stand more than 550 sales of kits each week. How does this affect the production schedule?

(vi) Suppose, following on from the last section, that total weekly sales are restricted to at most 450 kits. Would this affect the schedule?

Prepare a written report for the production manager which will answer all his queries and include suitable explanations.

NAPIER COLLEGE

DEPARTMENT OF MATHEMATICS

MATHEMATICS LABORATORY

Subject Area : INTEGER PROGRAMMING

Author : T.D. Scott

Date : 29th May 1985

TOPIC : INTEGER LINEAR PROGRAMMING

PROGRAM : LINPROG

OBJECTIVES: (i) To solve a 2-variable problem graphically.

(ii) To solve problems with 2 or more variables using the computer.

(iii) To examine mixed integer programming problems, and solve them using the branch and bound method.

(iv) To produce mathematical formulations of realistic I.P. problems.

1. Solve the following integer programming problem:

Maximise $z = x_1 + x_2$

subject to $14x_1 + 9x_2 \leqslant 51$

$-6x_1 + 3x_2 \leqslant 1$

$x_1, x_2 \geqslant 0$ and both integer

(a) graphically;

(b) using the computer.

2. Minimise $C = 3x_1 + 2x_2 + 1.5x_3$

subject to $x_2 - x_3 \leqslant 7.6$

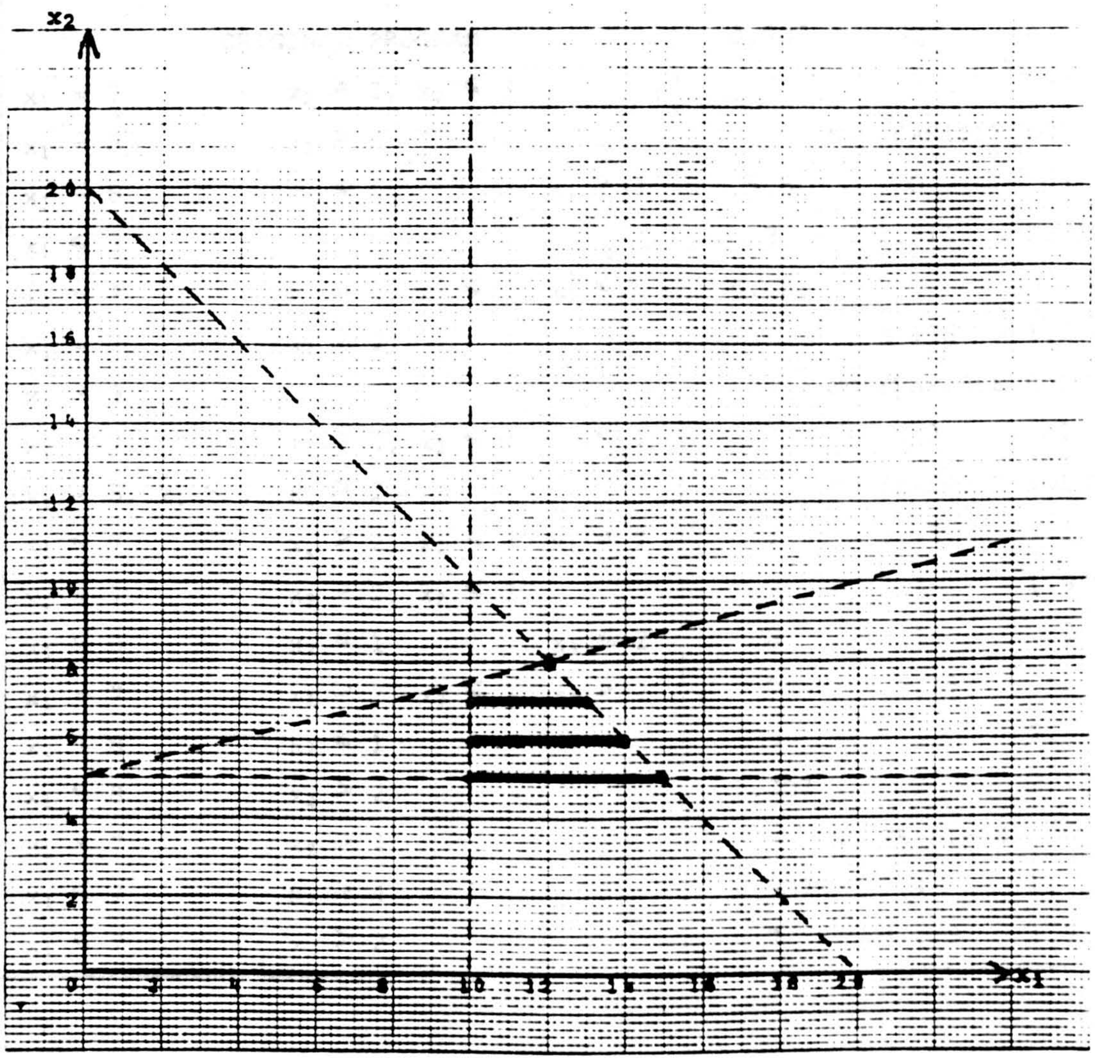
$2x_1 + x_2 \geqslant 18.3$

$x_1 + 2x_2 + x_3 \geqslant 26.9$

$x_1, x_2, x_3 \geqslant 0$

x_1, x_2, x_3 integers.

3. Develop the mixed integer constraint set for the following feasible region .



4. Given the integer programming problem:

$$\text{Maximise} \quad I = 13.2x_1 + 7x_2 + 5x_3$$

$$\text{subject to} \quad 2x_1 + x_2 - x_3 \leq 15.3$$

$$x_1 + 2x_2 + 7x_3 \leq 26.5$$

$$x_1 + x_2 \geq 4.6$$

$$x_1, x_2, x_3 \geq 0$$

$$x_1, x_2, x_3 \text{ integers,}$$

use data sheet Q.4 to construct the tree diagram obtained when the problem is solved using the branch and bound algorithm.

/Data sheet . . .

Data sheet Q.4

PROGRAM NUMBER	ADDITIONAL CONSTRAINTS			SOLUTIONS			
				x_1	x_2	x_3	I
1	ORIGINAL PROGRAM			8.907	0	2.513	130.135
2	$x_1 = 7$	$x_2 \geqslant 2$, $x_2 \leqslant 3$		7	3	1.929	123.043
3	$x_1 = 8$	$x_2 \geqslant 2$		NO FEASIBLE SOLUTION			
4	$x_1 \leqslant 8$	$x_2 \leqslant 1$	$x_3 \geqslant 3$	5.5	0	3	87.6
5	$x_1 \leqslant 8$			8	1.511	2.211	127.233
6	$x_1 \leqslant 7$	$x_2 \geqslant 2$, $x_2 \leqslant 3$	$x_3 \leqslant 1$	6.65	3	1	112.45
7	$x_1 = 7$	$x_2 \geqslant 2$		7	3.178	1.878	124.033
8	$x_1 \geqslant 9$			NO FEASIBLE SOLUTION			
9	$x_1 = 7$	$x_2 \geqslant 2$, $x_2 \leqslant 3$	$x_3 \leqslant 1$	7	2.3	1	113.5
10	$x_1 \geqslant 8$, $x_1 \leqslant 9$	$x_2 \geqslant 2$		NO FEASIBLE SOLUTION			
11	$x_1 \leqslant 7$	$x_2 \geqslant 2$, $x_2 \leqslant 3$	$x_3 \geqslant 2$	7	2.75	2	121.65
12	$x_1 \leqslant 7$	$x_2 \geqslant 2$, $x_2 \leqslant 3$		7	3	1.929	123.043
13	$x_1 \leqslant 8$	$x_2 \leqslant 1$	$x_3 \leqslant 2$	8	1	2	122.6
14	$x_1 = 7$	$x_2 \geqslant 2$, $x_2 \leqslant 3$	$x_3 \geqslant 2$	7	2.75	2	121.65
15	$x_1 \leqslant 8$	$x_2 \leqslant 1$		8	1	2.357	124.386
16	$x_1 \leqslant 8$	$x_2 \geqslant 2$		7.707	2	2.113	126.295
17	$x_1 = 7$	$x_2 \geqslant 4$		NO FEASIBLE SOLUTION			
18	$x_1 \leqslant 7$	$x_2 \geqslant 4$		6.507	4	1.713	122.455
19	$x_1 \leqslant 7$	$x_2 \geqslant 2$		7	3.178	1.878	124.033

Program: Maximise $13.2x_1 + 7x_2 + 5x_3$

subject to

$$2x_1 + x_2 - x_3 \leqslant 15.3$$
$$x_1 + 2x_2 + 7x_3 \leqslant 26.5$$
$$x_1 + x_2 \geqslant 4.6$$

x_1, x_2, x_3 integer and non-negative.

/5. . . .

5. Verify that the solutions listed in data sheet Q.4 are correct.

6. Determine <u>ALL</u> of the optimal solutions of the following problem:

$$\text{Minimise} \quad C = x_1 + 3x_2 + x_3$$

$$\begin{aligned} \text{subject to} \quad x_1 + 1.2x_2 + 0.9x_3 &\geq 53 \\ 1.2x_1 + 3.3x_2 &\leq 84 \\ x_2 + x_3 &\geq 13.8 \\ x_1, x_2, x_3 &\geq 0 \\ x_1, x_2, x_3 & \text{ integers.} \end{aligned}$$

7. Given the following mixed integer programming problem:

$$\text{Maximise} \quad z = 4x_1 + 5x_2 + 9x_3 + 5x_4$$

$$\begin{aligned} \text{subject to} \quad x_1 + 3x_2 + 9x_3 + 6x_4 &\leq 16 \\ 6x_1 + 6x_2 + 7x_4 &\leq 19 \\ 7x_1 + 8x_2 + 18x_3 + 3x_4 &\leq 44 \\ x_1, x_2, x_3, x_4 &\geq 0 \\ x_1, x_2, x_3 & \text{ integers,} \end{aligned}$$

investigate the effect (on the solution) of imposing the additional constraint that x_4 has to be an integer.

8. A large corporation is considering the investment possibilities that it currently has available:

Investment Possibility	Expected Net Present Value	Expenditure required
1. Build a new warehouse	p_1	a_1
2. Remodel the old warehouse	p_2	a_2
3. Buy automation for the new warehouse	p_3	a_3
4. Buy a company that supplies product A	p_4	a_4
5. Build a plant to manufacture product A	p_5	a_5
6. Refurbish corporate offices	p_6	a_6

Assuming that the firm has funds to the extent of a_0, formulate the problem of choosing among the projects as an integer programming problem. (Projects 1 and 2 are mutually exclusive, as are projects 4 and 5; project 3 can only be undertaken if project 1 is undertaken - the automation is only feasible for a new warehouse.)

9. An assembly line has eight jobs which are to be performed as indicated below:

Job	Time Required	Jobs that must be completed before starting this job
1	7 min.	-
2	6 min.	-
3	8 min.	-
4	8 min.	1,2
5	1 min.	2,3
6	6 min.	4,5
7	7 min.	5
8	8 min.	6,7

One worker is positioned at each station and performs certain jobs at his station. Formulate the problem of determining how many stations should be set up and which jobs to assign to each station as an integer programming problem. The objective is to minimise total worker idle time. (Hint: Let x_{ij} be a variable which is 1 if job i is assigned to station j, and zero otherwise.)

10. A corporation is considering buying two warehouses in a region. There are 20 possible sites. Each site has a fixed cost of purchase and development. In addition, there is an associated capacity of each warehouse and costs of servicing each customer from each warehouse. Formulate the problem of determining the two best warehouses as an integer programming problem.

APPENDIX 2

NODES worksheets

Reference Numbers	Topic	Objectives
DE2	Numerical Methods	To compare the behaviour of single step numerical methods of different order, for approximately solving ordinary differential equations
DE3	Numerical Methods	To investigate the effect on the errors of changing step length in single step numerical methods of first- and second-order differential equations
DE4	Numerical Analysis	To investigate the stability of single step numerical methods for first-order ordinary differential equations. The tutorial is restricted to the induced instability that arises when such methods are applied to "stiff" differential equations
DE5	First-order Differential Equations	To investigate the behaviour of an RL circuit under different initial conditions
DE7	Mechanical Vibrations I	(a) To construct a mathematical model of a simple vibrational system. (b) To examine the three modes of operation: (i) simple harmonic motion (ii) damped vibrations (iii) forced vibrations. (c) To illustrate the three types of damping: (i) underdamping (ii) critical damping (iii) overdamping. (d) To distinguish between the transient and the steady-state part of a solution. (e) To investigate the effects of changing model parameters and, in particular, the conditions for resonance.

Reference Numbers	Topic	Objectives
DE8	Mechanical Vibrations II	(a) To construct mathematical models of simple coupled systems. (b) To analyse the behaviour of each model for various specific conditions. (c) To investigate the effects on the solution of varying the parameters of each model. (d) To characterise the "frequency response" of a mass-spring system.
DE11	Phase Portraits	(a) To obtain solutions of non-linear second-order equations. (b) To construct phase portraits. (c) To investigate the limit cycle of the Van der Pol oscillator. (d) To investigate the effect of changing the parameters of the model.

APPENDIX 3

B.Sc. Science with Industrial Studies course details

B.Sc. Communications and Electronic Engineering course details

B.Sc Science with Industrial Studies

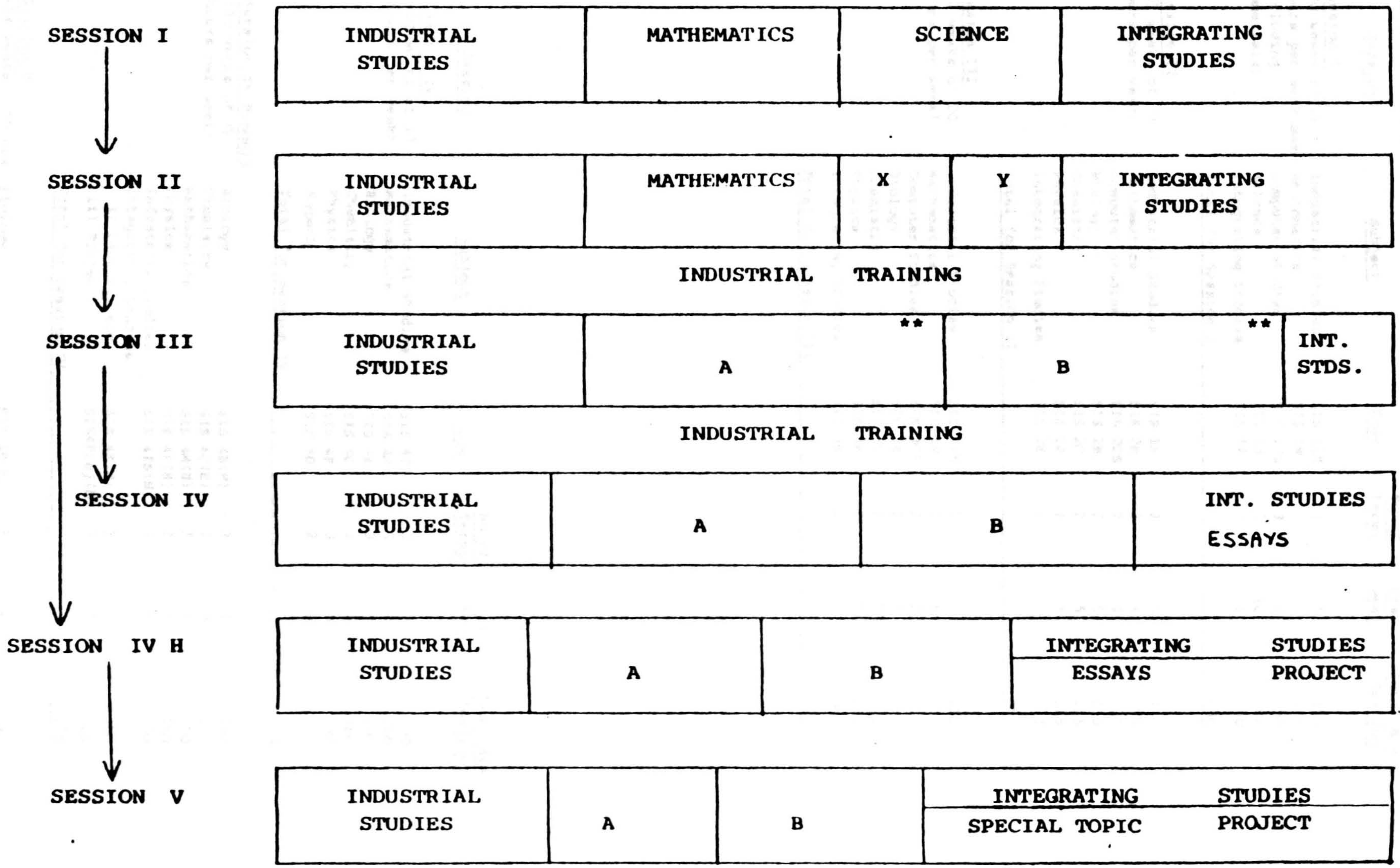
Course Structure

The Degree is a four session, and the Honours Degree a five session, thin sandwich course, with industrial placements between the second and third, and third and fourth sessions. The industrial placements are so situated to allow students the opportunity of attaining a consistent academic level despite variations in entry qualifications. After the second period of industrial experience Degree students undertake a final three terms and Honours Degree students a final five terms of academic study in the College. The periods of industrial training are seen as complementing the academic studies in the Sciences and in Industrial Studies. The organisation of the course is as follows:

Session 1	3 terms	Academic Study	-	30 weeks
Session 2	2 terms	Academic Study	-	23 weeks
		Industrial Placement	-	24 weeks
		out of		28 weeks
Session 3	2 terms	Academic Study	-	23 weeks
		Industrial Placement	-	24 weeks
		out of		28 weeks
Session 4	3 terms	Academic Study	-	30 weeks
Session 4H	3 terms	Academic Study	-	30 weeks
Session 5	2 terms	Academic Study	-	24 weeks

During the first year all students take a general science course, covering a wide range of scientific subjects. This provides a broadly based understanding of science. Mathematics with Computer Studies is taken by all students during the first two sessions of academic study. This provides all the students with the mathematical and computing skills required in other parts of the course. In the second and subsequent years choices of subject are made as indicated in the table. Interdisciplinary study is introduced by two special seminars in each of the first and second sessions and an introducton to Integrating Studies in the third session which leads to the Integrating Studies course in the fourth and fifth sessions for Honours Degree students. During the final session of academic study, a Degree student undertakes a dissertation on a subject of his own choice whereas the Honours Degree student undertakes a project during his last two sessions.

Structure of the Course and Possible Combinations



* X and Y are two subjects taken from Physics, Chemistry and Biology

** A and B are two subjects taken from X, Y and Mathematics.

Course Curriculum

Session	Subject	Code	No of Terms	Hrs/ Week	Total Hrs/ Session
Session I (30 weeks of 21 hours per week max excluding remedials)	Industrial Studies	SIS 1IS	3	5	150
	Mathematics	SIS 1M	3	4	120
	Computer Studies	SIS 1CS	3	1	30
	Science	SIS 1S	3	10	300
	Integrating Studies	SIS 1E	3	1	30
	Total for Session I				630
Session II (23 weeks of 21 hours per week)	Industrial Studies	SIS 1S	2	5	115
	Mathematics	SIS 2M	2	4	85
	Computer Studies	SIS 2CS	2	2	30
	Biology	SIS 2B	2	5	115
	Chemistry	SIS 2C	2	5	115
	Physics	SIS 2P	2	5	115
	Integrating Studies	SIS 2E	2	1	23
	Total for Session II				483
Session III (23 weeks of 20 hours per week)	Industrial Studies	SIS 31S	2	5	115
	Mathematics	SIS 3M	2	5	121
	Computer Studies	SIS 3CS	2	2	40
	Biology	SIS 3B	2	7	161
	Chemistry	SIS 3C	2	7	161
	Physics	SIS 3P	2	7	161
	Integrating Studies	SIS 3E	2	2	46
	Total for Session III				480

Session	Subject	Code	No of Terms	Hrs/ Week	Total Hrs/ Session
Session IV (30 weeks of 17 hours per week)	Industrial Studies	SIS 4IS	3	5	150
	Mathematics	SIS 4M	3	6	180
	Biology	SIS 4B	3	6	180
	Chemistry	SIS 4C	3	6	180
	Physics	SIS 4P	3	6	180
	Reports	SIS 4D	2	NA	
	Total for Session IV				510
Session IV Honours (30 weeks of 19 hours per week)	Biology	SIS 4B(H)	3	5	150
	Chemistry	SIS 4C(H)	3	5	150
	Mathematics	SIS 4M(H)	3	5	150
	Physics	SIS 4P(H)	3	5	150
	Industrial Studies	SIS 4IS(H)	3	5	150
	Integrating Studies				
	(i) Lectures etc	SIS 4E(H)	3	2	60
	(ii) Project	SIS4Proj(H)	3	2	60
	Total for Session IV				570
Session V (24 weeks : 20 hrs per week in term 1 and 15 hrs per week in term 2)	Biology	SIS 5B(H)	2	4	96
	Chemistry	SIS 5C(H)	2	4	96
	Mathematics	SIS 5M(H)	2	4	96
	Physics	SIS 5P(H)	2	4	96
	Industrial Studies	SIS 5IS(H)	2	5	120
	Integrating Studies				
	(i) Lectures etc	SIS 5E(H)	2	2	39
	(ii) Project	SIS5Proj(H)	1	5	60
	Total for Session V				411

Mathematics Syllabi

Session 1 . Total time 126 hrs

Functions and their graphs - domain of definition, examples of functions to include polynomials, rational functions, trigonometric functions, log, ln, exp, product and ratio of functions, inverse functions, composition of functions, logarithms, indices, radian measure and introduction to the use of simple trigonometric identities, cartesian and polar co-ordinates, graphs, use of semi-log and log-log graph paper. Converting to linear form and graphing equations of the forms $y = ae^{bx}$ and $y = ax^b$.

(25)

Calculus - Limits including limits of rational functions, (completing the graph at a point), gradients, the derived function, derivatives from first principles, use of table of derivatives, derivatives of sum, product quotient and composition of functions, turning points, the art of graph sketching including some simple multi-valued functions eg for conic sections. Simple integration and areas simple problems eg rectilinear motion and energy of stretched spring.

(25)

Numerical Mathematics - Absolute, relative and percentage errors. Error analysis for simple formulae eg $y = ab^{-1}c^{-1}+d$. Linear interpolation. Root finding techniques: Bisection and Newton-Raphson methods. Trapezoidal and Simpson's rule. Library programs for calculations relating to these numerical methods.

(14)

Series - Arithmetic and Geometric Series. Conditions for convergence. Binomial Theorem for a positive integral index.

(6)

Statistical Methods & Theory - Sample space concepts and laws of probability for statistically independent events. Random variables - discrete and continuous. Probably distribution for Uniform, binomial, Poisson and geometric distributions and density functions for normal and exponential distributions. Parameter estimates for these distributions from observational data. Methods of presenting statistical information. Use of normal probability paper.

(26)

Computer Studies

Introduction - overview of computer applications in business, commerce and industry. General outline of basic computer configuration.

(2)

Brief summary of computer systems operation (batch/on-line) and available computer languages.

(1)

A study of problem analysis and the use of flow charting.

(4)

A detailed study of BASIC - Let, Rem, Input, Read/Data, End, Print, If. Statements, looping and counting. Terminators. Functions and Subroutines. Arrays.

(23)

Session 1

Lectures	70
Tutorials	56
Revision and Examinations	24
Total	150

Session 2 Total time 98 hrs

Functions and Graphs - Hyperbolic and inverse trigonometric functions. Simple functions of two variables and contour diagrams for these.

(6)

Calculus - Derivatives of hyperbolic and inverse trigonometric functions. Partial derivatives and stationary values of functions of two variables. Maclaurin's Theorem. Power series for sin x, cos x, exp x, $(1 + x)^n$ and conditions for convergence. Taylor's Theorem for functions of one and of two variables. Integration by parts and by substitution.

(15)

Algebra - Complex numbers - cartesian, polar and exponential form. Euler's formulae. Matrices - elementary operations, transpose and inverse. Determinants.

(10)

Differential Equations - First-order with variables separable, second-order with constant coefficients - particular integral by trial: use in modelling physical and biological systems. Interpretation of transient and steady-state solutions. Graphical and computer solutions.

(9)

Numerical Mathematics

Gaussian elimination to solve linear simultaneous equations and to invert matrices. Solution of differential equations by Euler's method and using computer packages.

(9)

Statistical Method and Theory

The distribution of the sample mean in simple random sampling from a normal distribution. The central limit theorem studied experimentally. Hypothesis testing. Errors of Type 1 and Type 2. Confidence intervals for μ, $\mu_1 - \mu_2$ and μ_d. Statistical quality control. Acceptance sampling. Operating characteristics. Contingency tables using Chi-squared test. Linear regression.
(19)

Computer Studies

Further study of BASIC - Character handling. Microcomputer oriented instructions eg Peek, Poke.
(6)

Microprocessors. Architecture and functional components. Capabilities and limitations. Constraints on languages, programming and storage. Comparison with mainframes and mini's. Application and functions. Introduction to assembly language programming.
(12)

Conversion from BASIC to FORTRAN - assignment, READ, WRITE, FORMAT, IF, Comment statements. Arrays. A detailed study of imput/output.
(12)

Session 2

Lectures	58
Tutorials	40
Revision and Examination	17
Total	115

Total time 137 hrs

Session 3

Linear Mathematics

Vector Spaces - subspaces; linear dependence; bases; dimension.
(8)

Linear Transformations (operators) - products of transformations; null space; inverse transformations; linear transformations and matrices.
(8)

Linear differential Operators - linear ordinary differential equations - 1st order; 2nd order with constant coefficients; applications - S H M damped and forced oscillations, circuits.
(10)

Numerical Solutions of O D E's - Euler, Euler modified, Runge-Kutta methods.
(8)

Integral operators - the Laplace transform; properties and use in solution of O D E 's.
(8)

Difference equations - formation; solution; uses; similarity to O D E's.
(7)

Reduction of Differential Equations to the form $\underline{Ax} = \underline{b}$ by Finite Differences.
(4)

Statistics

More advanced design of industrial experiments: components of variance and linear hypothesis; randomised blocks and Latin squares.
(10)

Response surface methodology; analysis of the 2^n series.
(10)

Optimisation

Linear programming - a treatment of transportation and assignment algorithms, including consideration of industrial location, the Simplex algorithm and the concept of a dual program.

Critical path analysis - construction and analysis of networks.

Introduction to forecasting techniques - moving averages and exponential smoothing.
(24)

Computer Studies

A study of subprogramming in FORTRAN, library procedures including scientific subroutines and statistical procedures.
(12)

Advanced computer techniques - language, computer systems (batch, on-line, real-time), file processing. Instruction processing. Computer organisation/operating systems.
(12)

A detailed study of the use of computers (commercial, industrial, research applications).
(6)

Micros. Further assembly language programming. Current applications and trends.

(10)

Session 3

Lectures	74
Tutorials	63
Revision and Examination	24
Total	161

Session 4 (Degree) — Total time 160 hrs

Linear Mathematics

Eigenvalues and Eigenvectors - Eigenvalue problem for a linear transformation (matrix), orthogonality, diagonalisation, applications - vibrations, life-cycle matrices, simple economic systems etc. Caley-Hamilton Theorem. Use of computer to effect eigenvalue calculations.

(26)

Probability and Statistics

The generation and testing of random variates conforming to various distributions - uniform, normal, exponential, Poisson, and arbitrary observed. The use of these in simulation studies of queueing and other stochastic processes including birth and death processes in populating studies. Use of the computer to effect these simulations.

(36)

Theoretical investigation of the preceding topics:
Markov chains, definitions and examples.
General properties of Markov motion. The stability of a Markov system.

Introduction to queueing theory: single channel, Poisson arrival, exponential service.

Deterministic and stochastic models for the growth of populations. Elementary theory of epidemics: deterministic models with and without removal.

Decision theory in the context of industrial quality control.

(45)

Optimisation

Constrained optimisation - integer programming using branch and bound method, method of Lagrange multipliers, method of constraint elimination.

Unconstrained optimisation - direct search and gradient methods.

Use of the computer to solve realistic optimisation problems.

(53)

Session 4 (Degree)

Lecture	110
Tutorial	50
Revision and Examination	20
Total	180

Session 4 (Honours) — Total time 130 hrs

Linear Mathematics

Eigenvalues and Eigenvectors - Eigenvalue problem for a linear transformation (matrix), orthogonality, diagonalisation, applications - vibrations, life-cycle matrices, simple economic systems etc. Caley-Hamilton Theorem. Use of computer to effect eigenvalue calculations.

(25)

Probability and Statistics

The generation and testing of random variates conforming to various distributions - uniform, normal, exponential, Poisson and arbitrary observed. The use of these in simulation studies of queueing and other stochastic processes including birth and death processes in population studies and average run length characteristics in Cusum quality control schemes.

(18)

Theoretical study of the preceding topics pursued in greater depth than for the non-Honours course: a selection to be made from the following list, to suit student and staff interests related to project work and other interdisciplinary research activity relevant to the degree: Markov chains, queues, population dynamics, theory of epidemics including deterministic and stochastic models; mathematical demography.

In this work, in contrast with the non-Honours course, use will be made, in the theoretical treatment, of generating functions and the partial differential equations which these functions satisfy.

(30)

Decision theory in the context of industrial quality control. (7)

Optimisation

Constrained optimisation - integer programming using branch and bound method, method of Langrange multipliers including Kuhn-Tucker relations, methods of constraint elimination.

Unconstrained optimisation - direct search methods and gradient methods.

Use of the computer to solve realistic optimisation problems. (50)

Session 4 (Honours)

Lecture	94
Tutorial	36
Revision and Examination	20
Total	150

Session 5 — Total time 81 hrs

Survey of multivariate statistical methods with emphasis on problems arising in student projects and staff research relevant to work in industry. (17)

The statistical analysis of time series. The Box-Jenkins methodology and related forecasting procedures. The Spectral analysis of linear systems Transfer Function models. (24)

Further study of deterministic and stochastic models in the biological and social sciences. Birth and death processes, competing species, prey-predator, population age distributions, competing economies, spread of epidemics. Interpretation of typical computer prints out from statistical packages. (25)

Use of analytic and numerical methods in the solution of differential equations arising in industrial problems. (15)

Session 5

Lecture	65
Tutorial	16
Revision and Examination	15
Total	96

MATHEMATICS BOOKLIST

Author	Title	Publisher	Year
Reference			
Sherlock, A J Roebuck, E M Godfrey, M G	Calculus, Pure and Applied	Arnold	1, 2
Turner	Advanced Mathematics, Vols 1 and 2		1, 2
Matthews, G	Calculus	J Murray	1, 2
Owen, F Jones, R	Statistics	Polytech. Publishers	1, 2
Snedecor, J W Cochran, W G	Statistical Methods	Iowa State U P	1,2,3
Sprent, P	Statistics in Action	Penguin	1, 2
Emery, G	Elements of Computer Science	Pitman	1
Sanderson, P C	Interactive Computing in BASIC	Butterworth	1
Leventhal, L A	Introduction to Micro-processors	Prentice-Hall	2, 3
Peterson, T	Elementary Fortran	MacDonald & Evans	2, 3
Davies, O L	Design and Analysis of Industrial Experiments	Oliver & Boyd	2, 3
	Short Term Forecasting	ICI Monograph Oliver & Boyd	3
Conte & De Boor	Elementary Numerical Analysis	McGraw-Hill	3
Goldberg, S	Introduction to Difference Equations	J Wiley	3
Zionts, S	Linear & Integer Programming	Prentice-Hall	3, 4
Yaspan, Friedman & Sasieni	Operations Research	J Wiley	3,4, 4H

Author	Title	Publisher	Year
Reference			
Ackoff, Sasieni	Fundamentals of Operational Research	J Wiley	3, 4, 4H
Boyce, W E Di Prima, R C	Elementary Differential Equations & Boundary Value Problems	J Wiley	3, 5
Wagner, H M	Principles of Operations Research	J Wiley	4, 4H
Glaister, S	Mathematical Methods for Economists	Blackwell	4, 4H
Dixon, L C W	Non-Linear Optimis-ation	E U P	4, 4H
White, Donaldson & Lawrie	Operational Research Techniques	Business Books	4, 4H
Allen, A O	Probability, Statistics & Queueing Theory	Academic Press	4, 4H
Adby, P R Dempster, M A H	Introduction to Optimisation Methods	Chapman & Hall	4H
Tocher, K D	The Art of Simulation	E U P	4H
Coleman, R	Stochastic Processes	Allen & Unwin	4H, 5
Bailey, N T J	The Elements of Stochastic Processes	J Wiley	4H, 5
Kendall, M G	Multivariate Analysis	Griffin	5
Kendall, M G	Time Series Analysis		5
Anderson, O D	Time Series Analysis & Forecasting	Butterworths	5

SCHEME OF WORK (1985/1986)

Course : B.Sc. Science with Industrial Studies
Class : Year 4 (Honours)
Teaching Hours : 44 (2 hours per week) plus 1 tutorial hour per week
Recommended Reading : (1) Bunday, Basic Optimisation Methods
(2) Zionts, Linear and Integer Programming

WEEK	TOPIC	LOCATION*	PROGRAM	WORKSHEET
1	Revision of linear algebra	C		
2	Introduction to new concepts (definiteness, rank, etc.)	C		
3	Eigenvalues and eigenvectors I (classical method, etc.)	C		
4	Eigenvalues and eignevectors II (similarity transformations, modal matrices)	C		
5	Powers of a matrix (diagonalisation, Cayley-Hamilton theorem)	C		
6	The inclusion theorem and Gerschgorin's theorem	C		
7	Similarity transformations and Gerschgorin's theorem	C		
8	The power method and the Rayleigh quotient method	C/M	EIGEN	
9	Matrix deflation	L	EIGEN	
10	Applications of eigenvalues and eigenvectors	C		
11	Revision	C		
12	Unconstrained NLP problems (classical method)	C		
13	Constrained NLP problems (equality constraints, elimination)	C		
14	Constrained NLP problems (inequality constraints, Lagrange multipliers)	C		
15	Fibonacci search and golden section search)	L	NLP routines (weeks 15–19)	
16	Spendley, Hext and Himsworth's Simplex method	L		
17	Nelder and Mead's method (including original article)	L		
18	Cyclc co-ordinate descent, Cauchy's steepest descent	L		
19	Newton's method	L		
20	Integer programming problems with two variables	C		
21	The branch and bound method for IP and MIP problems	L	LINPROG	IP1
22	The branch and bound method for IP and MIP problems	L	LINPROG	

* C: classroom; C/M: classroom with monitors; L: laboratory

CNAA HONOURS DEGREE/DEGREE
IN ELECTRONIC AND
COMMUNICATION ENGINEERING

Total time in College

Degree	98 weeks
Honours	130 weeks

For all students

Skills Appreciation (EA1)	10 weeks
SWE	36 weeks

Total Duration of Course

Degree	108 weeks
Honours	140 weeks

COURSE CURRICULUM AND EXAMINATION SCHEDULE

Subject	Hrs/Wk Lect/Tut	Total Hrs/Yr	Exam Papers Ext assessed	Others
1st Year 36 weeks (including a 4 week EA1 block)				
Electrical Engineering I	3	96		1
Electronic Engineering I	3	96		1
Mathematics I	4.5	144		1
Engineering Science I	3	96		1
Computing I	1	32		CW
Communication Studies	1	32		CW
EA1 Engineering Drawing + Lab Familiarisation		64		
Formal Lab Work		64		
EA2 Lab Design Exercises		64		
EA2 Computing Assignments		32		
TOTAL Hrs/Wk 20.6				
2nd Year 28 weeks (including a 6 week EA1 block)				
Electrical Engineering II	3	66	1	
Electronic Engineering IIA	2	44	1	
Electronic Engineering IIB	2	44	1	
Mathematics II	4.5	99	1	
Engineering Science II	2	44	1	
Computing II	1	22		CW
Industrial Studies	1.5	33		CW
EA2 Computer Assignments		22		
EA2 Industrial Studies Assignments		11		
EA2 Open-ended Lab Assignments		80		
Formal Lab Work		30		
TOTAL Hrs/Wk 22.5				

1st Year - MATHEMATICS I

Aims

The student is given a broad foundation in Mathematics which builds naturally on the Higher Mathematics studied at school. In particular, the Calculus is extended to include techniques and applications relating to the engineering subjects of the course. The same is true of the Matrix Algebra.

The Mathematics teaching is enhanced by the use of computer programs and packages which relate to certain parts of the syllabus. As in the later years of the course, students will be given one assignment (EA2) of an engineering or practical nature, which will normally require investigation in the Mathematics Laboratory.

Syllabus

Basic Algebra

Revision of logarithms and indices. Transformation of formulae. Partial fractions.

(6)

Elementary Analysis

Exponential and logarithmic functions and their graphs. Circular functions and their inverses. Polar co-ordinates. Even, odd and periodic functions. Revision of APs and GPs. Convergence - simple tests. Binomial series.

(10)

Linear Algebra

Elementary matrix algebra, determinants, inverse of a matrix. Solution of simultaneous linear equations using Cramer's rule and Gaussian elimination - applications (eg, mesh and nodal analysis).

(12)

Vectors

Scalar and vector products; direction cosines. Applications.

(6)

Complex Numbers

Cartesian, polar, exponential forms. Argand diagram. De Moivre's theorem. Impedances and admittances.

(10)

Calculus

Continuity and limits. Differentiation from first principles. Rules of differentiation. Logarithmic, exponential and hyperbolic functions and their derivatives. Implicit, logarithmic and parametric differentiation. Stationary points.

Curve sketching, including asymptotes. First and second order partial derivatives - application to errors. Integration - general rule, simple substitutions leading to standard forms. Integration by parts. Integration as limit of a sum. Applications to areas, volume, mean, rms. Ordinary differential equations - variable separable, integrating factor. Second order linear with constant co-efficients. Applications. Maclaurin's series, Taylor series. Indeterminate forms. l'Hospital's rule.

(36)

Lectures	- 80
Tutorials	- 64
TOTAL HOURS	-144

BOOK LIST

K WELTNER, J GROSJEAN, P SCHUSTER & W J WEBER	Mathematics for Engineers and Scientists Thornes
SHERLOCK, ROEBUCK & GODFREY	Calculus, Pure and Applied Arnold
GRAHAM, GRAHAM & WHITCOMBE	A-level Mathematics Course Companion Letts

2nd Year - MATHEMATICS II

Aims

This syllabus follows through some of the topics introduced in the first year, with more emphasis on applications in electrical and electronic engineering. These will include impedance and admittance loci for ac circuits, applications of Fourier analysis to common waveforms and the behaviour of circuits modelled by systems of differential equations. Further use is made of computer packages in the work done on Numerical Analysis and Statistics. For example, the MICROTAB package is used extensively.

The EA2 assignment will normally take the form of an open-ended investigation requiring the use of statistics and/or numerical analysis.

Syllabus

Three-Dimensional Geometry

Vector triple products. Equations of straight lines and planes.

(5)

Complex Variables

Functions of a complex variable. Cauchy-Riemann equations. Transformations Loci. Applications.

(8)

Calculus

Simple introduction to multiple integrals (including change of order and change of variable). Partial differentiation - chain rule. Stationary points and their nature for a function of two variables. Laplace transforms - their use in solving differential equations and simultaneous differential equations. Fourier series - full and half-range, exponential form.

(22)

Probability/Statistics

Introduction to statistics. Collection and presentation of data. Measures of central tendency. Measures of dispersion. Permutations and combinations. Probability laws; Bayes' Theorem. Simple correlation and regression. Random variables. Probability density functions. Probability distributions, eg, Binomial, Poisson, negative exponential and Gaussian; expectation.

(12)

Numerical Methods

Errors and linear interpolation. Introduction to numerical integration (eg, Simpson's and Trapezoidal rules). Simple iteration. Newton-Raphson. Recurrence relations. Numerical solution of 1st order ordinary differential equations. Cholesky decomposition.

(8)

Lectures	- 55
Tutorials	- 44
TOTAL HOURS	- 99

BOOK LIST

K WELTNER, J GROSJEAN, P SCHUSTER & W J WEBER — Mathematics for Engineers and Scientists, Thornes, 1986

BOAS — Mathematical Methods in the Physics Sciences, Wiley

STEPHENSON — Mathematical Methods for Science Students, Longman

STROUD — Laplace Transforms, Programmes and Problems, Thornes

SCRATON — Basic Numerical Methods, Arnold

HUGILL — Advanced Statistics, Bell & Hyman

3rd Year - MATHEMATICS III (D)

Aims

Since this will be the final year of Mathematics for degree students, the overall aim of this syllabus is to provide a rounding-off of the students' mathematical education. To support the engineering subjects, an introduction to Vector Calculus is given, along with an elementary treatment of Bessel functions and partial differential equations. In addition, the Statistics covered in 2nd Year is taken a stage further and more work is done in Numerical Analysis. The EA2 assignment will involve mathematical modelling, including the use of the computer.

Syllabus

Calculus

Introduction to vector calculus. Simple treatment of Bessel functions - graphs. Simple partial differential equations arising in communication engineering.

(12)

Complex Variables

Poles, zeros, residues.

(8)

Probability/Statistics

Sampling. Distribution of sample mean. Confidence intervals. Significance testing.

(10)

Numerical Analysis

The method of least squares. Systems of 1st order ordinary differential equations, 2nd order ordinary differential equations. Numerical solution of partial differential equations.

(14)

Lectures	- 44
Tutorials	- 44
TOTAL	- 88

BOOK LIST

R J MADDOCK	Poles and Zeros in Electrical and Control Engineering Holt Rinehart
BOAS	Mathematical Methods in the Physical Sciences Wiley
SPIEGEL	Advanced Mathematics for Engineers and Scientists Schaum's Outline Series
KREYSZIG	Advanced Engineering Mathematics Wiley
SCHEID	Theory and Problems of Numerical Analysis Schaum's Outline Series
HUGILL	Advanced Statistics Bell & Hyman

3rd Year - MATHEMATICS III(R)

Aims

At this stage the Mathematics becomes rather more specialised and is designed to support the engineering topics of the third year. Some of the materials forms a basis for fourth and fifth year work, this being particularly true of the Vector Calculus. Relevant applications of Mathematics will include partial differential equations in an engineering context, eg. vibration, oscillation and wave equations. One such application will form the basis for the EA2 assignment.

Syllabus

Algebra

Complex variables - poles, zeros, residues. Introduction to contour integration.

(10)

Calculus

Series solution of ordinary differential equations. Simple treatment of Bessel functions - graphs. Simple partial differential equations arising in communication engineering. Orthogonal functions. Fourier transforms. Introduction to Vector Calculus. Line, surface and volume integrals. Vector identities. Gauss' theorem and Stokes' theorem. Conservative fields. Maxwell's equations. Z-transforms. Difference equations.

(35)

Special Functions

The gamma and beta functions, their properties and use in the evaluation of integrals. Ber and Bei functions and their properties.

(10)

Lectures	- 55
Tutorials	- 44
TOTAL	- 99

BOOK LIST

KREYZIG	Advanced Engineering Mathematics Wiley
R J MADDOCK	Poles and Zeros in Electrical and Control Engineering Holt Rinehart
BOAS	Mathematical Methods in the Physical Sciences Wiley
SPIEGEL	Advanced Mathematics for Engineers and Scientists Schaum's Outline Series
MEADE & DILLON	Signals and Systems, Models and Behaviour Van Nostrand Reinhold
OPPENHEIM & WILLSKY	Signals and Systems Prentice Hall

4th Year - MATHEMATICS IV (R)

Aims

This syllabus contains some of the Statistics and Mathematics involved in engineering topics and provides a broadening of the student's mathematical experience, with some further work in Numerical Analysis and an introduction to Optimisation techniques and applications.

The work on queuing systems includes an introduction to Simulation. Although the emphasis is on understanding Mathematics, extensive use of packages is made throughout.

The EA2 assignment will be based on the simulation of a queuing system or an investigation in the field of OR.

Syllabus

Probability/Statistics

Moments, moment generating functions, characteristic functions, sums of random variables, central limit theorem, applicaitons to the distribution of sample mean and to finding confidence intervals of parameters. Random processes, stationarity, autocorrelation and cross-correlation.

(16)

Numerical Analysis

Butterworth and Chebychev polynomials and their properties. Approximations. Solution of polynomial equations (eg. Bairstow's method). Special matrix methods. Eigenvalues and eigenvectors. Levinson-Durbin recursion. Applications to linear prediction. Finite difference schemes. Solution of systems of equations having banded matrices of co-efficients. Solution of boundary value problems, including the use of computer packages. Discrete Fourier transforms.

(14)

Optimisation

Linear programming, including the simplex method. Integer programming including the transportation problems. Non-linear programming, including the method of Lagrange multipliers. Use of appropriate computer packages.

(8.5)

Queuing Systems

The Poisson process. Introduction to the M/M/1 queue Various modifications to the M/M/1 queue, with applications. Computer simulation of queuing systems.

(7)

Lectures	- 45.5
Tutorials	- 22
TOTAL	- 67.5

BOOK LIST

ZIONTS	Linear and Integer Programming Prentice Hall
BUNDAY	Basic Optimisation Methods Arnold
COX & MILLER	Theory of Stochastic Processes Methuen
BALFOUR & McTERNAN	Numerical Solution of Equations Heinemann
LAM	Analogue and Digital Filters Prentice Hall
BLAKE	An Introduction to Applied Probability Wiley
PEEBLES	Probability, Random Variables and Random Signal Principles McGraw Hill

SCHEME OF WORK (1985/1986)

Course : B.Sc. Communication and Electronic Engineering
Class : Year 4 (Honours)
Teaching Hours : 26 (1 hour per week) plus 1 tutorial hour per week
Recommended Reading : (1) Cox & Miller, Theory of Stochastic Processes, chapter 1
(2) Zionts, Linear and Integer Programming
(3) Bunday, Basic Optimisation Methods

WEEK	TOPIC	LOCATION*	PROGRAM	WORKSHEET
1	Introduction to the Poisson process and some results from Queueing Theory	C		
2	The M/M/1 queue	C		
3	Various modifications and extensions of the M/M/1 queue	C		
4	Uniformly distributed random nos. and the generation of exponentially distributed r.n.s	L	EXPRND	EXPRND
5	Simulation of the M/M/1 queue	L	QUEUE	
6	Experiments with the M/M/1 simulation	L	QUEUE	Q1
7	Graphical solution of 2-variable linear programming problems	C/M	LINPROG	
8	The general LP problem	C		
9	The Simplex method	C		
10	Duality and complementary slackness	C		
11	LP problem solving	L	LINPROG	LP1
12	Sensitivity analysis	L	LINPROG	LP2
13	Parametric programming	L	LINPROG	
14	Graphical solution of 2-variable integer programming problems	C		
15	Branch and bound method for IP and MIP problems (3 or more variables)	C		
16	IP and MIP problem solving	L	LINPROG	IP1
17	Further IP and MIP problem solving	L	LINPROG	
18	Unconstrained non-linear programming (classical methods)	C		
19	Constrained non-linear programming (constraint elimination)	C		
20	Constrained non-linear programming (Lagrange multipliers)	C		
21	Investigation of search methods	C		
22	Investigation of gradient methods	C		
23	Solution of tridiagonal systems of equations	C		
24	Solution of boundary value problems (2nd order linear ODEs)	C		
25	Investigation of finite difference schemes	L		
26	Solution of problems by finite differences	L	OWN PROG	

* C: classroom; C/M: classroom with monitors; L: laboratory

APPENDIX 4

Student questionnaires

Mathematics Questionnaire

1. **Background**

Name of course	
Year of study	
Qualifications in mathematics (including grades) eg SCE Higher B	

2. **Mathematics**
In the following questions please tick the most appropriate answer.

a) Would you describe your mathematical abilities as?

good	
average	
poor	

b) How would you rate your enjoyment of mathematics?

enjoy it very much	
okay	
dislike	

c) In relation to the rest of your course, do you see the mathematics content as

very useful	
fairly useful	
irrelevant	

3. **Computers**

a) Have you used computers before ?

i) at college:

regularly	
occasionally	
not at all	

ii) at school:

regularly	
occasionally	
not at all	

iii) elsewhere (eg at home)

regularly	
occasionally	
not at all	

b) Have you written your own programs ?

c) As an aid to learning mathematics, do you think that the use of computers at school and/or college is

very useful	
fairly useful	
not useful	

d) Do you enjoy (have you enjoyed) using computers at college (school)?

very much	
okay	
not at all	

4. It is entirely optional but it would be helpful to have your name (or pseudonym) for comparison with future questionnaires . The information you give is confidential and will be used only for research purposes. If you use a pseudonym <u>please</u> use the same one each time.

Name

Thank you for your help.

Student Questionnaire 1985/86

(All replies are confidential and are for research purposes only)

Where a multiple choice question is asked, it is best to reply according to your first instinct. Where reasons are asked for, try to give the question more thought and give a useful response.

1. Is mathematics your

favourite subject	
least favourite subject	
neither	

2. Do you enjoy using computers in mathematics

very much	
okay	
not at all	

3. Tick any of the following computer packages which you have used in the mathematics laboratory this session:

GRAPH NEWTON NUMINT KUTTA

GAUSS LINPROG MEI(REG1, REG2) SURF(3D pictures)

4. For each package used, how would you rate the following aspects: (1 - very highly; 2 - highly; 3 - okay; 4 - poor; 5 - very poor)

Name of package	Ease of use	Reliability	Usefulness of output

5. Which package did you find most useful?

Why?

6. Have you used any PRIME computer packages this session (e.g. MINITAB)?

If YES, do you prefer working on (a) the PRIME?

(b) the BBC micros?

Why?

7. In what ways, if any, do you think computer packages can help you in mathematics (e.g. solving realistic problems, understanding graphs, less arithmetic, experimenting with different parameters in a problem, working at your own pace, etc.)?

8. During your mathematics course this sesson, do you think computer packages were used:

too much	
about right	
not enough	

9. Do you think some computer-based work should be included as part of your overall assessment?

10. Can you suggest any ways in which the mathematics laboratory could be improved (e.g. more computers, lineprinters or deskspace, more time for tutorials, more help available, more open access, better worksheets, etc.)?

11. In the mathematics laboratory, do you prefer:

(a) supervised sessions?

(b) working on assignments in your own time?

12. In the mathematics laboratory, do you like to work:

on your own	
in pairs	
in a small group	

13. In relation to the rest of your course, do you find the mathematics content:

very useful	
fairly useful	
not at all useful	

14. Do you own a micro (wholly or shared)?

If you gave your name (or pseudonym) on the first questionnaire, please give it here also (the same one!):

Name:

Male / Female

Many thanks for your help.

Student Questionnaire SQ2

Computer packages for Mathematics

NODES

1. Title of course
   Year of course
   Sex

2. Do you enjoy mathematics ?

very much	
okay	
not at all	

3. Have you previously used computer packages for mathematics ?

a. on a mainframe

frequently	
occasionally	
not at all	

b. on a micro

frequently	
occasionally	
not at all	

4. Do you enjoy using computer packages in mathematics?

very much	
okay	
not at all	

5. For how many sessions have you used NODES?

1	
2-5	
>5	

6. Have you enjoyed using NODES ?

very much	
okay	
not at all	

7. How would you rate NODES for the following features?
   1 very highly 2 highly 3 okay 4 poor 5 very poor

ease of use	
usefulness of screen output	
reliability	
flexibility/options offered	

8. For which of the following reasons, if any, would you recommend using NODES?

a. to enhance understanding of the numerical solution of 1st order d.e's and systems of d.e's	
b. to check hand-calculated results	
c. to obtain numerical solutions of 1st order d.e's and systems of d.e's	
d. to investigate the effect on the solution of varying the method, step size, initial conditions etc	
e. to obtain graphical solutions, of d.e's or systems of d.e's	
f. to obtain phase plots of systems f d.e's	
g. to determine and analyse the behaviour of a model, involving d.e's	
h. to investigate the accuracy of numerical solutions	
i. other reasons	

9. Which features do you like most about NODES ?

10. Which features do you like least about NODES?

11. Do you consider numerical solution of d.e's to be a useful topic to be included in your course?

12. Do you think some computer-based work should be included as part of your overall assessment?

13. In the mathematics micro-laboratory, do you prefer

supervised sessions	
working on assignments in your own time	
both equally	

14. In the mathematics micro-laboratory, do you prefer to work

on your own	
in pairs	
in a small group	

....

Thank you very much for your help in filling in this questionnaire.

sq3

Student Questionnaire 1986/87

(All replies are confidential and are for research purposes only)

Where a multiple choice question is asked, it is best to reply according to your first instinct. Where reasons are asked for, try to give the question more thought and give a useful response.

1. Do you enjoy mathematics

very much	
okay	
not at all	

2. Do you enjoy using computers

very much	
okay	
not at all	

3. Tick any of the following computer packages which you have used in the mathematics laboratories this session:

GRAPH	SURF	NUMINT	GAUSS	QUEUE
NODES	EIGEN	LINPROG	LPROG2D	MICROTAB

4. If you have used any of the packages mentioned below, how would you rate the following aspects:
(1 - very highly; 2 - highly; 3 - okay; 4 - poor; 5 - very poor)

Name of package	Ease of use	Reliability	Usefulness of output
GRAPH			
NUMINT			
LINPROG			
NODES			
QUEUE			

5. Which package (of all the ones you have used) did you find most useful? Why?

6. Have you used any PRIME computer packages this session (e.g. MINITAB)?

If YES, do you prefer working on (a) the PRIME?

(b) the BBC micros?

Why?

7. In what ways, if any, do you think computer packages can help you in mathematics (e.g. solving realistic problems, understanding graphs, less arithmetic, experimenting with different parameters in a problem, working at your own pace, etc.)?

8. During your mathematics course this sesson, do you think computer packages were used:

too much	
about right	
not enough	

9. Do you think some computer-based work should be included as part of your overall assessment?

10. Can you suggest any ways in which the mathematics laboratories could be improved (e.g. more computers, lineprinters or deskspace, more time for tutorials, more help available, more open access, better worksheets, etc.)? (Please state which laboratory M Merchiston C Craiglockhart)

11. In the mathematics laboratory, do you prefer:

(a) supervised sessions?

(b) working on assignments in your own time?

12. In the mathematics laboratory, do you like to work:

on your own	
in pairs	
in a small group	

13. In relation to the rest of your course, do you find the mathematics content:

very useful	
fairly useful	
not at all useful	

14. Do you own a micro (wholly or shared)?

If you gave your name (or pseudonym) on the first questionnaire, please give it here also (the same one!):

Name:

Male / Female

Many thanks for your help.

Student Questionnaire SQ4

Computer packages for Mathematics

1.

Title of course
Year of course
Male / Female

2. How would you rate your enjoyment of mathematics ? (Circle the answer which corresponds most closely to your opinion.)

enjoy it very much -------------------------> not at all
1 2 3 4

3. Does the use of the maths. science lab. packages contribute towards your enjoyment of mathematics ?

very much ------------------------------> not at all
1 2 3 4

4. Tick any of the following computer packages which you have used in the mathematical sciences laboratories this session:

GRAPH	NODES	NUMINT	LINPROG	EIGEN
SURF	DEPLOT	NEWTON	LPROG2D	FOURIER
MEI	TRIG	EXPRND	QUEUE	COMPLEX
ALEVEL	MICROTAB	VIEW	VIEWSHEET	

5. For each of the packages mentioned below, which you have used, can you rate the given aspects on a scale 1.......4 as follows:

very highly ----- highly ----- okay ---- poor
1 2 3 4

package	ease of use	reliability of software	usefulness of screen	range of options
GRAPH				
LINPROG				
NODES				
SURF				
MICROTAB				

6. Do you think that the use of computer packages in mathematics has helped you in any of the following ways:- (Tick for YES)

	Tick	If YES, which package(s) ?
better understanding of a topic/ algorithm/method	☐	
able to visualise solution more clearly	☐	
solving problems	☐	
experimenting/ investigations	☐	
working at own pace	☐	
checking results	☐	
report writing	☐	

7. Can you think of any other ways use of packages has helped?

8. Do you think some computer-based assessment should be included as part of your overall assessment?

9. As an aid to learning mathematics, do you think that the use of computer packages is

very useful ------------------------------> not at all useful
1 2 3 4

....

Thank you very much for your help in filling in this questionnaire.

March 1988

Computer packages for Mathematics

1.

Title of course	
Year of course	
Male / Female	

2. How would you rate your enjoyment of mathematics ?
(Circle the answer which corresponds most closely to your opinion.)

enjoy it very much -----------------------> not at all
1 2 3 4

3. Does the use of the maths. science lab. packages contribute towards your enjoyment of mathematics ?

very much ------------------------------> not at all
1 2 3 4

4. Use of the mathematical sciences laboratory packages can involve (amongst other things)

a) investigating a mathematical model
b) analysing the sensitivity of a solution
c) report writing

Can you rate each of these aspects under the given headings in the table below on a scale 1........4

eg 1 ------------2------------3---------> 4
very interesting not at all interesting

	interesting	enjoyable	useful towards the rest of your course
investigating a model			
analysing sensitivity			
report writing			

5. Do you think some computer-based work should be included as part of your overall assessment?

6. Does use of the lab. enhance your understanding of any aspects of mathematics ?

yes, very much ------------------------------> not at all
1 2 3 4

If yes, which aspects ?

The following questions refer to your use of the NODES package during the current session, 1978/88.

7. For how many sessions have you used NODES? 1
   2-5
   >5

8. How would you rate NODES for the following features? (Circle your answer)

	very highly	----------->		very poor
ease of use	1	2	3	4
usefulness of screen output	1	2	3	4
reliability of software	1	2	3	4
range of options	1	2	3	4
suitability for investigative/ experimental use	1	2	3	4

9. For which of the following reasons, if any, would you recommend using NODES?

   a. to obtain graphical solutions of d.e's or systems of d.e's

   b. to enhance your understanding of the solution of 1st order d.e's and systems of d.e's

   c. to determine and analyse the behaviour of a model, involving d.e's

   d. to obtain phase plots of systems of d.e's

   e. to obtain numerical solutions of 1st order d.e's and systems of d.e's

   f. to investigate the effect on the solution of varying the method, step size, initial conditions etc

   g. to check hand-calculated results

   h. to investigate the accuracy of numerical solutions

   i. other reasons

10. Which features do you like most about NODES ?

11. Which features do you like least about NODES?

....

Thank you very much for your help in filling in this questionnaire.

LINPROG

Student Survey

For each of the following statements, tick the column which corresponds most closely to your own views, according to the following scale:

5	strongly agree	(✓✓)
4	agree	(✓)
3	not sure or doesn't apply	(?)
2	disagree	(×)
1	strongly disagree	(××)

	5 ✓✓	4 ✓	3 ?	2 ×	1 ××
1. I enjoyed using LINPROG					
2. I couldn't understand what the program was doing					
3. Using LINPROG helped me understand the Simplex method					
4. Using LINPROG helped me understand the concept of duality					
5. I prefer conventional tutorials to using LINPROG					
6. It was interesting to investigate the sensitivity of the solutions of problems					
7. I learn more by working it out by myself on paper					
8. I like to experiment with different parameters and see how they affect the solutions					
9. LINPROG reinforced my understanding of linear programming					
10. I had difficulty using LINPROG					
11. Using LINPROG helped me to understand the branch and bound method for integer programming					
12. Using LINPROG made the topic more interesting					
13. I did not learn anything new about linear programming					

/14. . . .

LPAQ

LINPROG

	5 ✓✓	4 ✓	3 ?	2 ×	1 ××
14. Knowledge of linear programming techniques would be useful when working in industry or commerce					
15. I am only really interested in passing my exams					
16. Similar computer programs should be used to enhance the learning of other topics					
17. I prefer to work by myself on the computer					
18. The program was user friendly					

Please add any other comments you wish to make about LINPROG here:

NAQ

NODES

Student Survey

For each of the following statements, tick the column which corresponds most closely to your own views, according to the following scale:

5	strongly agree	(✓✓)
4	agree	(✓)
3	not sure or doesn't apply	(?)
2	disagree	(×)
1	strongly disagree	(××)

	5 ✓✓	4 ✓	3 ?	2 ×	1 ××
1. I enjoyed using NODES					
2. I couldn't understand what the program was doing					
3. Using NODES helped me understand numerical methods for solving differential equations					
4. Using NODES helped me understand stiff equations					
5. Using NODES helped me understand the effect of errors on the solution					
6. I prefer conventional tutorials to using NODES					
7. It was interesting to investigate the sensitivity of solutions to d.e.s					
8. I learn more by working problems out on paper					
9. I like to experiment with different parameters in the program and see how they affect the solution					
10. Graphical solutions of d.e.s are easier to understand than numerical values					
11. I had difficulty using NODES					
12. NODES reinforced my understanding of the behaviour of solutions of d.e.s					
13. Using NODES made the subject more interesting					
14. Investigating a model using NODES is a useful exercise					

/15. . . .

NODES

	5 ✓✓	4 ✓	3 ?	2 ×	1 ××
15. I did not learn anything new about solving differential equations					
16. I am only really interested in passing my exams					
17. Experience of investigating mathematical models would be useful in industry					
18. Similar computer programs should be used to enhance the learning of other topics					
19. I prefer to work by myself at the computer					
20. The program was user friendly					

Please add any other comments you wish to make about NODES here:

APPENDIX 5

Staff questionnaires

Staff Questionnaire 1984/85

I. During the 1984/85 session, did you make use of the facilities in the mathematics laboratory:

(a) for a formal class meeting ☐

(b) by setting assignments for students to complete in their own time ☐

(c) on your own ☐

(d) not at all (complete Section IV onwards) ☐

II. If your answer to I includes (a), (b) or (c) then, for each of the packages listed below, which you have used, please indicate your rating and any comments:

Package Name	Rating: Very Good → 1	2	3	4	5 Poor	Comments
GRAPH						
TRIG						
NEWTON						
NUMINT						
KUTTA						
EIGEN						
DEPLOT						
QUEUE						
BM.STAT						

III. If your answer to I above includes (a) or (b),

1. List classes involved:

2. How often, on average, did you use the lab for formal class meetings? (tick one answer)

(a) more than 2 hours per week ☐

(b) 1 to 2 hours per week ☐

(c) less than 1 hour per week ☐

(d) less than 1 hour per week ☐

Staff Questionnaire 1984/85

3. For which of the following purposes did you choose to use the packages available in the laboratory:

(a) an improved teaching method ☐

(b) a means of solving more realistic problems ☐

(c) to carry out investigations and/or experiments ☐

(d) to carry out (mini) projects ☐

(e) any other (please specify) ☐

4. For each of the categories in (3) above, indicate how successful you rate the experience:

	Very successful	Fairly successful	Not successful
(a)			
(b)			
(c)			
(d)			
(e)			

IV. Answers to the following questions would be much appreciated:

1. Can you suggest improvements to the existing software?
2. What additional software would you like to see provided?
3. What other lab facilities would you like improved?
4. In what way does the availability of the packages in the lab affect your approach to teaching?

/V. . . .

Staff Questionnaire 1984/85

V. If your answer to I above is either (c) or (d), please indicate which of the following reasons apply:

(a) use of the microcomputer considered inappropriate ☐

(b) appropriate software not yet available ☐

(c) back-up materials (e.g. worksheets) not available ☐

(d) lab already booked ☐

(e) teaching not at Merchiston ☐

(f) other - please specify ☐

VI. Please add any other comments which you feel may be helpful:

LQ2

Staff Questionnaire 1985/86

1. During the 1985/86 session, did you make use of the facilities in the mathematics laboratory:

   (a) for a formal class meeting ☐

   (b) by setting assignments for students to complete in their own time ☐

   (c) on your own ☐

   (d) not at all (please complete question 7) ☐

2. For each of the packages listed below, which you have used, please rate the package for the aspects listed.

   (1 very highly; 2 highly; 3 fair; 4 poor; 5 very poor)

Package	reliability	ease of use	usefulness of output
GRAPH			
TRIG			
NEWTON			
NUMINT			
KUTTA			
EIGEN			
DEPLOT			
QUEUE			
GAUSS			
LPROG2D			
LINPROG			
EXPRND			
SURF			
M.E.I.			

3. How often, on average, during the first 2 terms, did you use the lab for formal class meetings? (tick one answer)

   (a) more than 2 hours per week ☐

   (b) 1 to 2 hours per week ☐

   (c) less than 1 hour per week ☐

   (d) less than 1 hour per month ☐

4. For which of the following purposes did you choose to use the mathematics laboratory?

(a) to re-inforce/ enhance understanding of an algorithm/ method ☐

(b) as a means of solving more realistic problems ☐

(c) to carry out investigation and/or experiments ☐

(d) to set student assignments using packages ☐

(e) to set student assignments/ projects involving students writing their own programs ☐

(f) any other (please specify) ☐

5. For each of the categories in (4) above, indicate how successful you rate the experience

	very successful	fairly successful	not successful
(a)			
(b)			
(c)			
(d)			
(e)			
(f)			

6. Can you suggest any improvements to the organisation of the mathematics laboratory (eg layout, peripherals, technical support) ?

7. Have any of the following reasons prevented you from making use of the mathematics laboratory when you would have liked to?

(a) class size too large ☐

(b) appropriate software not available ☐

(c) back-up materials (eg worksheets) not available ☐

(d) lab already booked ☐

(e) teaching not at Merchiston ☐

(f) other - please specify ☐

LQ3

Staff Questionnaire 1986/87

1. During the 1986/87 session, did you make use of the facilities in either of the mathematics laboratories:

for a formal class meeting	
by setting students assignments to complete in their own time	
on your own	
not at all (please complete question 10)	

2. **Packages**

Circle any of the following computer packages which you (or your classes) have used in the mathematics laboratories this session:

GRAPH SURF TRIG EIGEN NEWTON NUMINT M.E.I.

NODES KUTTA DEPLOT GAUSS EXPRND QUEUE MICROTAB

LINPROG LPROG2D COMPLEX BM-STAT VIEW VIEWSHEET

3. **Software**

For each of the packages listed below, which you have used, please rate the software for the aspects listed.

(1 very highly; 2 highly; 3 fair; 4 poor; 5 very poor)

Package	reliability	ease of use	usefulness of output	flexibility / options
GRAPH				
LINPROG				
NODES				
MICROTAB				
SURF				
COMPLEX				

4. **Worksheets**

For each of the packages below, please list any lab. worksheets which you have used this session (by number or topic) and give a rating for each:

	Worksheets used	Rating (1 very highly....5 very poor)
GRAPH		
NODES		
LINPROG		
MICROTAB		

5. Would you like more worksheets to be provided in the laboratories? If yes, for what topics?

6. Which package(s)(of all the ones you have used) did you find most useful? Why?

7. How often, on average, during the first 2 terms (2nd term only at Craiglockhart) did you use the laboratories for formal class meetings?

	Merchiston	Craiglockhart
more than 2 hours per week		
1 to 2 hours per week		
1 to 4 hours per month		
less than 1 hour per month		

8. For which of the following reasons did you choose to use the mathematics laboratory?

to reinforce/ enhance understanding of an algorithm/ method	
as a means of solving more realistic problems	
to carry out investigations and/ or experiments	
to set student assignments using packages	
to set student assignments/ projects involving the student writing programs	
preparation of classwork/examinations/worksheets etc	
other (please specify)	

9. Improvements to laboratories

a) Do you think permanent technical support is required in or near the laboratories?

b) Do you think the mathematics department requires
   i) larger laboratories
   ii) more laboratories

c) Do you think instruction sheets are required for
   i) each package in lab.
   ii) general use of packages, printers etc

d) What other improvements ?

10. Have any of the following reasons prevented you from making use of the mathematics laboratories when you would have liked to ?

class size too large	
appropriate software not available	
worksheets not available	
insufficient class time	
lab already booked (please state approx. number of occasions)	
lab out of order (" " " " ")	

Staff Questionnaire 1987/88

1. During the 1987/88 session, did you make use of the facilities in either of the mathematics BBC laboratories:

for a formal class meeting	
by setting students assignments to complete in their own time	
on your own	
not at all (please complete question 8)	

2. How often, on average, during the first 2 terms did you use the laboratories for formal class meetings?

more than 2 hours per week	
1 to 2 hours per week	
1 to 4 hours per month	
less than 1 hour per month	
OR total no. of hours during session	

3. Packages

Circle any of the following computer packages which you (or your classes) have used in the mathematics laboratories this session:

GRAPH	NODES	NUMINT	LINPROG	EIGEN
SURF	DEPLOT	NEWTON	LPROG2D	FOURIER
MEI	TRIG	EXPRND	QUEUE	COMPLEX
ALEVEL	MICROTAB	GAUSS	VIEW	VIEWSHEET

4. For which of the following reasons did you choose to use the mathematics laboratory (for supervised sessions or to set assignments) ?

to reinforce/ enhance understanding of an algorithm/ method	
as a means of solving more realistic problems	
to carry out investigations and/ or experiments	
to set student assignments/ projects involving the student writing programs	
preparation of classwork/examinations/worksheets etc	
other (please specify) (eg student motivation)	

5. Would you like more worksheets to be provided in the laboratories? If yes, for what topics?

6. What additional computer packages would you like provided in the BBC laboratories ?

7. Did you include any computer-based work as part of the student's overall assessment for any course this session ?

8. Have any of the following reasons prevented you from making use of the mathematics laboratories when you would have liked to ?

class size too large	
appropriate software not available	
worksheets not available	
insufficient class time	
lab already booked (please state approx. number of occasions)	
lab out of order (" " " " ")	
class not based at Merchiston or Craiglockhart	

....

Thank you very much for your help in filling in this questionnaire.

Computer packages for Mathematics

LINPROG

1. For which course did you use LINPROG:
   Title of course:
   Year of course :

2. For which of the following reasons, if any, did you choose to use LINPROG?

   a. to enhance understanding of the Simplex method

   b. to enhance understanding of duality

   c. to solve realistic L.P. problems

   d. to carry out a post-optimal analysis

   e. to solve integer programming problems

   f. other reasons

3. For each of the categories above, indicate how successful you rate the experience.

	very successful	--------------	-------------->	not successful
a.	1	2	3	4
b.	1	2	3	4
c.	1	2	3	4
d.	1	2	3	4
e.	1	2	3	4
f.	1	2	3	4

4. How would you rate LINPROG for the following features?

	very highly	-----------	----------->	very poor
ease of use	1	2	3	4
usefulness of screen output	1	2	3	4
reliability	1	2	3	4
flexibility/options offered	1	2	3	4

5. Which features do you like most about LINPROG?

6. Which features do you like least about LINPROG?

7. Can you suggest any improvements ?

8. What was the students reaction to using LINPROG ?

9. What kind of support material did you use with LINPROG ?
eg handouts, worksheets etc.
(A copy of of any such tutorials or worksheets would be much appreciated.)

10. Did you include some computer-based work using LINPROG as part of the overall assessment for the course?

11. Can you suggest any other topics for which a similar package would be beneficial ?

....

Thank you very much for your help in filling in this questionnaire.

LQN2

Staff Questionnaire - NODES

1. For which of the following purposes did you choose to use NODES

   a to obtain graphical solutions of d.e.'s or systems of d.e's.

   b to obtain numerical solutions of d.e's or systems of d.e's

   c to investigate the accuracy of numerical solutions

   d to enhance understanding of the solution of 1st order d.e's or systems of d.e's

   e to investigate the effect on the solution of varying method, step size, initial conditions etc

   f to obtain phase plots of systems of d.e's

   g to determine and analyse the behaviour of models involving d.e's

   h any other (please specify)

3. For each of the categories above, indicate how successful you rate the experience.

	very successful	--------------	--->	not successful
a.	1	2	3	4
b.	1	2	3	4
c.	1	2	3	4
d.	1	2	3	4
e.	1	2	3	4
f.	1	2	3	4
g.	1	2	3	4
h.	1	2	3	4

4. How would you rate NODES for the following features?

	very highly	--------	--->	very poor
ease of use	1	2	3	4
usefulness of screen output	1	2	3	4
reliability	1	2	3	4
flexibility/options offered	1	2	3	4

5. How often did you use NODES for

a) supervised sessions

b) student assignment

c) classroom demonstration

6. Which features do you like most about NODES ?

7. Which features do you like least about NODES ?

8. Can you suggest any improvements ?

9. What was the students reaction to using NODES ?

10. What kind of support material did you use with NODES eg handouts, worksheets etc ? (A copy of any tutorials or worksheets would be much appreciated.)

11. Did you include any computer-based work using NODES as part of the overall assessment for the course ?

. . . .

Thank you very much for your help in filling in this questionnaire.

APPENDIX 6

External questionnaires

Student Questionnaire

Computer packages for Mathematics
LINPROG

1.

Title of course	
Year of course	
Sex	

2. How would you rate your enjoyment of mathematics ?
(Circle the answer which corresponds most closely to your opinion.)

enjoy it very much -------------------------> not at all
1 2 3 4

3. Have you previously used computer packages for mathematics ?

a. on a mainframe

frequently	
occasionally	
not at all	

b. on a micro

frequently	
occasionally	
not at all	

4. Do you enjoy using computer packages in mathematics?

very much -------------------------> not at all
1 2 3 4

5. For how many sessions have you used LINPROG?

1	
2-5	
>5	

6. Have you enjoyed using LINPROG?

very much ----------------------------> not at all
1 2 3 4

7. How would you rate LINPROG for the following features?

	very highly	----------->		very poor
ease of use	1	2	3	4
usefulness of screen output	1	2	3	4
reliability	1	2	3	4
flexibility/options offered	1	2	3	4

8. For which of the following reasons, if any, would you recommend using LINPROG?

a. to enhance understanding of the Simplex method

b. to solve realistic L.P. problems

c. to solve integer programming problems

d. to enhance understanding of duality

e. to check hand-calculated results

f. to carry out a post-optimal analysis

g. other reasons

9. Which features do you like most about LINPROG?

10. Which features do you like least about LINPROG?

11. Do you think some computer-based work should be included as part of your overall assessment?

12. In the mathematics micro-laboratory, do you prefer

supervised sessions	
working on assignments in your own time	
both equally	

13. In the mathematics micro-laboratory, do you prefer to work

on your own	
in pairs	
in a small group	

14. Can you suggest any other topics for which a similar package would be beneficial ?

....

Thank you very much for your help in filling in this questionnaire.

Staff Questionnaire

1. For which of the following purposes did you choose to use LINPROG ?

(a) to re-inforce/ enhance understanding of the Simplex method	
(b) to solve realistic L.P. problems	
(c) to solve integer programming problems	
(d) as a means of carrying out post-optimal analysis	
(e) to set student assignments	
(f) any other (please specify)	

2. For each of the categories above, indicate how successful you rate the experience

	very successful	fairly successful	not successful
(a)			
(b)			
(c)			
(d)			
(e)			
(f)			

3. How would you rate LINPROG for each of the following features ?
1 **very** highly 2 highly 3 okay 4 poor **5** very poor

reliability	
flexibility/options offered	
ease of use	
usefulness of screen output	

4. How often did you use LINPROG for

a)	supervised sessions	
b)	student assignment	
c)	classroom demonstration	

5. Which features do you like most about LINPROG ?

6. Which features do you like least about LINPROG?

7. Can you suggest any improvements ?

8. What was the students reaction to using LINPROG ?

9. What kind of support material did you use with LINPROG eg handouts, worksheets etc ? (A copy of any tutorials or worksheets would be much appreciated.)

10. Can you suggest any other topics for which a similar package would be beneficial ?

. . . .

APPENDIX 7

Summary of student interviews, 1986

Student A (male), AC1

Chemistry is his favourite subject but mathematics is mostly interesting and he expects it to prove useful. He enjoys the use of computers in mathematics very much because it involves 'learning by doing' and allows more work to be done because calculations are so fast. During lectures, which he finds long and boring, he is too busy taking notes to learn much. He enjoyed doing the computer-based assignment and would welcome more investigative work of that nature.

Student B (female), AC1

Her favourite subjects are computing and mathematics, which she finds both interesting and enjoyable. She thinks that the use of computer packages has helped her to understand some aspects of mathematics better. Graphical output is particularly helpful. All the packages used this year were helpful but she would have liked to spend more time using them and to do more investigative work. She thinks that computer packages should have been used in mathematics in 6th year at school.

Student C (male), AC1

He is not interested in mathematics and does not enjoy it at all, but believes that it might be of use to him some day. He enjoys the use of computers and thinks that some packages can help in learning mathematics, for example, by drawing graphs or for numerical integration. He did not pay attention during laboratory sessions and found the assignment difficult. Nevertheless he thinks that more use should be made of computers both during tutorial time and for setting assignments to be done in their own time.

Student D (male), CEE4

He enjoys mathematics, more so now because of the practical applications. It is useful, relevant and interesting. Computer packages are good for checking results. Their use can also aid understanding by allowing more examples to be solved and by illustrating results graphically (e.g. Fourier analysis). He found LINPROG especially useful when learning the Simplex method and would like more investigative work. He enjoys writing his own programs too.

Student E (male), CEE4

He does not find mathematics interesting or enjoyable but thinks it would be if it were more practical. More help is needed to get started when using computers or packages. He needed frequent assistance during laboratory sessions, as he does in normal tutorials also. Using LINPROG to reinforce the Simplex method for linear programming problems was "far superior to the lecture". He returned to the laboratory on his own several times to use the package. It helped with the branch and bound method for integer programming problms also. An introductory course on the facilities in the laboratory would be useful, as would handing out worksheets in advance of the session.

Student F (male), CEE4

His favourite subject is computer engineering but mathematics is quite interesting. He derives satisfaction from solving a problem. Computers can help in the learning of mathematics because "they allow what if questions". In general, he thinks that writing his own program is more beneficial than the use of packages but he found both LINPROG and QUEUE helpful. He thinks that the worksheets should contain more information about the methods being used and the program. They could be given out in advance.

Student G (male), SIS4

Mathematics is his favourite subject at college though he did not enjoy it much at school. He finds college mathematics more relevant and practical. He enjoys using computers, too, but prefers to write his own programs for mathematics. He has written two programs during the year, one for random numbers and one for a search method. On the whole, he sees more value in these exercises than using prepared packages, with the exception of KUTTA and LINPROG. In particular, LINPROG helped him to understand the branch and bound method for integer progamming problems. "It was not clear from the lecture and notes".

Student H (female), SIS4

This student is older than average. Mathematics is her favourite subject and always has been. She considers it to be a practical subject but derives her satisfaction from getting results. She has no confidence in the use of computers. Her initial experience of computing in first year was discouraging and she has never recovered. She was one of only four in the class who had no previous computing experience, and the lecturer's attitude was that he was not going to hold up the rest of the class for them. She was not offered any additional or remedial help. She does not see how computers can help towards learning mathematics but cannot dissociate use of computer packages from programming. She finds micros less intimidating than the mainframe, and is happier working on her own or with a friend with no lecturer present so that nobody sees her "silly mistakes". She is scared of experimenting.

Student J (male), SIS4

He likes mathematics more now than two years ago because it seems more directly relevant - for example, when modelling real-life situations. The use of computers in mathematics helps a lot towards this. He feels confident about using a computer package to solve a problem. Solving 'hard' problems in the laboratory helps him to solve 'easier' ones by hand later. LINPROG helped particularly when using the branch and bound method to solve integer programming problems. He did not really understand the method from the lecture but did after using LINPROG and constructing a tree diagram. Writing one's own programs also aids understanding - for example, the algorithm for a search method. He considers all his work in the mathematical sciences laboratories to have been helpful, including the use MINITAB the previous year, and a positive contribution towards the course. He has twice used the laboratory in his own time. He cannot remember computers having been used much in other subjects.

Student K (female), SIS4

Chemistry is this student's favourite subject but she enjoys mathematics too. She thinks that computers are useful for getting answers quickly but do not help you to learn mathematics as they do not show how they arrive at the answer. "I must know what is happening". Computers can be useful for drawing graphs and 3-D surfaces. She does not consider mathematics to be very useful.

APPENDIX 8

Results of questionnaires SQ0 (1986, 1987), SQ1, SQ3, SQ4

Results of initial questionnaire SQ0, 1986, 1987

		1986			1987	
		AC1	SIS4	CEE4	SIS4	CEE4
Mathematical ability:	good	1	4	3	7	10
	average	16	10	10	12	13
	poor	2	0	2	1	0
Enjoyment of mathematics:	very much	4	7	4	2	7
	OK	12	7	11	17	14
	not at all	3	0	0	1	2
Mathematics course:	very useful	9	8	8	4	16
	fairly useful	9	6	7	14	7
	irrelevant	0	0	0	2	0
	don't know	1	0	0	0	0
Previous use of computers:						
at college:	regularly	1	12	5	13	5
	occasionally	7	2	10	7	18
	not at all	11	0	0	0	0
at school:	regularly	1	2	3	0	3
	occasionally	7	2	5	6	9
	not at all	11	10	7	14	11
elsewhere:	regularly	2	1	5	2	3
	occasionally	5	4	6	6	11
	not at all	12	9	4	12	9
Written programs?	yes	6	14	12	16	19
	no	13	0	3	4	4
Use of computers as aid to learning maths:	very useful	11	4	4	4	5
	fairly useful	7	8	6	12	10
	not useful	0	2	5	3	4
	don't know	1	0	0	1	4
Enjoyment of use of computers:	very much	5	5	3	3	4
	OK	1	5	9	9	12
	not at all	2	4	3	8	5
	don't know	11	0	0	0	2
Number of replies		19	14	15	20	23

Results of questionnaire SQ1, 1986

		AC1	SIS4	CEE4
Enjoyment of mathematics:	favourite subject	0	5	0
	least favourite subject	6	0	1
	neither	1	3	12
Enjoyment of computers in mathematics:	very much	2	3	1
	OK	4	4	12
	not at all	1	1	0
Amount of use of computer packages:	too much	1	0	0
	about right	3	8	13
	not enough	3	0	0
Should some computer-based work be assessed?	yes	5	5	5
	no	2	3	8
Preferred mode of working:	supervised sessions	2	4	7
	working in own time	2	3	3
	no preference	3	1	3
Preferred mode of working:	on own	4	3	8
	in pairs	2	5	4
	in small group	1	0	1
Mathematics course:	very useful	1	3	7
	fairly useful	3	5	6
	not at all useful	3	0	0
Own a micro:	yes	0	3	7
	no	7	5	6
Packages used: (GR = GRAPH, GA = GAUSS, N = NEWTON, K = KUTTA, L = LINPROG, S = SURF)		GR 7 N 4 L 2	GR 5 N 3 L 7 K 4 S 2	L 13 K 1 GA 1
Number of replies:		7	8	13
Male		4	5	13
Female		3	3	0

Results of questionnaire SQ3, 1987

		SIS4	CEE4
Enjoyment of mathematics:	very much	1	6
	OK	15	17
	not at all	0	0
Enjoyment of computers in mathematics:	very much	4	7
	OK	9	16
	not at all	3	0
Amount of use of computer packages:	too much	3	3
	about right	12	18
	not enough	1	2
Should some computer-based work be assessed?	yes	5	15
	no	10	8
	don't know	1	0
Preferred mode of working:	supervised sessions	10	14
	working in own time	3	6
	no preference	3	3
Preferred mode of working:	on own	6	1
	in pairs	8	19
	in small group	2	3
Mathematics course:	very useful	4	13
	fairly useful	11	9
	not at all useful	1	1
Own a micro:	yes	4	1
	no	12	22
Packages used: (G = GRAPH, L = LINPROG, Q = QUEUE, E = EIGEN, M = MICROTAB, S = SURF, L2 = LPROG2D)		G 4	G 23
		L 16	L 23
		Q 15	Q 18
		E 14	L2 8
		M 5	
		S 6	
Number of replies:		16	23
Male		10	21
Female		6	2

Results of questionnaire SQ4, 1988

			SIS4	CEE4
Enjoyment of mathematics:	very much	4	2	4
	↓	3	8	12
		2	0	1
	not at all	1	1	0
Enjoyment of computers in mathematics:	very much	4	1	3
	↓	3	5	9
		2	5	5
	not at all	1	0	0
Should some computer-based work be assessed?	yes		6	8
	no		5	9
Use of packages as an aid to learning mathematics	very useful	4	1	5
	↓	3	6	8
		2	4	4
	not at all useful	1	0	0
Packages used:	G = GRAPH	S = SURF	G 5	G 13
	L = LINPROG	Q = QUEUE	L 11	L 17
	N = NEWTON	L2 = LPROG2D	N 9	N 15
	E = EIGEN		E 8	Q 7
			S 11	L2 8
Number of replies:			11	17
Male			5	17
Female			6	0

APPENDIX 9

Results of questionnaires SQO (1987), SQ2, SQN2

Results of questionnaire SQ0, 1987

		CEE1	%	CEE2	%
Mathematical ability:	good	9	15	10	18
	average	47	77	37	67
	poor	5	8	8	15
Enjoyment of mathematics:	very much	3	5	7	13
	OK	48	79	44	80
	not at all	10	16	4	7
Mathematics course:	very useful	38	62	26	47
	fairly useful	23	38	28	51
	irrelevant	0	0	1	2
Previous use of computers:					
at college:	regularly	47	77	25	45
	occasionally	11	18	29	53
	not at all	3	5	1	2
at school:	regularly	13	21	7	13
	occasionally	21	35	20	36
	not at all	27	44	28	51
elsewhere:	regularly	17	28	6	11
	occasionally	23	38	24	44
	not at all	21	34	25	45
Written programs?	yes	55	90	54	98
	no	6	10	1	2
Use of computers as aid to learning maths:	very useful	20	33	9	16
	fairly useful	27	44	30	55
	not useful	8	13	12	22
	don't know	6	10	4	7
Enjoyment of use of computers:	very much	19	31	10	18
	OK	32	52	33	60
	not at all	4	7	11	20
	don't know	6	10	1	2
Number of replies		61		55	

Results of NODES questionnaire SQ2, 1987
(All results are given as percentages)

		CEE1	CEE2	combined
Enjoyment of mathematics:	very much	2	17	8.5
	OK	88	76	83
	not at all	10	7	8.5
Enjoyment of computers in mathematics:	very much	6	17	11
	OK	67	71	69
	not at all	27	12	20
Previous use of computer packages for mathematics:	frequently	0	5	2
	occasionally	35	9	24
	not at all	65	86	74
No. of sessions using NODES:	1	4	7	5
	2-5	71	62	67
	>5	25	31	28
Enjoyment of using NODES:	very much	0	14	6.5
	OK	75	76	75.5
	not at all	25	10	18
Is numerical solution of d.e.s a useful topic?	yes	63	81	71
	no	25	7	17
	don't know	12	12	12
Should some computer-based work be assessed?	yes	65	57	61
	no	29	38	33.5
	don't know	6	5	5.5
Preferred mode of working:	supervised sessions	12	21	16
	working in own time	45	29	38
	both equally	43	50	46
Preferred mode of working:	on your own	39	43	41
	in pairs	43	28.5	36
	in a small group	18	28.5	23
Number of replies:		51	42	93
Male		51	41	
Female		0	1	

Results of NODES questionnaire SQN2, 1988
(All results are given as percentages)

		CEE2		
Enjoyment of mathematics:	very much 4	4		
	3	60		
	2	32		
	not at all 1	4		
Does use of laboratory contribute towards your enjoyment of mathematics?	very much 4	0		
	3	26		
	2	58		
	not at all 1	16		
Ratings for aspects of laboratory work: (4 very highly → 1 not at all)		interesting	enjoyable	useful
investigating a model:	4	11	4	15
	3	62	35	45
	2	27	53	38
	1	0	8	2
analysing sensitivity:	4	4	0	7
	3	35	23	38
	2	50	68	49
	1	11	9	6
report writing:	4	2	0	6
	3	10	4	34
	2	46	41	34
	1	42	55	26
No. of sessions using NODES:	1	6		
	2-5	44		
	>5	50		
Does use of laboratory enhance your understanding of any aspect of mathematics?	very much 4	11		
	3	50		
	2	35		
	not at all 1	4		
Should some computer-based work be assessed?	yes	78		
	no	22		
Number of replies:		50		
Male		50		
Female		0		

APPENDIX 10

Staff interviews, 1989

A. Donald

1. Classes involved:

Class	No.of students	Description of use of laboratories
CEE1	45	5 hours mathematics per week 1 introductory laboratory session 1 optional session using MUMATH (computer algebra) 1 EA assignment, based on GRAPH package, to investigate wave-forms and introduce Fourier series
EWM1	30	4 hours mathematics per week 2 supervised sessions 2 assignments (a) investigation of exponential functions, using GRAPH (b) 2-variable linear programming
BSc Building 1	30	1 laboratory tutorial on numerical integration
CEE2		encouraged to use FOURIER series package and SURF (3-D graphs) in own time

2. Teaching approach

He has found it difficult to find time to fit in laboratory sessions and would like to do more, as he feels that the use of computer packages is often a good approach. The CEE1 assignment worked well, being both useful and relevant. He frequently relates work in class to packages available in the laboratories and encourages the students to go and try them out. Students ask more questions both of each other and of lecturers when working in the laboratory. He enjoys teaching more as a result of using computers. With recent improvements in hardware and software it is difficult to see how one could avoid using computers. Other resources used are a video and a guest lecturer.

3. <u>Impact on the mathematics curriculum</u>

The use of computer packages has resulted in a change of emphasis in some areas. He uses computers to reinforce ideas and to explore them further, for example, with numerical integration. More use is made of graphical output and more time spent on experimental and investigative work. However, there is at present insufficient time to fit in laboratory time on a regular basis and syllabi should be redesigned to accommodate this.

4. <u>Assessment</u>

Computer-based assignments have been used as part of overall assessment for 3 years. Style of examination questions has not altered.

5. <u>Future developments</u>

Software and hardware will continue to improve and be more readily available. Syllabi must be changed to include regular time-tabled laboratory work. Computers should be available for use at any time during lectures.

B. **John**

1. **Classes involved**

Class	No.of students	Description of use of laboratories
SIS3	15	2 hours mathematics per week Approximately 6 laboratory tutorials for numerical analysis and modelling. Laboratory lectures using MATHCAD
SIS5	10	2 hours mathematics per week Approximately 8 ad hoc laboratory tutorials at intervals throughout the year
BSc App. Physics 1	12	4 hours mathematics per week plus 2 hours for special entry students Weekly laboratory session for special entry students (but standard entry came too!) Laboratory tutorials for graphical work, trigonometry and matrices
MET2	12	4 hours mathematics per week Used weekly; laboratory tutorials using NODES for numerical analysis; stand-alone course using MUMATH + assignment

2. **Teaching approach**

Use of laboratory is integrated into lecture material - i.e. use packages when that part of syllabus is reached. Packages are used to investigate both methods and models, to enhance understanding and to eliminate tedious arithmetic. He relies heavily on graphical output. Computers are used to do things that cannot easily be done by hand, for example, graph sketching, numerical analysis, modelling. He encourages the students to work in pairs because they learn more that way.

The mathematics students (SIS, MET) are good at techniques and manipulation, but have trouble with concepts. For example, the idea of steady state is difficult to get across analytically, but is obvious when seen graphically on a computer.

He uses questions during laboratory sessions to probe understanding. The questions asked are more demanding conceptually. He often leaves the student to think about it and comes back later. Laboratory sessions mean continual work for the supervising lecturer, usually with one pair of students at a time. Students are more responsive. It is not a suitable environment for whole class discussion. Weaker students sometimes do better in the laboratory - the differences are not so noticeable. The potential is there to provide greater challenge for the more able students, but he hasn't achieved this yet. Use of computers makes learning more self-paced and passes more responsibility to the students. The students want to get stuck into it by themselves.

He feels that with several classes he has 'broken the barrier'. Students are 'quite familiar with the lab and happy to use it'. Physics students use the GRAPH package for physics work during their own time. Other students use packages by themselves and go and view videos a second time in the Open Learning Unit.

His use of the laboratories has gradually increased from one ten-minute classroom demonstration a few years ago to occasional laboratories and now, for most classes, to a regular laboratory session replacing a conventional tutorial as an integrated part of the teaching.

The use of computers has increased his enjoyment of teaching greatly. Certain topics, such as the phase solutions of differential equations, have become alive for him, too, through the new approach.

In the classroom his teaching style is changing to a lecture/tutorial mix using interactive handouts. For these, he uses overhead projector slides. The students get a copy of the notes with some details missing which they fill in as the lecture proceeds. The handout also contains examples for the students to do. He stops frequently to allow them to complete these.

3. Impact on the mathematics curriculum

There has been a considerable impact on the curriculum. In numerical analysis, for example, he no longer teaches 4th-order Runge-Kutta methods by hand but does now include the stability of numerical methods, using NODES to investigate the solution as the step length varies. He has introduced more discrete mathematics, such as non-linear difference equations (leading to chaos). Leslie matrices have been reintroduced in SIS5 because the difficult mathematics can be done by computer. He uses computer algebra packages to do mathematics that they could not otherwise do, thus showing the power of such packages.

In general, syllabi are changing to include topics which lend themselves to computers. Techniques are played down and more time is spent on applications and investigating models involving the use of differential equations. Some topics have been dropped, for example, stochastic processes. Differential equations now form almost two-thirds of the SIS5 course whereas, formerly, they occupied one-third.

4. Assessment

Most laboraotry tutorials are assessed as part of coursework. He feels that students are already overburdened with coursework and report writing so these are usually completed in class time. Some are extended assignments but, again, mostly done in class time. Assignments may include both analytical and numerical work. The exception is SIS5 which has no assessed coursework yet, though some is planned for the session beginning Autumn, 1990.

He has used computer-generated exercises to replace class tests (for statistics and differential equations). Numerical methods are assessed mainly by coursework. Examination questions on numerical methods are concerned with the behaviour of methods, conceptual questions based on work done in the laboratories. He uses printer output from both MINITAB, MUMATH and MATHCAD in examination questions, particularly for SIS5.

He feels that laboratory sessions where the students work through and fill in a worksheet which is then assessed for coursework have been the most successful. They provide good feedback on the student's grasp of concepts and mathematics involved due to the nature of the questions asked. Set assignments don't reveal a student's understanding of mathematics so well.

5. Student attitudes

As a result of using computers to aid their learning, he believes his students enjoy their mathematics course more, are more interested and attentive, more motivated and more mathematically confident. Their ability to communicate improves dramatically. Some students show more desire to experiment - NODES is particularly good for this. The mathematics students tend to ask more questions and discuss more with each other, though at first the questions are concerned with the operation of the computer and package.

6. Future developments

There will be better computer algebra and scratchpad type packages, such as DERIVE and MATHCAD, which could be used to help students understand mathematical techniques. Calculus syllabi must change, particularly for engineers who will use packages extensively.

The laboratory environment could be improved by arranging small groups of desks and computers in U-shapes. This would allow a more relaxed atmosphere and encourage co-operation within a group but insulation from other groups.

C. Tom

1. Classes involved

Class	No.of students	Description of use of laboratories
CEE2	45	1 laboratory session using NODES 1 EA assignment using NODES
CEE3	24	3 hours mathematics per week Laboratory lectures, about 50% of time using a computer-illustrated text for statistics and 50% using NODES for numerical analysis
CEE4 (Hons)	26	2 hours mathematics per week 6 laboratory lectures and tutorials for linear programming 2-3 laboratory tutorials for non-linear programming methods
SIS4	13	3 hours mathematics per week Laboratory tutorials, usually one hour per week, for linear algebra, integer programming and non-linear programming. Uses EIGEN, SURF, GAUSS and LINPROG

2. Teaching approach

Several classes are time-tabled in the laboratory so that the use of computers can be fully integrated into the course, to be used whenever suitable. In practice they are used for about 50% of the class time. Packages are used for graphic illustration, to eliminate tedious arithmetic, enhance understanding, increase motivation, to check results (of algorithms) and for investigative work. For example, by using a 3-D graphical package to draw a surface, students can see the minimum point or saddle point previously located using calculus. This boosts their confidence. The use of computers allows students to solve problems which would take far too long to do otherwise, such as integer programming problems using the branch and bound method. This is a particularly successful use of computers which combines the use of a package (LINPROG) with pen and paper work (to construct a tree diagram). The investigation of many mathematical models would not be possible without computers. NODES has been used very effectively to investigate the behaviour of given models.

He encourages students to work in pairs at the computer. Often one is happy to use the machine whilst the other acts as scribe.

Other resources used include the overhead projector, videos on mathematical modelling and a self-paced learning text to replace conventional notes.

Some of his classes have tutorial time shared with another lecturer. As he is not so free to use this time as he would wish, computer use with these classes replaces lecture time. Other classes are time-tabled wholly in the laboratory and there is no distinction between lectures, tutorials and computer use. Often a class is a mixture of these. The students spend more time working by themselves; there is less formal lecturing. He spends less time than previously teaching techniques, for which they now use the computer, and uses the saved time to extend some topics, for example, linear programming. He asks students more questions in the laboratory and students are more responsive. They spend more time doing interesting things. Investigations are presented in an open-ended way to encourage students to take them as far as they like.

He enjoys teaching more as a result of using computers. There is less drudgery, and it is more interesting. Sometimes, when lecturing, he wishes a computer were available to demonstrate something. When in the laboratory, he spends more time talking to students on an individual basis, which promotes a closer relationship.

3. <u>Impact on the mathematics curriculum</u>

The syllabus has not changed. The same topics are taught but the emphasis within many topics has changed. In many cases, time saved on computation is used to extend the topic. For example, in linear programming, whereas students used to spend a lot of time solving examples by hand, they now solve more problems on the computer and more time is available for post-optimal analysis. In the past, numerical analysis consisted solely of learning techniques. Now there is no need to do lots of examples by hand using methods such as 4th-order Runge-Kutta. The students spend more time on error analysis which is much more easily understood using graphical output from NODES. More time is spent doing experimental and investigative work. The teaching of probability and statistics has also changed.

4. <u>Assessment</u>

30% of overall assessment for CEE classes is now covered by laboratory assignments (Engineering Applications assignment). Starting next session (1989-90), CEE3 will do a formal laboratory assessment under examination conditions.

In examinations, he no longer asks arithmetic-type questions in topics for which computer packages have been used. For example, students would not be asked to solve a linear programming problem by the Simplex method. They might be given a solution and be asked to interpret it or analyse it further. SIS4 still get a data sheet for the branch and bound method to tackle integer programming problems.

5. <u>Student attitudes</u>

A few students do not like using computers but the majority enjoy it and are happy to be doing less tedious computation. Most have a natural instinct to experiment. Their problem solving skills and investigative skills improve, probably because they spend more time talking about the problem and its solution. They do not have to wade through a series of manipulative and computational steps, and thus spend more time thinking about the problem. If their formulation is wrong, it doesn't take too long to reformulate and get a new solution.

There is more discussion amongst students and the questions they ask the lecturer tend to be more complex, often concerning the meaning of a result rather than the techniques used to obtain it.

6. Future developments

There will be more and better software, probably on PC-compatible computers. Some topics must be pruned from the syllabus. More discrete mathematics is required for engineers and less traditional calculus.

There will be a move towards student-centred learning. Chalk and talk for an hour is seldom effective, but students still need formal contact time. Groups of students could work on given materials with staff present to help when requested - not unlike a primary school atmosphere.

D. **Robin**

1. **Classes involved**

Class	No.of students	Description of use of laboratories
MET1	15	2½ hours mathematics per week (avge) Use mainframe for MINITAB tutorials for 50% of tutorial time. Demonstration to whole class then set assignments
MET2	20	3 hours mathematics per week Use mainframe for MINITAB for regression and student t-tests, about 6 hours in total
MET3	20	Stochastic processes
MSc Water Management	10	MINITAB: analysis of real data sets, 6 hours
SIS Hons. Biology		2 hours, mathematical modelling with sets of simultaneous non-linear differential equations

2. **Teaching approach**

Most of his teaching is statistics and he uses MINITAB with most classes. He explains concepts in class, hands out exercises during a tutorial session, then follows this up with a laboratory session. The use of packages saves much routine calculation and reinforces concepts more effectively than chalk and talk. Packages are also useful for illustrating some ideas such as confidence intervals, and to give students experience of setting up and analysing real data sets. Tutorial sessions are split about 50:50 between classroom and laboratory. The latter are less formal. The students normally work singly but interact a lot. He always has asked a lot of questions, but probably even more so in the laboratory.

He feels that a few years ago he tended to use computers too much and too soon, but has achieved a better balance now.

3. <u>Impact on mathematics curriculum</u>

Some statistical topics which used to be taught very theoretically can now be approached in a practical way using real data sets, for example, Box-Jenkins ARIMA models, and confidence intervals. Students use real data sets for project work. For an SIS Honours Biology project the student fitted a model using real data obtained from laboratory experiments.

4. <u>Assessment</u>

More extensive projects are undertaken as a result of using packages. These may involve using several packages and generating numerical solutions.

For computer-based assignments he generates individual data sets. This reduces 'coursework by committee'. Students can discuss the work with each other, but not copy results.

5. <u>Future developments</u>

MINITAB is not necessarily the best package for teaching statistics. There is a need to investigate other ones. There will be more emphasis on experimental data analysis and non-parametric statistics.

It would be nice to have a good classroom layout for lectures with terminals on hand but unobtrusive. Coursework needs updated. He would like notes, exercises and sample output incorporated into single booklets, appropriate to the different classes taught.

E. **Sandra**

1. **Classes involved**

Class	No.of students	Description of use of laboratories
SIS2	30	3 hours mathematics per week Laboratory tutorials using MINITAB
SIS3	10	2 hours mathematics per week Laboratory lectures using LINPROG Set assignments using LINPROG and MINITAB. 10 hours total
BSc Biology	50	2 hours mathematics per week Laboratory tutorials, using MINITAB, integrated into curriculum; 30 hours continuously assessed
BSc Life Sciences	50	Laboratory tutorials using MINITAB, one per week
Quantity Surveyors	50	Laboratory tutorials using MINITAB, two hours per week

2. **Teaching approach**

Computer packages are used to eliminate tedious arithmetic and thus allow the students to understand more and 'do' more. They use data in context ('real' data) and spend more time on interpretation. Familiarisation with packages can also be important for their future careers. She encourages students to co-operate but prefers them to work at their own terminal where possible. Other resources used include overhead projector slides (with overlaps) and MINITAB workbooks which she has written. These booklets are also used by other members of staff.

Lectures are less formal than they used to be. She uses the MINITAB workbooks and hopes that the students will have previously looked at the notes. The use of computers is integrated into the curriculum. In the laboratory, she asks students more questions, which tend to be more demanding and take longer to respond to. There is more support for weaker students as the lecturer can keep prodding the student to interpret the results. This is particularly true in BSc Biology which has continuous assessment. The minority who have a mental block against computers do not necessarily benefit. There is more challenge for more able students, as they can experiment more and discover more possibilities than with pencil and paper.

Computer-based learning is more student-centred. During laboratory sessions, students work through MINITAB booklets at their own pace and the computer package offers them more freedom to explore a topic in different ways, to choose from a variety of outputs or to seek additional help when necessary. She uses computers with most classes and feels hampered when not able to do so. Teaching with the aid of computers is more interesting and enjoyable; it generates interesting discussions with students on points which would not have come up otherwise. The use of packages has benefitted her own understanding of the practical side of handling data. This cannot be gained from theoretical study.

3. Impact on mathematics curriculum

The use of computer packages has led her to a completely new approach to teaching statistics, i.e. starting from data rather than starting from theory. The balance of importance between topics has changed. There is more emphasis on regression and hypothesis testing. Students used to spend time working out means, standard deviations and frequency tables. Now the emphasis is more on understanding. Within topics the balance has shifted from calculation to interpretation.

There is more experimental and investigative work - most statistics comes into this category. Hopefully, students get a better understanding by doing investigations.

Project work often involves collecting data from the student's own discipline (for example, biology, chemistry, business studies) and using statistical packages to analyse it.

4. Assessment

Two courses have continuous assessment which is mainly computer-based. In other courses, the coursework component is usually computer-based, for example, SIS3 has one assignment using LINPROG and one using MINITAB.

The types of question asked in examinations have changed. She would no longer ask a student to calculate a mean and standard deviation or carry out a t-test. The question would either include all or part of the answer, or computer print-out is given and the student is required to pick out the solution and interpret it. For example, several tableaux of linear programming using the Simplex method might be given.

5. Student attitudes

Some enjoy their mathematics course more as a result of using computers, some less. Students tend to ask more questions, some of which are very basic ('How do you log in?'), and some which are more to the point - they are not sidetracked by the arithmetic. They are more likely to work co-operatively and discuss work amongst themselves.

6. Future developments

New, better packages will become available on PC-compatible computers. The use of packages will be written into statistics curricula.

Classes are moving away from traditional lecturing and assessment methods towards a more student-centred approach.

APPENDIX 11

Student interviews, 1988

Summary of student interviews with CEE2, March 1988

Student A

This student enjoys mathematics and finds it interesting. The computer assignment was reasonably straightforward. It was very interesting to obtain a visual representation of the solution of a model and to observe the effect on the solution of changing the parameters of the model. He did not consider the subject matter relevant to the course since they had not spent much time on differential equations during the session and had not previously learnt about phase-plane trajectories. He did not appreciate that there was any wider benefit to be gained other than learning about the Van der Pol oscillator. However, he did feel that observed effects such as transient solutions, limit cycles and damping had a wider relevance.

The student's previous experience of using NODES the previous year enabled him to get started on the project quickly, but he spent a long time on it and produced a great many hard-copy graphs, many of which showed only small changes in the solution. When he came to write his report he could only use some of the graphs and found he ran out of things to say about slightly different solutions. He does not "really know what they're looking for in a write-up", i.e. a description of what happened in the package or a write-up based on knowledge gleaned from books. Nevertheless, he reckons he has developed a good style of report writing.

He usually works alone in the laboratory but collaborated with a friend to produce graphs more quickly as that process is time-consuming and tedious. He found that varying one parameter at a time and superimposing many graphs aided his understanding of the model. The slowness of the package when calculating results was frustrating.

Student B

Student B doesn't like computers! He feels disadvantaged compared to some of his fellow students since he had no experience at school and did not enjoy or cope well with the PASCAL programming course in first year. He does not consider computing studies to be as important as 'examinable' subjects, therefore does not devote much time to them. He cannot see any benefit in using computer packages in the mathematics laboratory. The assignment was neither interesting nor relevant to the course. Nothing similar was done in class. The first year differential equations project was a bit more relevant.

This student does not feel he has gained anything from the use of computers this year but thinks he will do so in later years of his course. He works on his own in the laboratories and elsewhere as he prefers to stick to his own ideas. He spent the minimum amount of time in the mathematics laboratory doing his assignment but quite a long time doing background reading and the writing-up. "All I want out of it is a good mark". He thinks that the majority of his class would agree that they just want a good mark and are not really interested in the assignment. It would have to play a much greater role in the overall assessment before he took any more interest in it, for example, if there had been a question on Van der Pol oscillators in the final examination.

Student C

This student considers his course to be a good one and enjoyed this year's mathematics laboratory assignment. He found it particularly interesting to investigate the effect on the model solution of varying single coefficients in a differential equation. He considers NODES to be "a good visual aid to understanding differential equations" and that its use has enhanced his general background understanding of differential equations and what they represent. He also appreciates that differential equations play an important role in electrical work.

He finds the report-writing stage of an assignment important as it forces him to draw conclusions from his laboratory work (i.e. pile of graphs), and thus builds up a complete picture of the model. He believes that the standard of his report-writing has improved since last year. This is due both to practice and greater knowledge enabling him to describe the behaviour of the model in more technical terms and to relate computer results back to the physical model.

Investigative assignments are designed to "build up an overall picture of what is going on in a field rather than just the theory that is thrown at you for exams", and thus he would not expect the model chosen for investigation necessarily to have been covered in classwork. There is "no point in knowing all the theory and how to solve all the equations if you've never come across practical applications".

APPENDIX 12

Results of attitude surveys, 1989

LINPROG : Attitude Survey Data

MTB > print c1-c9

ROW	C1	C2	C3	C4	C5	C6	C7	C8	C9
1	3	4	3	3	4	3	4	4	4
2	3	4	3	3	4	3	3	4	4
3	4	4	3	2	4	4	4	4	4
4	4	4	4	3	4	4	3	4	3
5	4	5	3	3	4	3	2	3	3
6	4	4	4	4	4	4	4	4	4
7	4	5	3	5	2	4	3	4	5
8	4	4	5	4	4	4	5	4	4
9	5	3	4	4	2	5	4	3	4
10	4	4	2	2	4	3	2	5	4
11	3	4	4	4	2	2	2	2	4
12	4	4	4	4	4	2	2	3	4
13	4	4	4	4	3	3	4	4	4
14	4	5	4	4	3	4	3	4	4
15	4	5	4	4	1	4	1	4	4
16	5	4	2	1	4	4	2	4	2
17	4	4	4	2	2	4	2	5	4
18	4	5	5	4	4	2	1	2	3
19	3	4	3	3	2	3	2	3	3
20	4	4	4	4	4	•	2	4	4
21	4	4	3	3	4	3	2	3	3
22	4	3	3	3	2	4	1	4	3
23	4	5	3	3	3	4	4	4	4
24	3	5	2	2	2	5	1	4	3
25	3	5	1	1	4	4	2	4	4
26	4	4	4	3	4	3	4	2	3
27	3	5	2	4	2	2	1	2	4
28	4	4	4	4	3	3	3	4	4
29	4	2	3	3	2	4	1	4	3
30	4	4	4	4	3	3	3	4	3
31	4	4	3	3	5	3	•	2	4
32	4	5	2	2	4	4	3	5	2
33	4	4	2	2	4	4	2	4	4
34	4	4	3	2	4	2	2	3	2
35	3	4	2	2	4	3	2	3	3

MTB > print c10-c18

ROW	C10	C11	C12	C13	C14	C15	C16	C17	C18
1	4	4	4	3	4	4	4	4	3
2	4	4	4	4	4	4	4	3	3
3	4	4	4	4	5	4	4	2	4
4	4	4	4	4	4	4	4	3	4
5	5	5	5	5	4	3	4	3	4
6	4	4	4	4	4	3	4	2	4
7	5	4	5	5	4	5	5	4	4
8	4	4	4	5	4	4	5	2	3
9	4	5	5	2	5	3	5	2	5
10	2	3	4	4	5	1	4	4	2
11	4	5	4	5	3	2	4	2	4
12	4	4	4	4	3	2	5	4	4
13	4	3	4	4	•	•	•	•	•
14	5	5	4	5	3	1	3	4	2
15	5	4	4	1	4	1	4	4	4
16	5	2	4	1	4	1	4	2	2
17	4	2	4	2	•	•	•	•	•
18	5	3	4	5	•	•	•	•	•
19	4	3	3	2	3	3	3	4	4
20	4	4	4	2	4	4	4	2	1
21	3	3	4	3	4	3	4	4	4
22	4	3	4	2	5	4	4	3	4
23	5	4	4	4	•	•	•	•	•
24	4	2	2	4	3	3	4	4	3
25	5	3	4	3	3	1	4	2	1
26	4	4	3	4	3	1	4	4	4
27	5	3	3	4	4	1	4	4	3
28	5	3	4	4	•	•	•	•	•
29	4	3	2	2	5	2	3	4	4
30	2	4	4	3	4	2	4	3	3
31	5	2	4	4	3	1	4	4	3
32	3	5	4	5	3	2	4	5	3
33	4	3	4	4	2	2	4	2	4
34	4	2	4	2	4	2	4	2	4
35	5	3	2	2	3	1	3	3	4

* missing data items

LINPROG

Total scores for individual statements

Statement	Total score (column total)
1	134
2	146
3	113
4	108
5	115
6	116*
7	86*
8	126
9	124
10	146
11	123
12	134
13	121
14	113**
15	74**
16	120**
17	95**
18	101**

Note: * 1 missing data item
** 5 missing data items

LINPROG : Principal Component Analysis

```
MTB > pca c1-c18
```

Eigenanalysis of the Correlation Matrix

28 cases used   7 cases contain missing values

Eigenvalue	4.1938	2.6153	2.4441	1.6606	1.4534	1.1338
Proportion	0.233	0.145	0.136	0.092	0.081	0.063
Cumulative	0.233	0.378	0.514	0.606	0.687	0.750
Eigenvalue	1.0260	0.8132	0.5981	0.4817	0.4137	0.3489
Proportion	0.057	0.045	0.033	0.027	0.023	0.019
Cumulative	0.807	0.852	0.885	0.912	0.935	0.955
Eigenvalue	0.3054	0.2031	0.1545	0.0846	0.0556	0.0142
Proportion	0.017	0.011	0.009	0.005	0.003	0.001
Cumulative	0.972	0.983	0.991	0.996	0.999	1.000

Variable	PC1	PC2	PC3	PC4	PC5	PC6
C1	0.187	0.394	-0.029	0.027	0.118	-0.539
C2	-0.038	-0.405	-0.307	-0.177	0.140	-0.223
C3	0.363	-0.012	0.226	0.148	-0.111	-0.127
C4	0.348	-0.226	0.291	-0.134	-0.108	-0.101
C5	-0.011	0.082	-0.428	0.489	-0.037	0.101
C6	0.064	0.277	-0.150	-0.486	0.104	0.029
C7	0.329	0.053	-0.238	0.222	-0.061	0.152
C8	-0.003	0.270	-0.376	-0.386	-0.255	0.011
C9	0.253	-0.243	-0.026	-0.282	0.137	0.263
C10	-0.048	-0.200	0.088	-0.189	0.615	0.026
C11	0.350	-0.201	-0.047	0.013	-0.132	-0.230
C12	0.335	0.057	-0.219	0.006	0.221	-0.257
C13	0.210	-0.341	-0.289	0.049	-0.202	0.110
C14	0.146	0.384	0.140	-0.162	-0.096	0.061
C15	0.284	0.112	-0.011	-0.138	-0.079	0.566
C16	0.323	0.040	-0.123	-0.004	0.205	-0.036
C17	-0.091	-0.212	0.081	-0.263	-0.547	-0.248
C18	0.197	0.074	0.426	0.149	0.018	0.104

NODES : Attitude Survey Data

ROW	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
1	4	4	4	3	4	4	4	3	4	4
2	4	5	4	4	4	2	3	2	4	4
3	4	4	4	5	4	3	4	3	4	5
4	4	4	3	3	4	3	4	2	3	2
5	4	4	3	3	4	3	4	2	3	4
6	4	4	4	3	4	2	4	2	3	4
7	4	4	4	3	4	2	4	2	4	5
8	3	4	3	2	4	5	3	2	4	4
9	4	4	4	3	4	4	3	2	4	5
10	4	5	5	3	5	3	4	3	4	4
11	4	4	3	3	4	2	4	2	4	4
12	2	2	1	3	4	1	2	1	2	4
13	4	4	4	3	4	4	4	4	4	4
14	4	2	3	2	3	4	4	2	4	3
15	4	4	4	3	4	*	3	4	4	5
16	3	4	4	3	4	3	3	2	2	3
17	4	4	3	3	3	2	3	2	4	4
18	5	5	5	3	4	4	5	2	5	5
19	4	3	4	3	3	3	3	4	3	5
20	5	5	4	3	3	2	4	2	5	5
21	4	4	2	3	3	2	5	2	4	5
22	4	3	3	3	3	4	4	*	4	4
23	1	4	3	4	3	5	3	1	5	5
24	4	4	2	3	3	4	4	4	4	4
25	4	2	4	3	3	3	4	2	4	4

ROW	C11	C12	C13	C14	C15	C16	C17	C18	C19	C20
1	4	4	4	4	4	4	4	4	2	4
2	4	3	2	4	4	2	4	3	4	4
3	5	4	4	5	5	5	5	5	4	4
4	4	3	3	3	4	3	4	4	2	4
5	4	3	3	4	4	2	3	4	2	4
6	4	4	4	4	4	4	4	2	3	4
7	3	4	4	4	4	4	4	4	3	4
8	4	3	4	4	4	3	4	4	2	3
9	5	4	4	5	5	4	5	4	4	2
10	5	4	4	4	5	5	5	4	3	3
11	4	4	4	4	2	1	2	4	2	4
12	4	2	2	2	2	1	1	2	4	4
13	2	4	4	4	2	2	4	4	4	2
14	4	3	4	4	3	2	4	4	3	4
15	4	3	4	4	3	1	3	4	3	4
16	2	4	4	4	2	2	4	4	2	4
17	3	4	4	4	2	3	5	5	4	4
18	5	5	5	5	5	5	5	5	5	4
19	4	4	4	4	3	2	4	4	4	4
20	4	4	4	4	4	4	4	4	4	4
21	5	4	4	4	2	4	4	4	4	5
22	3	4	4	4	4	3	3	4	3	4
23	5	5	4	5	5	2	5	5	5	2
24	4	4	4	5	4	2	4	4	2	4
25	4	4	4	3	2	1	4	5	4	4

NODES

Total scores for individual statements

Statement	Total score (column total)
1	95
2	96
3	87
4	77
5	92
6	74*
7	92
8	57*
9	95
10	105
11	99
12	94
13	95
14	101
15	88
16	71
17	98
18	100
19	82
20	93

Note: * 1 missing data item

NODES : Principal Component Analysis

```
MTB > pca c1-c20
```

Eigenanalysis of the Correlation Matrix

23 cases used   2 cases contain missing values

Eigenvalue	6.6892	2.5284	2.3004	2.0603	1.2942	0.9339
Proportion	0.334	0.126	0.115	0.103	0.065	0.047
Cumulative	0.334	0.461	0.576	0.679	0.744	0.790
Eigenvalue	0.8234	0.7209	0.6333	0.5458	0.3415	0.3262
Proportion	0.041	0.036	0.032	0.027	0.017	0.016
Cumulative	0.831	0.868	0.899	0.926	0.944	0.960
Eigenvalue	0.2025	0.1842	0.1482	0.1016	0.0790	0.0514
Proportion	0.010	0.009	0.007	0.005	0.004	0.003
Cumulative	0.970	0.979	0.987	0.992	0.996	0.998
Eigenvalue	0.0245	0.0112				
Proportion	0.001	0.001				
Cumulative	0.999	1.000				

Variable	PC1	PC2	PC3	PC4	PC5	PC6
C1	-0.137	-0.481	-0.070	-0.201	-0.035	-0.020
C2	-0.228	-0.180	0.297	0.033	-0.110	0.040
C3	-0.251	-0.205	0.111	0.128	-0.316	-0.318
C4	-0.117	0.198	0.284	-0.205	-0.354	0.472
C5	0.003	-0.229	0.470	0.274	-0.073	-0.175
C6	-0.187	0.164	-0.171	0.468	0.267	0.115
C7	-0.197	-0.353	-0.145	-0.204	0.244	0.028
C8	-0.107	-0.248	-0.166	0.166	-0.303	0.537
C9	-0.296	0.114	-0.101	-0.116	0.170	-0.107
C10	-0.213	0.224	0.059	-0.319	-0.145	-0.025
C11	-0.157	0.173	0.232	-0.234	0.513	0.122
C12	-0.317	0.055	-0.172	-0.045	-0.197	-0.078
C13	-0.280	-0.097	-0.327	0.058	0.043	-0.210
C14	-0.322	0.072	-0.016	0.094	0.017	0.306
C15	-0.247	0.053	0.376	0.129	0.279	0.136
C16	-0.267	-0.204	0.263	-0.098	0.115	-0.155
C17	-0.322	0.028	-0.026	0.113	-0.064	-0.034
C18	-0.244	0.099	-0.304	0.023	0.009	0.071
C19	-0.140	0.356	0.018	-0.326	-0.258	-0.283
C20	0.115	-0.310	-0.081	-0.455	0.134	0.203

APPENDIX 13

Results of questionnaire SQL1

Results of questionnaire SQL1, Dundee, 1988

Enjoyment of mathematics:	very much	4	0
		3	8
		2	3
	not at all	1	0
Enjoyment of computers in mathematics:	very much	4	0
		3	6
		2	5
	not at all	1	0
Previous use of computer packages for mathematics:	frequently		2
	occasionally		9
	not at all		0
No. of sessions using LINPROG:	1		2
	2-5		9
	>5		0
Enjoyment of using LINPROG:	very much	4	0
		3	9
		2	2
	not at all	1	0
Should some computer-based work be assessed?	yes		1
	no		9
Do you prefer:	supervised sessions		6
	working in own time		1
	both equally		3
Do you prefer working:	on your own		1
	in pairs		3
	in a small group		5
	no preference		1
Number of replies:			11
Male			5
Female			6

APPENDIX 14

Publications

D. Mackie

EXPLORING LINEAR PROGRAMMING WITH A BBC MICRO

Background

The Linear Programming Learning package which I shall desribe and demonstrate is one of a number which have been developed at Napier College as part of a research project in the field of computer-based mathematical education. It is intended for use by students on a variety of degree and diploma courses in which linear programming and/or integer programming is studied.

Early versions of the package were used with students last session with encouraging results. This latest version will be used more widely during this session and an evaluation exercise will be carried out.

Aims

It is NOT a self-learning package. The user is assumed to have been introduced to linear programming in his course before using the package.

The aims of the package are:

(i) to enhance student understanding of linear programming and, in particular, the Simplex Method;

(ii) to facilitate problem solving which involves the use of linear programming;

(iii) to facilitate the solving of integer programming problems by the branch and bound method;

(iv) to encourage investigative work.

In more general terms, computer-based packages of this nature can be used to increase student interest and generate discussion. Questions of the 'What happens if ...?' sort are encouraged because the result can usually be readily demonstrated.

Design Philosophy

1. The user must feel that he is in charge of the situation and choosing which path to follow, or which facilities to use. The programs are thus menu-driven.

2. Input of data should be straightforward - in as familiar a format as possible - with an opportunity to correct mistakes.

3. There is never too much text on the screen at one time - this is discouraging - but, rather, the different options are described briefly and a help facility is included to provide more detail when necessary for the inexperienced user.

4. There is an explanation of user errors whenever possible.

5. The package enables different levels of use.

6. To accompany the computer programs we have prepared worksheets for the students. The material included in the worksheets ranges from straightforward exercises through realistic problems to opportunities for investigative work, e.g. a sensitivity analysis.

7. Above all, the package must be ROBUST and RELIABLE.

Facilities

The package copes with three main classes of problem:

(a) The restricted class of linear programming problems where one is required to

either Maximise $f(\underline{x})$
subject to $g_i(\underline{x}) \leq b_i$

or Minimise $f(\underline{x})$
subject to $f_i(\underline{x}) \geq b_i$

There are many realistic problems of this nature. For these problems the user can follow a tableau by tableau display through the Simplex Method. The user is asked to select the pivot element, to decide when the solution is optimal and to extract the solution from the final tableau.

Minimisation problems are solved by first forming the dual problem. The algorithm used by the program follows that taught to our students.

(b) General linear programming problems where any combination of constraints is allowed.

For these problems, the program uses a BIG-M method but only the solution is given to the user.

(c) Integer and mixed integer programming problems - the initial problem is solved as for a general linear programming problem. The user can then add further constraints and solve the new problem thus formed or return to the initial set of constraints. Thus the branch and bound method can be implemented with ease.

I shall demonstrate the program with some examples selected from the worksheets.

Computer-based Mathematics in Further Education

D. Mackie

1. Background

The mathematics department at Napier College has been involved in innovative work with computers for several years now. The main usage has gradually shifted from main-frame to microcomputers, firstly PET and APPLE micros, and, now, BBC micros.

In September, 1984, our first mathematics laboratory was installed. This contained a network of 10 BBC micros linked to a 30 megabyte Winchester disk and a lineprinter. Later a graph plotter was added.

This year we have purchased 20 BBC Masters and a second larger laboratory will soon be in operation. This second laboratory is necessary to cope with larger classes and multi-site operation. We also have several stand-alone micros which are available to staff only for classroom use and software development.

The laboratory is used by many different classes, from 1st to final year students studying a wide range of diploma and degree courses. It is also available to students on an open access basis when not being used by a class.

The purpose of the mathematics laboratory is not to teach students programming but to assist in the learning of mathematics. To this end a variety of software is provided, the software being stored on the large central disc. This includes many programs developed in-house as well as some commercial packages and programs obtained from other academic institutions. Topics covered include graph plotting, various numerical methods, linear programming, queue simulation and statistics.

2. Who, how, why ?

So much for the set up. Who uses it and why?

Any lecturer may choose to use the laboratory with a class or to set work to be completed in the student's own time. A class held in the mathematics laboratory normally replaces a convential tutorial session at which the student has a sheet of problems to work through. In the laboratory he is given a worksheet related to the topic being studied and requiring the use of a particular program or programs.

Alternatively, the lecturer may take a stand-alone machine into the classroom for a demonstration during a lecture.

It is not always appropriate to use a computer. Many different teaching aids and media are available to teachers today and the computer should only be used if it has been selected as the best method for the teaching purpose intended. To justify its use, it is essential that a computer-based lesson should have clearly defined objectives.

The most frequent objectives are

a) to enhance student understanding of an algorithm or method
b) to solve more realistic problems
c) to carry out investigations or experiments
d) to obtain a graphical solution to a problem
e) to carry out simulations.

Having formulated an objective, the extent to which we can achieve it depends on two things:-

1. the design of the available software
2. the way in which we use it.

3. Software design considerations

Consider, first, the qualities required of software designed for laboratory use :-

It must be reliable and easy to use. The user must be confident in his or her use of the package in order to concentrate on the mathematical aspects of the topic being studied.

The screen layout is important. It should encourage easy assimilation of the information it conveys. It must be as concise as possible whilst leaving the user in no doubt as to range of options open to him at that stage. Colour can be used to enhance the screen presentation and highlight particular words or figures.

There should be good use of graphics - a pictorial representation of a problem or a result can greatly improve a student's understanding of that problem or solution.

The program must be flexible, with the ability to vary paramters, to re-run, and, where appropriate, to offer a choice of outputs. This is essential for investigative work.

The program should be interactive - that does not mean the 'push-space-bar' syndrome but genuine interaction with the user which requires him to make decisions.

Different modes of use of a program should be offered where possible eg step by step through a method, or , solution only - the former is usually an aid to understanding whilst the latter aids investigations.

All these design features will be illustrated by the program NODES, which is currently under development at Napier College and demonstrated here.

This program solves ordinary differential equations by a choice of Runge-Kutta methods or a predictor-corrector method. The method and step size can be varied and the program re-run to compare 2 or more solutions. Output is graphical or tabular and, if the analytic solution is known, it, too, can be plotted to be compared with the numerical solution. Control of the program is by menu making it very easy to change parameters of the original problem and re-run it.

NODES greatly facilities problem solving and investigations and can enhance a student's understanding of the methods used.

The design of worksheets which accompany computer programs is also important. Again, the objectives of each worksheet should be clearly thought out and the content constructed towards achieving them.

Given appropriate software and worksheets, the hand's on laboratory approach allows the students to solve problems, to experiment, to discover and to deduce results. One of the advantages of a computer over a teacher is that the student is not embarrassed by his mistakes and is therefore more likely to persist in his attempts to find a solution. The laboratory environment encourages the development of an investigative approach towards mathematics.

Packages can be used for both classroom demonstrations and laboratory sessions with just slight modification or change of emphasis. For a classroom demonstrations, it is preferable if the package allows pre-prepared data from a disc file - for example, a matrix - to be used to avoid lengthy input during a lesson. It is important to have tried out examples beforehand and chosen suitable input data values eg a starting point for finding a root by Newton-Raphson iteration. The package must also allow the teacher to step through the output (particularly tables of results) at his own pace so that he may stop and explain interesting features when he wishes.

In general, the program designer must use the computer to do what it is good at but leave the teacher the freedom to direct it. Teachers are superior to machines when it comes to presenting the overall picture, promoting discussion and drawing conclusions. The teacher should, therefore, be in sufficient control to make the mathematical points he wishes to, when he wishes, using the functions and parameters he considers appropriate.

4. Evaluation

Evaluation of the impact of the use of computers in mathematics is currently being undertaken at Napier College. Results so far indicate that most students enjoy using computers in Mathematics. (93% of sample) . The proportion of students who considered that the use of computer packages can greatly assist in the learning of mathematics (highest rating) increased from 43% to 57% during the session, and the proportion who considered they were of no use (lowest rating) decreased from 14% to 4%. During interviews with individual students, the main advantages offered were improving understanding of particular algorithms or methods, saving tedious calculations, the opportunity to experiment and obtaining graphic output. It is satisfying to note that these relate closely to the intended objectives.

Use of the laboratory within the department is increasing, as shown by the table below:-

Type of use	No. of staff 1984/85	1985/86
for a formal class meeting	7	11
set assignment to be completed in students own time	3	6
used for 1 or more hours per week on average	3	5

The linear programming package [1] was particularly popular with students as, by removing the tedious arithmetic but not the decisions, it aided understanding of the methods involved. Staff found that, by using this package, their students could tackle post-optimal analysis and integer programming problems which were not previously possible.

5. Conclusions

The use of a computer can greatly enhance a students mathematical experience.

To maximise that experience it is essential to use appropriate, well-designed software.

I believe that the impact on the students learning of mathematics will be determined by the quality of the software and the way in which we use it.

Reference

1. "Exploring Linear Programming with a BBC micro", Proceedings of "Mathematics Teaching 85" conference, University of Edinburgh, September 1985.

Diana Mackie

Diana Mackie

Computer-based investigations in mathematics

Introduction

The desirability of including investigative and experimental work in the mathematics curriculum has been accepted for some time now [1]. Investigative work in groups assists the development of teamwork and cooperation between students whilst also encouraging initiative and application of a broad range of skills. Recent changes in school syllabi and examinations reflect this new approach, as, for example, in the new Scottish standard grade, and appropriate course material is becoming more widely available within the school sector.

However the need for investigative work applies at all levels of mathematical education, including tertiary level. The demands made on mathematicians and engineers working in industry are such that they must be able to apply their mathematics to a wide range of problems in the real world, interpret the results of their work and present them in a form that is understandable to non-mathematical colleagues. Despite restrictions of timetable, class size or time, it is important to include problem solving, experimental and investigative work in the mathematics curriculum of higher education courses.

Use of computer packages can greatly extend the nature and scope of such work. This papers offers a guide to choosing software suitable for student investigations. The use of one particular package, the assessment of the students performance and their reactions to it are described. The impact on the mathematics curriculum is also discussed.

Which computer package?

The most immediate advantage of using computers for student investigations is the enormous increase in the range and relevance of the problems that can be tackled, due to the elimination of tedious and often lengthy calculations. However, repeatibility, that is the ability to vary the parameters of a problem and observe the effect on the solution when the calculations are repeated, is equally important. It is by experimenting that the student develops a feel for the properties of the system being studied and the physical significance of the results [2]. Each student can work at his or her own pace and meet each new step in the investigation as and when ready. Yet another benefit arises if alternative outputs can be obtained from the same problem, eg graphs, tables of results, a statistical chart etc.

Computer-based mathematics packages may be designed with one or more of a number of educational objectives in mind. The most common ones are demonstration/ electronic blackboard use, drill and practice, computer-aided instruction, simulation, investigations and modelling. In fact, a list of objectives should be the starting point of the design of any software unit [3]. Not all mathematics packages, therefore, are suitable for student investigations, and use of an inappropriate one is at best frustrating and inefficient but may even result in loss of motivation and failure to complete the investigation.

Some basic requirements for computer programs suitable for investigative work are listed below. They must be

1. reliable and easy to use

   Any educational software unit must be user-friendly and robust to enable the user to gain confidence in its use.

2. flexible

   The ability to experiment, ie, to modify the original input data, initial conditions or any parameter interactively and re-run the program is absolutely crucial.

3. user-controlled

   A menu-driven program can provide a flexible route through a program offering a variety of methods, where applicable, and a choice of outputs. The user is thus in control of the situation and can choose which program path to follow.

4. able to provide graphical output wherever possible.

The package NODES, developed at Napier College, has been specifically designed to facilitate investigative work and problem solving. The package solves single or systems of first order differential equations numerically. It satisifies all the criteria mentioned above and, in particular, its highly flexible and interactive structure makes it easy to alter problem parameters and examine the effect on the solution either graphically or numerically.

Student Investigations

Mathematics students at Napier College are introduced to computer-based investigations in the Mathematical Sciences Laboratory, a networked system of BBC micros. A wide range of software is available, backed up by carefully constructed worksheets. The laboratory is used for both supervised tutorial sessions and unsupervised open access work by students.

The NODES package has been used by first and second year students on the Communications and Electrical Engineering (CEE) degree course at Napier to carry out investigative assignments in connection with the engineering applications content of their mathematics course. A typical assignment has involved constructing a methematical model of an electrical or mechanical system, using NODES to analyse the behaviour of the model and to investigate the effect on the solution of varying the parameters of the model, and, finally, presenting a written report of the results.

One investigation designed for use with NODES directs the student to model the Van der Pol oscillator and analyse its behaviour by constructing phase portraits. For this investigation the students are given the non-linear Van der Pol equation

$$y'' + \mu y'(y^2-1) + y = 0 \quad \ldots\ldots\ldots\ldots\ldots(1)$$

where $\mu > 0$.

Letting $y_1 = y$ and $y_2 = y'$, equation (1) becomes

$$y_1' = y_2$$
$$y_2' = \mu y_2(1-y_1^2) - y_1$$

This pair of first order equations is the input to NODES.

For $\mu = 0.75$, figure 1 shows solutions for 2 sets of initial conditions

(i) $y(0) = 0,\ y'(0) = 1$

(ii) $y(0) = 3,\ y'(0) = 3$

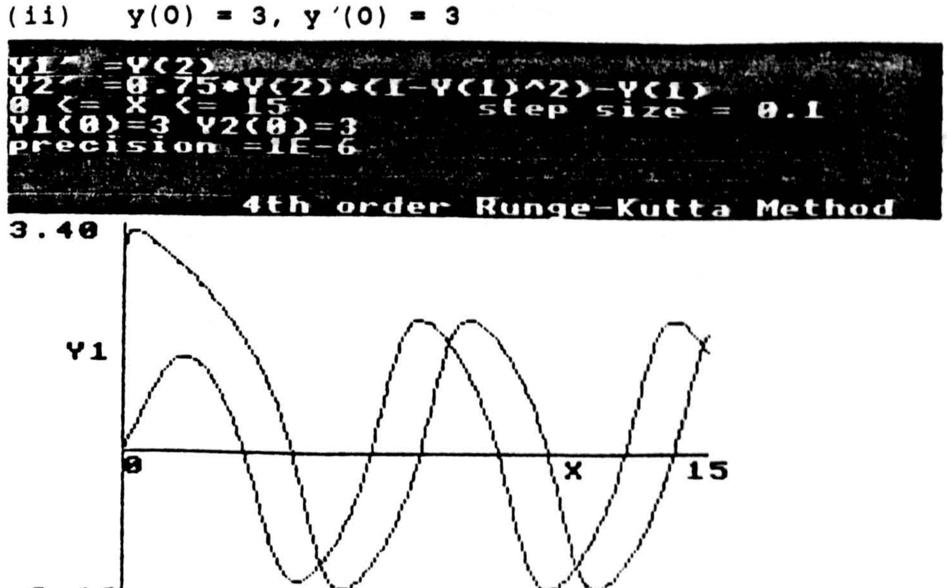


Figure 1

Figure 2 shows the phase-plane trajectory, ie y plotted against $\dot{y}$, for the first of these solutions. If the phase plot of the second solution is superimposed on the first they are seen to tend towards the same limit cycle as shown in figure 3. Phase plane trajectories are not covered in the teaching, prior to doing the exercise . The students 'discover' them in the course of working through the assignment aided by a carefully structured worksheet.

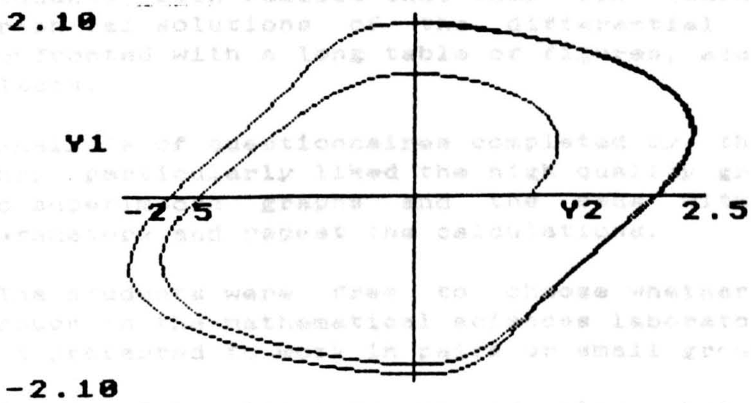


figure 2

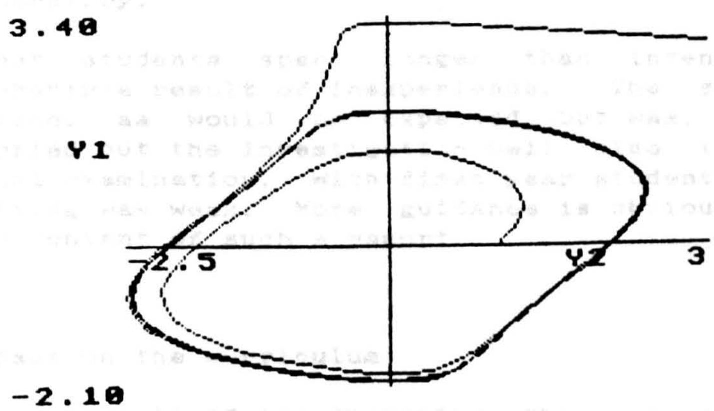


figure 3

Although students would be familiar with the formulation and analytical solution of differential equations, it is not necessary for them to have studied numerical solutions of differential equations before using NODES.

Student reaction and assessment of performance

A survey of two classes which used the package during the 1986-87 session has shown that, in spite of teething problems with hard-copy output from the program, 81% of the students enjoyed using the package and 72% considered the investigations to be a useful part of their course. The fact that, by using a computer, they are able to solve and analyse realistic problems makes the course seem more relevant. The students soon realise that they can learn more from studying the graphical solutions of the differential equations than from being confronted with a long table of figures, accurate to several decimal places.

Analysis of questionnaires completed by the students revealed that they particularly liked the high quality graphical output, the ability to superimpose graphs and the ease with which they could modify parameters and repeat the calculations.

The students were free to choose whether to work on their own or in groups in the mathematical sciences laboratory. The survey found that 57% preferred to work in pairs or small groups.

Some useful criticisms emerged, also, but these were mainly concerned with the restrictions in size of and access to the mathematics laboratory.

Most students spent longer than intended on the investigation, probably a result of inexperience. The standard of work submitted varied, as would be expected, but was, in general, good. Those who carried out the investigation well also tended to score well in the final examination. With first year students, in particular, the report writing was weak. More guidance is obviously needed as to the length and content of such a report.

Impact on the curriculum

As a result of incorporating the use of computer packages into the students mathematics curriculum, they are given access to a wide range of problems that are modelled mathematically. They are able to carry out investigations of important systems which they would have been unable to tackle otherwise, either because they are too difficult or cannot be solved analytically.

For instance, second year CEE students carried out a phase-plane analysis of the Van-der-Pol oscilletor. Work such as this is considered as a structured introduction to modelling. Without a computer package such as NODES, obtaining even a single phase-plane trajectory is a lengthy and complicated task. The idea of then constructing a phase portrait and analysing the sensitivity of the solution would be unthinkable.

The interactive nature of the software encourages the students to experiment. The graphical output and, in particular, the ability to superimpose graphs, gives students a qualitative feel for the model being studied and this leads to a greater appreciation and understanding of its behaviour.

The requirement for the student to submit a written report of his investigation is considered to be an important component of the exercise. Although collaboration is allowed, and indeed encouraged, when using the computer for the actual investigation, each student is required to produce his own report. Copying is easily detected.

The majority of students chose to work in small groups, giving them valuable experience of cooperative effort and team working techniques.

Conclusions

The laboratory environment encourages the development of an investigative approach towards mathematics. Given appropriate software and structured worksheets, the students are able to concentrate on the mathematical model, to experiment, to discover and to deduce results [4]. Students enjoy the laboratory work which thus leads to increased motivation.

The computer packages used are not specifically designed to teach any new material but to assist in the application of existing knowledge. Understanding in mathematics implies an ability to make use of a concept in a variety of settings [1]. Use of computer software as a teaching aid for investigative work and realistic problem solving can enhance student understanding of the underlying mathematical concepts.

References

1. Cockroft, W.H, Report of the Committee of Inquiry into the teaching of Mathematics in Schools, London, HMSO, 1982.

2. Harding, R.D., Int. J. Math. Educ. Sci. Tachnol., Vol 5, 447.

3. Bajpai, A.C., Fairley, J.A., Harrison, M.C., Mustoe, L.R., Walker, D., and Whitfield, A.H., 1985, Int. J. Math. Educ. Sci. Technol.,Vol 16, 407.

4. Mackie, D.M., Scott, T.D., Int. J. Math. Educ. Sci. Technol., to be published in 1987.

Finding the Unstable Orbit - a tertiary level investigation

D. Mackie

Introduction

The desirability of including investigative and experimental work in the mathematics curriculum has been accepted for some time now [1]. Investigational work is not only an excellent vehicle for learning but also encourages initiative, co-operation between students, and the application of a broad range of skills. Recent changes in schools' curricula reflect this new approach.

Although, at higher education level, restrictions of timetable, class size or time may limit the scope for such work, it is important to include some investigational work in the mathematics curriculum. The demands made on mathematicians and engineers working in industry are such that they must be able to apply their mathematics to a wide range of problems, to interpret the results of their work and to report their findings.

In the Mathematics Department at Napier Polytechnic, computers play a significant role in introducing science and enginering students to meaningful investigations. Use of computer packages can greatly extend the nature and scope of such work. This paper describes a typical exercise given to second year students of the B.Eng. degree in Communications and Electrical Engineering.

The Laboratory Environment

The mathematical sciences laboratories at Napier consist of a network of BBC micros, a library of carefully selected software and a large collection of associated worksheets, many of which are of an open-ended nature, thus creating an environment ideally suited to experimentation and discovery [2]. Students on the Engineering degree course (and many other courses) use the laboratory both for supervised tutorial sessions and to complete exercises in their own time.

The computer package, NODES, which is used for the investigation described in this paper, has been developed at Napier Polytechnic [3] specifically for investigative work. It solves single or systems of first-order differential equations numerically and produces both tabular and graphical solutions. The highly flexible and interactive nature of the program allows problem parameters to be easily modified and the resultant effect on the solution observed. Successive graphical solutions may be superimposed.

Using the NODES package as a sophisticated tool, the students are able to model realistic and important systems which they would be unable to solve analytically.

The Investigation

For this exercise the student is given the following second-order differential equation:

$$y'' + \epsilon(1 - y^2 + \mu y^4)y' + y = 0 \quad \text{------------} \quad (1)$$

where $\epsilon > 0$ and $0 < \mu \leqslant \frac{1}{8}$.

This equation is an extension of the van der Pol oscillator and is essentially a harmonic oscillator with a non-linear damping coefficient. (Setting $\mu = 0$ gives the standard van der Pol equation.)

After an initial discussion of phase-plane trajectories and phase portraits, the students are ready to tackle the following investigation:

(a) Obtain the solution to the given equation when $\epsilon = 1$ and $\mu = 0.1$, using the following sets of initial conditions:

	y(0)	y'(0)
(i)	4	-5
(ii)	2	2
(iii)	1	1

(b) Plot the phase plane trajectories associated with each of the above solutions, and hence construct the phase portrait for $\epsilon = 1$ and $\mu = 0.1$.

(c) Construct the phase portrait for $\epsilon = 2$ and $\mu = 0.1$.

The first task is to convert the second-order differential equation to a pair of simultaneous first-order equations as follows:

$$\text{Let } y_1 = y$$
$$y_2 = y'.$$

Then equation (1) may be written as:

$$y_2' = \epsilon(y_1^2 - 1 - \mu y_1^4)y_2 - y_1.$$

With $\epsilon = 1$ and $\mu = 0.1$, the problem may be restated as:

$$\text{Solve } y_1' = y_2$$
$$y_2' = (y_1^2 - 1 - 0.1y_1^4)y_2 - y_1 \quad \text{-------} \quad (2)$$

When this pair of equations (2) is input to NODES, the solutions for the three given sets of initial conditions can be plotted as shown in Figure 1. The ability of the software to superimpose graphs greatly facilitates the comparison of these solutions. Whilst (i) and (ii) exhibit periodic solutions, the different behaviour of problem (iii) is immediately apparent.

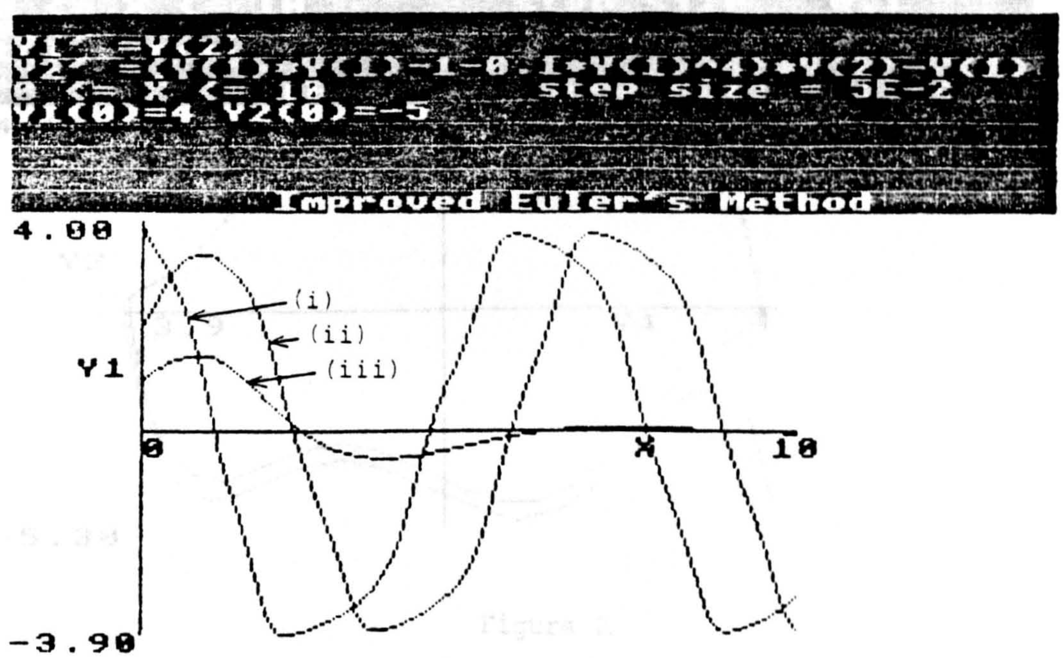


Figure 1

By plotting y against y' (y_1 against y_2 in NODES), phase plots are obtained. The phase plot of problem (i) settles down to a closed trajectory (Figure 2).

Figure 3 shows that, whereas the plot of problem (ii) spirals out to join the same stable orbit as problem (i), the phase plot of problem (iii) spirals in towards the origin.

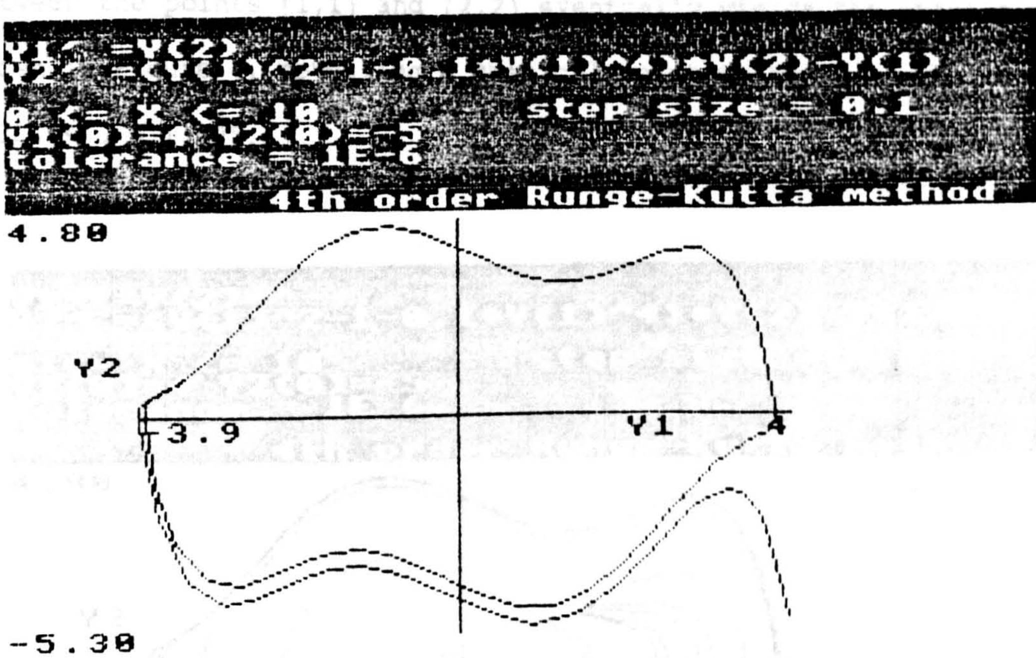


Figure 2

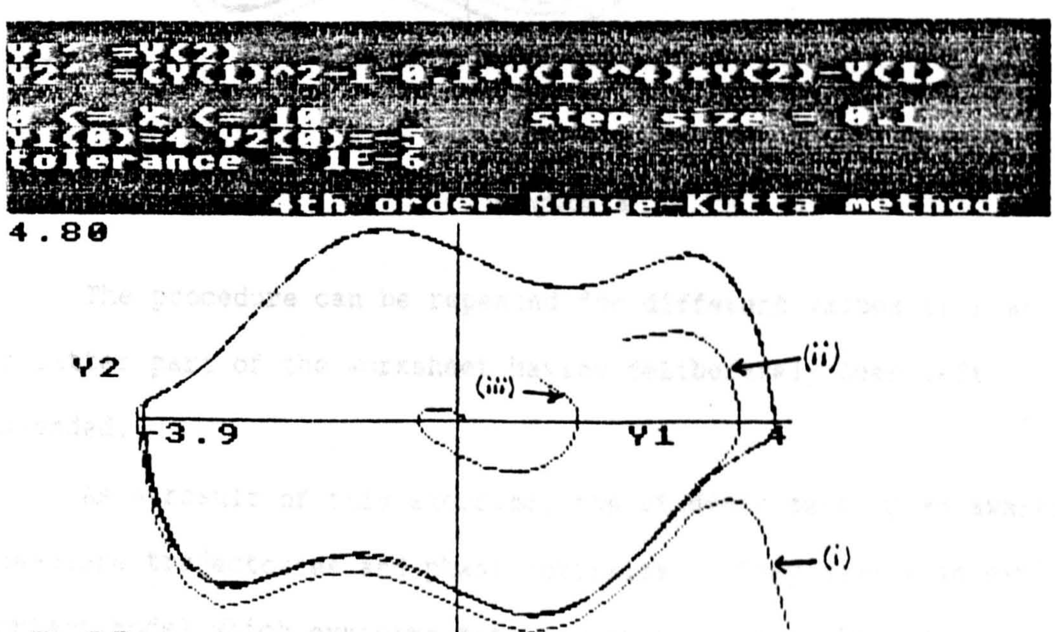


Figure 3

These results suggest a stable limit cycle within which there is a focus at the origin. The interesting task now is to find the critical unstable trajectory. Experimentation with different starting values between the points (1,1) and (2,2) eventually yields the phase portrait shown in Figure 4, revealing a critical point to be (α,α) where $1.304 < \alpha < 1.305$.

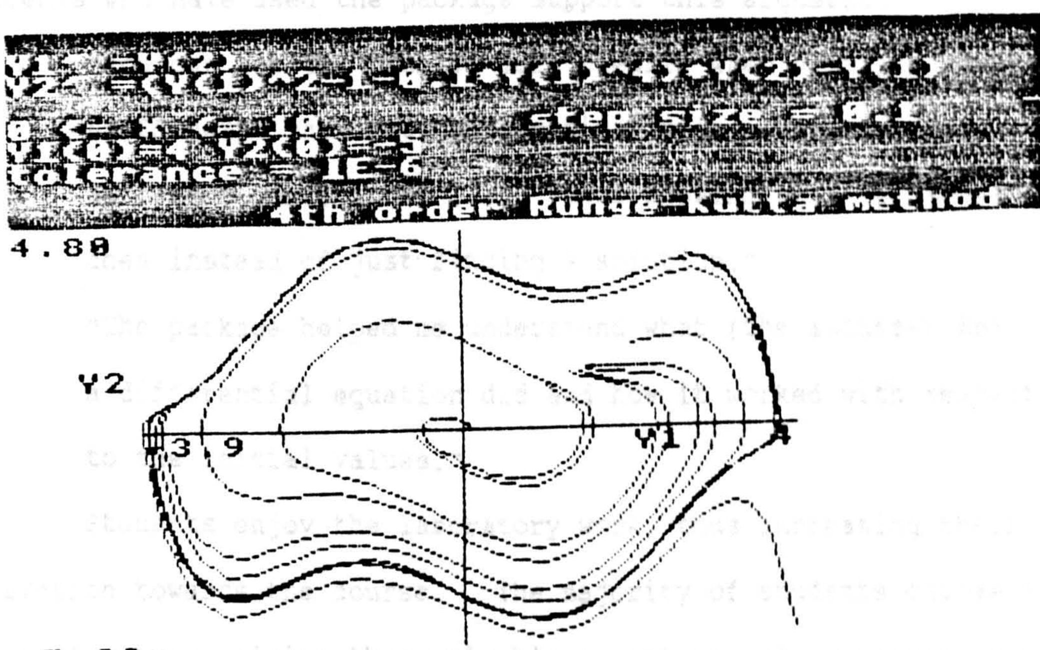


Figure 4

The procedure can be repeated for different values of ϵ and μ, this latter part of the worksheet having deliberately been left open-ended.

As a result of this exercise, the students develop an awareness of phase-plane trajectories and phase portraits. They have also explored an important model which exhibits both a stable limit cycle and an unstable orbit.

Impact on the Student Learning

Use of a computer package is essential for an exercise of this nature. By experimenting, students develop an understanding of the properties of the system being studied and the physical significance of the results [4]. In particular, the graphical output gives the user a qualitative feel for the solution and enables him to build up a comprehensive picture of the behaviour of the model. Comments from students who have used the package support this argument:

"You can see the problem more clearly when visually displayed."
NODES helps show "how mathematical models behave under different conditions" ..... "thus showing what the model does instead of just finding a solution."
"The package helped me understand what (the solution to) a differential equation did and how it worked with respect to the initial values."

Students enjoy the laboratory work, thus increasing their motivation towards the course. The majority of students choose to work in small groups, giving them valuable experience of co-operative effort and team-working techniques. However, each student is required to submit his own report of the investigation.

In conclusion, investigative work in the mathematics laboratory helps students to develop an experimental approach towards problem solving. In addition, it enriches and extends their knowledge and understanding of a topic.

References

1. Cockroft, W.H., Report of the Committee of Inquiry into the teaching of Mathematics in Schools, London, HMSO, 1982.

2. Mackie, D.M. and Scott, T.D., Int. J. Math. Educ. Sci. Technol., Vol. 19, 83-88, 1988.

3. Mackie, D.M. and Scott, T.D., "NODES, A differential equations package for the BBC micro", Napier Polytechnic of Edinburgh, 1988.

4. Harding, R.D., Int. J. Math. Educ. Sci. Technol., Vol. 5, 447, 1974.

Scottish Mathematics Council Journal No. 17, 1987

COMPUTER-BASED INVESTIGATIONS

by
Diana Mackie and Tom Scott
Dept of Mathematics, Napier College

At long last, amongst recent intakes of first-year students at Napier College, a significant proportion have some experience of investigations as part of their school Mathematics curriculum. This is to be applauded, and it is hoped that the current trend towards more investigative work in our schools will continue. Evidence has been collected [1] which confirms that as a vehicle for learning, the investigation is more effective (and efficient) than traditional methods, although different areas of the syllabus require different types of investigation.

It has long been recognised that an important medium for investigative work is the microcomputer, and it is disappointing to note that in Scottish schools there is still a severe shortage of computing facilities for Mathematics classes. Even on an individual basis, which might be quite adequate for some types of investigation, it seems to be very difficult to gain access to a microcomputer. Of a sample of 116 students entering college in September 1986, only 15% had used computers regularly at school, 32% occasionally and the remaining 53% not at all. Fortunately, however, the provision of computing facilities for Mathematics students in Colleges and Universities is much better. It is therefore relatively easy to organise computer-based assignments in Mathematics, and even more appropriate when there is good software available for the students to use.

In the department of Mathematics at Napier College, there are two Mathematical Sciences Laboratories, each of which contains a network of BBC microcomputers. The most important usage of these facilities is in the field of Mathematical and Statistical investigations. Although Statistics packages have been purchased from external sources, most of the Mathematics software which is commonly used has been developed within the department. The resulting library of software is therefore highly suitable for investigative work, having been produced (or acquired) with this type of usage in mind.

Against this background, laboratory exercises have been generated by several of the staff in the department and a large collection of worksheets which utilise the software has now been compiled. The general aim of all of the worksheets is to direct the student's learning experience in a given direction. With this is mind, authors are required to state their objectives at the beginning of each sheet. By far the most common type of objective is one involving some sort of investigation of a mathematical or statistical model, including perhaps a sensitivity analysis of the solution. It is important to emphasise that, with an appropriate worksheet, a single laboratory session is sufficient to allow useful and realistic problems to be explored. The student is encouraged to experiment with model parameters and investigate their effect on particular solutions. It is this latter analysis, in particular, which enriches the student's experience; and the laboratory environment is essential for the work involved.

With the recent conversion of many engineering degree courses from BSc to BEng, the inclusion of "Engineering Applications" across the curriculum has given a new purpose to laboratory-based assignments. Many of the worksheets are suitable for the Mathematics component of engineering applications, and more are being produced for use with first through to final year students. The arrangements for any group of students would typically include a series of lectures and tutorials on the relevant section of the syllabus, followed by an introduction to the software to be used, with some further explanation of the Mathematics as part of a supervised laboratory session. Thereafter the investigation would be carried out by the students during periods of open access to the laboratory.

The situation is ideal for extending the student's learning through discovery and experimentation with new ideas. For example, an assignment which is given to a second year class of electrical engineering students involves the phase-plane analysis of the Van der Pol oscillator. Prior to tackling this investigation, the group has received lectures on solving ordinary differential equations using Laplace Transforms. They are familiar with initial value problems and how to express higher order equations as a system of 1st order equations. They have derived the simultaneous equations which model coupled circuits, and solved them analytically. But that is all !

With this background, the students are provided with the non-linear Van der Pol equation:

$$\ddot{y} + \mu\dot{y}(y^2 - 1) + y = 0 \qquad \text{...........(1)}$$

and an extension:

$$\ddot{y} + \epsilon(1 - y^2 + \mu y^4)\dot{y} + y = 0 \qquad \text{..........(2)}$$

and asked to:

(a) obtain solutions for both models;

(b) construct phase portraits;

(c) investigate the limit cycle of the Van der Pol oscillator;

(d) investigate the effect of changing the parameters of model (2);

(e) submit a written report of their findings within a prescribed period of time.

As part of their introduction to the software, the students learn that the package is designed for solving systems of 1st order equations, so that both models have to be expressed as pairs of simultaneous equations before part (a) can be done.

During the (initial) supervised laboratory session, some discussion of phase-plane trajectories and phase portraits takes place. Following this discussion, the students are ready to tackle the investigation as prescribed by the following two experiments:

EXPERIMENT 1
(The Van der Pol Oscillator)

The non-linear van der Pol equation is

$$\ddot{y} + \mu\dot{y}(y^2 - 1) + y = 0 \quad \ldots\ldots\ldots\ldots\ldots(1)$$

(a) For $\mu = 0.75$, obtain the solution to (1) using the following sets of initial conditions:

	$y(0)$	$\dot{y}(0)$
(i)	0	1
(ii)	3	3

(b) Plot the phase-plane trajectories associated with each of the above solutions, and hence construct the phase portrait for $\mu = 0.75$.

(c) For μ = <15, plot graphs against μ of:

(i) the periodic time;
(ii) the maximum displacement y;
(iii) the maximum velocity $\dot{y}$;

and comment on your results.

EXPERIMENT 2

For the differential equation

$$\ddot{y} + \epsilon(1 - y^2 + \mu y^4)\dot{y} + y = 0 \quad \ldots\ldots\ldots(2)$$

where $\epsilon > 0$ and $0 < \mu < \frac{1}{8}$,

(a) obtain the solution when $\epsilon = 1$ and $\mu = 0.1$, using the following sets of initial conditions:

	$y(0)$	$\dot{y}(0)$
(i)	4	-5
(ii)	2	2
(iii)	1	1

(b) plot the phase-plane trajectories associated with each of the above solutions, and hence construct the phase portrait for $\epsilon = 1$ and $\mu = 0.1$.

(c) investigate the behaviour of the solution of (2) for various values of the parameters ϵ and μ.

For this assignment, the software used is a program called NODES, which has been developed by the authors. Having been designed specifically for investigative work, the package produces graphical solutions as well as the usual tables of values. It also allows the user to easily change parameters and obtain new solutions, with the option of superimposing successive graphs. Students obviously require a hard-copy of their results, and this too is a feature of the package.

This is only one example of an investigation. There are many others currently in use and the signs are that computer-based assignments will become an essential ingredient in most courses. The main educational purpose is to exploit the effectiveness of investigations. The authors believe that such projects reinforce and extend the students' understanding of the topics to which they relate.

REFERENCES:

[1] Harding, R., "Microcomputers in University Mathematics Teaching", Proceedings of the Conference: Trends in Computer Assisted Education, University of Lancaster, 1986.

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Using Computers to enhance the Learning of Mathematics

D. Mackie

T.D. Scott

Department of Mathematics,

Napier College,

Colinton Road,

Edinburgh.

Summary

Using computers to support the teaching and learning of mathematics requires careful planning, good software and appropriate back-up material. This paper describes the role of micros in the Department of Mathematics at Napier College, and outlines the design philosophy of the learning packages being used. The students benefit from improved facilities for realistic problem solving and investigative work, leading to a significant enhancement of their mathematical experience.

1. Introduction

For some considerable time now, it has been recognised that teachers/lecturers should devote some time to planning, as part of their lesson/lecture preparation; and, in particular, careful selection of the most appropriate teaching method has become an essential prerequisite for effective learning. It is certainly true that such an approach is necessary for successful learning of mathematics, within which we include the levels studied at colleges and universities.

Until fairly recently, mathematics lecturers have had only limited resources available for attempting to improve (or even sustain) their effectiveness in the classroom. Now, however, teachers/lecturers have a wide choice of resource to support their teaching: tape/slide programmes, video programmes, self-instruction texts, computer-based materials, etc. It has become essential, therefore, for the teacher/lecturers to devote time to the selection of teaching medium for each part of the course and to the planning of the total learning 'package' [1]. Because of the demands for higher standards of achievement, at school and college, such an approach is necessary at all levels of mathematics teaching.

In this paper the role of computers in this educatonal 'package' is examined, and a development at Napier College is described.

2. Computers in Mathematical Education

As computers have become cheaper and available to larger numbers of students, more attention has been paid to using them to assist in the teaching of mathematics. Computers have been in use in the mathematics departments of colleges and universities since the 1960s. In the fields of numerical analysis and statistics, in particular, they have proved themselves invaluable by removing much of the numerical drudgery.

One approach towards using computers which is sometimes taken requires students to write their own programs. This can be a useful exercise for straightforward algorithms (e.g. Newton-Raphson iteration) as it encourages the student to think logically and reinforces his understanding of the method. Writing good computer programs, however, is time-consuming and, for some students, may be tedious. In addition, there is a danger in seeing the program as an end in itself rather than a tool to be used to further his understanding of the topic being studied.

An alternative approach is to use existing packages. Unfortunately, much of the available software is badly written, unreliable and difficult to use, having changed little from the days of batch-processing. Given such software and a lack of suitable support material, their use is often limited and unimaginative. In this situation students can quickly become frustrated and resort to the more familiar calculator or even pencil and paper. Better programs, which do take advantage of the computer's interactive capabilities, are slowly becoming available.

3. A Teaching Approach

At Napier College, Edinburgh, the Mathematics Department has been involved in the use of computers in teaching for many years [2]. At first, teletype terminals to a mainframe were used. These were slow and unreliable. With the advent of the microcomputer, the authors believed that improved methods of enhancing the teaching/learning of mathematics were possible. The use of carefully and appropriately designed learning packages could promote better student understanding of the underlying mathematical concepts and encourage him towards problem solving and investigative work.

Two important methods have evolved. The first of these involves using computer-based demonstrations within lectures, and has been successfully introduced over a range of course with a wide variety of topics. This set-up requires a microcomputer (or mainframe terminal) linked to monitors (or a display screen) in such a way that the interaction with the computer is clearly visible to all. Using this arrangement the lecturer can demonstrate important concepts, and solve problems in a manner which time and conventional facilities would not permit. For example, good graphics is ideal for illustrating the convergence of Fourier Series.

The second method followed naturally from the first, as the interest generated by this use of the computer made it evident that students would benefit from using the packages themselves. In the interests of efficiency, it is desirable to locate all of the computer equipment inside one room, the mathematics laboratory.

At Napier College a "mathematics laboratory" was established, consisting of a networked system of 10 BBC microcomputers linked to a file server, a 20 megabyte Winchester disc, a line printer and a graph plotter. The laboratory is available both for supervised sessions and for periods of open access to students. It is used regularly by science and engineering students. The majority of the software used has been developed within the department.

The first package to be made available in the laboratory was a Graph Plotting package. Students quickly gained confidence when using this package and, being of such a general nature, it has proved useful in a wide variety of applications.

Other packages include Numerical Integration, solution of differential equations by Runge-Kutta methods, Newton-Raphson iteration, simulation of a single server queue, linear and integer programming, and a statistical analysis package. Many of these are first issues and extensions are planned.

4. Design of Packages

In line with views of others working in the field, the authors believe that the design of the software is extremely important [3]. It is essential for each package to have clearly defined objectives. A common feature of the programs is that, once loaded (a single command), they are very simple to use and, in general, do not require a user manual. They should have a uniformity of approach and be flexible in use, making full use of the computer's interactive and graphics capabilities. They must allow the student to try any apparently reasonable "experiment" and, if the algorithm fails, or the program produces error messages, it should tell him why. The reasons for getting it wrong are often more important than the error messages. One of the advantages of a computer over a teacher is that the student is not embarrassed by his mistakes, and is therefore more likely to persist in his attempts to find a solution [4]. The student, thus, has at his disposal a flexible piece of "apparatus" with which he can conduct "experiments" in mathematics.

Of course, not many students will progress sufficiently if left to experiment completely freely. Just as in any other science laboratory, direction must be provided via a worksheet [5]. Such worksheets must be carefully designed and well prepared. Their design should reflect the aims and objectives of the laboratory session. Depending on the lecturer's perception of the exercise, the written material may take the form of a set of questions similar to the ones encountered in a traditional tutorial/problem class. However, it is more likely that a lab sheet would include some "open-ended" exercises, of the type that lend themselves to investigation in the laboratory. Indeed, some of the work to be undertaken might be of an experimental nature, leading to written reports for submission to the lecturer concerned.

A typical package is LINPROG which is described in the Appendix. It is intended for use by students on a variety of degree and diploma courses in which linear and/or integer programming is studied. Extracts from accompanying worksheets are included in the Appendix.

5. Summary

It must be emphasised that the student should have been taught the underlying mathematical concepts before using the package. Its use will then enhance his understanding and enable him to apply his mathematics to a greater variety of problems.

Inappropriate use of the computer is to be avoided. More specifically, it can only be effective if it has been selected and properly evaluated as the best method available for a given learning situation [6].

In conclusion, the role of the computer, whether in the classroom or in the laboratory, is to extend the repertoire of the mathematics teacher. The authors believe that it has become an important tool in the field of mathematical education, and deserves proper consideration as such at the planning stage of the student's learning experience. Much work has still to be done in the area of computer assisted learning of mathematics, including the preparation of new material and, of equal importance, the proper evaluation of such material.

References

1. Searl, J.W., "Mathematics for engineers and scientists: some teaching approaches", Int. J. Math. Educ. Sci. Technol., Vol. 16, No. 2, 275-283.
2. MATLAB Project supported by National Development Programme in Computer Assisted Learning.
3. Bajpai, A.C., Fairley, J.A., Harrison, M.C., Mustoe, L.R., Walker, D. and Whitfield, A.H., "Mathematics and the micro", Int. J. Math. Educ. Sci. Technol., Vol. 16, No. 3, 407-412.
4. O'Shea, T. and Self, J., "Learning and teaching with computers", Harvester Press, 1983, Chapter 2.
5. Tawney, D.A., "Learning through computers", Macmillan, 1979.
6. Cockcroft, W.H., Report of the Committee of Inquiry into the teaching of mathematics in schools, London, HMSO, 1982.