

Statistics of Local and Global Flame Speed and Structure for Highly Turbulent H₂/Air Premixed Flames

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Abstract

A statistical analysis is conducted for turbulent hydrogen-air premixed flames at a range of Karlovitz numbers up to 1,126 by direct numerical simulations (DNS) with detailed chemistry. The local and global burning velocities are evaluated and the deviation from the laminar flame speed is assessed. It is found that the global turbulent flame speed is largely determined by the integral length scale than the turbulent Karlovitz number, due to the flame surface area enhancement. The turbulent flame speed in all examined cases correlates well with the flame surface area, according to Damköhler's first hypothesis; even at Karlovitz number well above 1,000, reaction zones stay intact and only the preheat zone is broadened by the strong turbulence level. The statistical analysis with the probability density function (PDF) for the displacement speed shows that the highest probability of the local flame speed coincides with the one-dimensional unstretched flame speed. Despite some deviations, the mean flame structures and reaction rate of hydrogen of the higher Ka cases are found to resemble those of the laminar flame, and this further confirms that the turbulent flame brush topology is mainly determined by the large scale turbulence behavior. The results also suggest that the engineering modeling based on the flamelet concept may be valid for a wider range of Ka conditions.

Keywords: Direct numerical simulation, High Karlovitz number, Turbulent premixed flame, Turbulent flame speed, Turbulent flame structure

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11 1. Introduction

12 Computational fluid dynamics (CFD) plays a vital role in the design and development
13 of practical combustion devices. Due to prohibitive computational costs, the device-level
14 simulations rely on Reynolds-averaged Navier–Stokes (RANS) and large eddy simulation
15 (LES) with various mixing and combustion closure/subgrid models. As such, the predictive
16 capability of the simulations depends strongly on the fidelity of the underlying sub-models.
17 For premixed combustion, a common approach is the laminar flamelet concept [1, 2], as-
18 suming that the turbulent flame brush is composed of a collection of laminar flamelets that
19 are wrinkled and stretched by the action of turbulent eddies. The modeling framework is
20 implemented, for example, in the form of the flame surface density (FSD) or coherent flame
21 model (CFM) [3, 4].

22 FSD estimates the mean/filtered chemical source term as the product of the flame surface
23 density (Σ) and the local consumption rate ($\rho_0 \langle S_c \rangle_s$), where ρ_0 is the unburned gas density
24 and $\langle S_c \rangle_s$ is the local consumption speed which is further broken down to $S_L^0 I_s$ with S_L^0
25 and I_s as the unstretched laminar flame speed and stretch factor, respectively. At typical
26 low Karlovitz numbers ($Ka < 1$), I_s has been developed by the laminar flame theory. At
27 higher Karlovitz numbers ($1 < Ka < 100$) in the thin reaction zone regime, the laminar
28 flamelet model may still be applicable by incorporating the enhancement of transport in
29 the preheat zone by turbulent eddies, according to Damköhler’s second hypothesis [5]. At
30 $Ka > 100$, the theoretical “distributed combustion regime” condition is expected, requiring
31 a drastically different reaction closure model to account for a different mode of the strong
32 turbulence-chemistry interaction.

33 Premixed combustion at high Ka has attracted substantial research interest in recent
34 years as modern combustion devices operate at extreme conditions in pursuit of higher
35 efficiencies. To gain fundamental understanding, laboratory-scale flames at $Ka > 100$ con-
36 ditions have been studied experimentally and numerically. Experiments were conducted in
37 swirl [6–10] and pilot [11–15] configurations, while direct numerical simulation (DNS) stud-
38 ies are largely limited to the turbulence-in-a-box configuration, except for a few studies of
39 laboratory-scale combustors [10, 16–21], due to high computational costs. A number of re-
40 view papers provided a detailed account of recent progress on turbulent premixed combustion

41 research [22–25].

42 Recent experimental investigations on high Ka turbulent flames mostly focused on flame
43 structures [6, 7, 9, 11, 12, 14]. Wabel et al. [11, 14] proposed a revised turbulent combustion
44 regime diagram by introducing a metric relating the turbulent diffusivity with the molecular
45 one. Zhou et al. [6, 7, 9] measured reactive radicals such as CH and HCO, in addition
46 to CH₂O and OH, which are conventionally used to assess the broadening of preheat or
47 reaction zones. The reaction layers identified by CH and HCO profiles were found to become
48 broadened at high Ka conditions. On the other hand, the scope of DNS studies has been
49 wider by taking advantage of detailed spatially and temporally resolved information, such
50 as the analyses of the turbulent flame speed [24, 26–28], flame structure [29–31], heat release
51 characteristics [32, 33], differential diffusion effects [26, 31, 32, 34–36].

52 A common conclusion from both experiments and numerical simulations at high Ka
53 conditions is that the preheat zone is broadened considerably by turbulence, whereas the
54 reaction zone stays nearly unaffected or broadened only at significantly high Ka conditions
55 ($Ka \gg 1,000$) [37]. While many of these studies examined the detailed flame structure in
56 comparison with the reference laminar flames, few studies exist on the statistical analysis
57 of the differences in turbulent burning velocity distributions at low and high Ka conditions,
58 especially for a wide range of the parametric space of the turbulent intensities and integral
59 length scales, which have direct implications in RANS and LES submodels.

60 Although the turbulent flame speed is affected by both the turbulent intensity (u') and
61 integral length scale (l_T), most of the discussion has focused on the former parameter. The
62 general conclusion for the dependence of the turbulent flame speed (S_T) on u' is that S_T ini-
63 tially increases with u' and becomes saturated at sufficiently large u' as discussed theoretically
64 in [38], numerically in [39], and experimentally in [40], showing the so called “bending effect.”
65 The effects of l_T on S_T have so far remained largely unexplored and have been partly inves-
66 tigated in turbulent jet flames [20, 21], spherically expanding flames [41], and turbulence-
67 in-a-box flames [42]. Although these studies commonly reported that S_T has a proportional
68 relation with l_T , the dependence of the stretch factor (defined by $(S_T/S_L)/(A_T/A_L)$, with
69 A denoting flame surface area and the subscripts T and L corresponding to the turbulent
70 and laminar counterparts) on the integral length scale was reported to increase in [21] but

71 remained constant in [41, 42]. Moreover, discussion on flame structure over a wide range of
 72 parametric spaces is insufficient in the literature despite its importance to turbulent com-
 73 bustion modeling. Hence, for better understanding of turbulent combustion characteristics
 74 such as the flame speed and structures, a wide range of the parameters u' and l_T should be
 75 considered.

76 To address these issues, the present study investigates the statistical aspects of the local
 77 and global turbulent flame speed by using DNS data for turbulent hydrogen-air flames prop-
 78 agating into forced turbulent flows in a periodic box, at a range of Ka conditions (14–1,126)
 79 that cover the thin reaction zone and distributed combustion regime. The global turbulent
 80 flame speed is directly compared with the surface area growth that is due to the interaction
 81 of the flame with turbulent eddies and the effect of the integral length scale on the global
 82 flame speed is carefully examined, while local displacement speed is analyzed in a statistical
 83 manner and the results are compared with the unstretched laminar flame speed. Finally, the
 84 structural changes in the statistical average are evaluated in two different ways: spatially
 85 averaged and conditionally averaged against temperature, and modeling implications are
 86 discussed.

Table 1: Parameters of the current simulations at $P = 1$ atm, $T_u = 300$ K, and $\phi = 0.7$ under the uniform grid system ($\Delta x = \Delta y = \Delta z$). The Kolmogorov length scale η is evaluated at the unburned condition.

Case	u'/S_L	l_T/δ_L	L_y/δ_L	η [μm]	Δx [μm]	$\delta_L/\Delta x$	Re	Da	Ka
F1	5	5.65	28.24	14.91	20	17.7	686	1.13	23
F2	35	0.82	4.11	2.14	2.6	136.2	700	0.02	1126
F3	2.6	0.86	4.29	15.15	20	17.7	55	0.33	22
F4	18.3	0.12	0.59	2.14	2.6	136.2	52	0.01	1126
F5	5	0.83	4.16	9.24	11.5	30.8	101	0.17	60
F3'	2.6	2.08	10.39	18.90	20	136.2	132	0.80	14
F4'	18.3	0.29	1.47	2.67	2.6	17.7	131	0.02	722

87 2. Numerical method and selected conditions

88 Direct numerical simulations of propagating flames in a periodic box are carried out using
89 the KAUST Adaptive Reacting Flow Solver (KARFS) [43, 44], which solves the conservation
90 of mass, momentum, energy, and species equations in the compressible formulation by uti-
91 lizing an eighth-order central-difference scheme for spatial discretization and a fourth-order
92 explicit Runge–Kutta method for the time integration. The nonreflecting Navier–Stokes
93 characteristic boundary conditions (NSCBC) [45, 46] are applied to the outflow boundary
94 of the box configuration, while periodic boundary conditions are applied to the transverse
95 directions.

96 To initialize the simulations, a one-dimensional premixed flame solution (equivalence
97 ratio $\phi = 0.7$, fresh mixture temperature $T_u = 300$ K, and pressure $P = 1$ atm) is mapped
98 onto the three-dimensional domain and turbulent velocity fluctuations are superimposed.
99 The initial isotropic turbulence field is generated by specifying the energy spectrum in the
100 spectral space [47] and is further fed at the left boundary $x = 0$. While the turbulent flame
101 propagates to the unburned gas side, the mean inflow velocity is properly adjusted based on
102 the fuel consumption speed, so that the flame is anchored around a specified location within
103 the computational domain, following Bell et al. [48]. To retain the initial turbulence level,
104 the linear turbulent forcing scheme developed in [49] for incompressible flows, is applied in
105 the upstream region starting from 10% of the streamwise domain length. To ensure that
106 the turbulence-flame interaction is realistic, the forcing is turned off as the flow approaches
107 the flame base, defined by a temperature cutoff value of 320 K, which corresponds to the
108 temperature-based progress variable $c_T \approx 0.01$. Although the cutoff value is somewhat
109 arbitrary, it ensures that the applicability of the forcing scheme remains reasonably valid. It
110 is also noted that the employed turbulent forcing scheme restricts the integral length scale
111 statistically to approximately 1/5 of L_y . The detailed chemical kinetic model by Burke et
112 al. [50] is used, which consists of 9 species and 23 reactions.

113 For a systematic investigation of the parametric effects at different turbulent combustion
114 regimes, a range of cases have been designed as marked and listed in Fig. 1 and Table 1,
115 respectively. The cases F1 to F4 are located in a parallelogram such that the turbulent
116 Reynolds number (Re) of F1 and F2 is 700, while that of F3 and F4 is 50, yet cases with the

117 same Re have significantly different Karlovitz number (Ka). Fixing the Reynolds number
 118 in essence keeps constant the competition between turbulent kinetic energy and the work
 119 of the viscous forces. On the other hand, F1 and F3 have the same Ka of approximately
 120 20, while that of F2 and F4 is about 1,126, but cases with the same Ka have significantly
 121 different Re. Fixing Ka number essentially fixes the Kolmogorov length scale, therefore,
 122 keeping constant the interaction between the flame and the small scale turbulent eddies. To
 123 examine the turbulent burning behavior at the same integral length scale l_T and different
 124 velocity fluctuation levels u' , the case F5 is designed such that F3, F5, and F2 are on the
 125 same vertical line. Furthermore, to study the scaling of the turbulent flame speed, two extra
 126 cases, F3' and F4' were added which have the same u' as F3 and F4, respectively, but larger
 127 integral length scales. Note that F3' exhibits the same turbulent combustion characteristics
 128 as other cases that fall in the same regime do, hence its general discussion is omitted. Cases
 129 F1, F3, F3', and F5 fall into the thin reaction zone regime, while F2, F4, and F4' fall into
 130 the distributed combustion regime, according to the Borghi diagram [5].

131 The relevant nondimensional numbers are expressed as follows:

$$132 \quad \text{Re} = \frac{u' l_T}{\nu}, \quad \text{Da} = \frac{l_T u'}{\delta_L / S_L}, \quad \text{Ka} = \frac{\delta_L / S_L}{(\nu / \epsilon)^{1/2}} = \frac{\sqrt{\text{Re}}}{\text{Da}}, \quad (1)$$

133 where u' , l_T , and ϵ are the root-mean-square turbulent velocity fluctuation, integral length
 134 scale of turbulence, and turbulent dissipation rate, respectively, while S_L , δ_L , and ν refer to
 135 the laminar flame speed, laminar thermal thickness, and kinematic viscosity, respectively.
 136 The definition of the flame thickness follows the thermal thickness, which is based on the
 137 maximum temperature gradient ($\delta_L = (T_{\max} - T_{\min}) / (dT/dx)_{\max}$), where T_{\max} and T_{\min} refer
 138 to the maximum and minimum temperature values at the reference one-dimensional laminar
 139 premixed flame condition. The values of the laminar flame speed and thermal thickness
 140 are 1.356 m/s and 0.354 mm, respectively. The configuration of the simulation is shown
 141 in Fig. 2 where L_x , L_y , and L_z are differently chosen such that approximately five integral
 142 length scales fit in the L_y and L_z directions for all cases.

143 Grid resolutions are determined based on the Kolmogorov length scale (η) and satisfy the
 144 criterion $\Delta x = \Delta y = \Delta z \leq 2\eta$ [51], with the values for each case listed in Table 1. The time
 145 step is determined by the strict condition of the Courant–Friedrichs–Lewy (CFL) number

146 being less than unity. The total number of grid points for F1, F2, F3, F3', F4, F4', and F5
147 are respectively about 250, 516, 1.3, 15.6, 8.2, 52, and 6.3 million, which led to the simulation
148 computing times of approximately 6.1, 6.3, 0.03, 0.39, 0.14, 0.8, and 0.25 million core-hours,
149 respectively, at the KAUST Supercomputing Laboratory.

150 3. Results and discussion

151 Figure 2 shows instantaneous snapshots of the temperature field from the various simu-
152 lations, after the flames reached a fully developed state. The time scale, τ_{eddy} , in the caption
153 is the eddy turnover time defined as $\tau_{\text{eddy}} = l_T/u'$. Fig. 2 clearly shows different responses of
154 the flame front to turbulence depending on the conditions. From the visual inspection, F1
155 shows the strongest presence of cellular structure on the flame surface, which is attenuated
156 approximately in the order of F5, F3, F2, F4', and F4. Although the Karlovitz number of
157 F4 is as high as that of F2, F4 resembles a laminar flame without any significant level of
158 corrugation. This may be attributed to three reasons: first, the size of the energy-containing
159 turbulent eddies is too small to wrinkle the flame noticeably; second, the turbulent dissi-
160 pation rate ($\epsilon \approx u'^3/l_T$) is remarkably high, dissipating the eddies rapidly; and third, the
161 constraints imposed by the size of the lateral and transversal dimensions of the domain.
162 Therefore, when discussing the effects of the integral length scale, F4' will be used instead
163 of F4. As shown in Fig. 2(d') and will be discussed later, although F4' involves a limited
164 domain size in the transversal direction ($L_y/\delta_L \approx 1.5$), turbulent eddies wrinkle the flame
165 front, leading to the expected presence of the turbulence-flame interaction. Nevertheless,
166 readers should be aware of the limitations imposed by the small domain size for F4', and
167 hence F4' should be interpreted with caution. Overall, different responses on the flame front
168 suggest that the flame wrinkling has a closer relation with the integral length scale rather
169 than the Karlovitz number alone.

170 It is thus evident that the size of large scale turbulent eddies is a crucial factor affecting
171 the level of the flame surface corrugation, thereby directly impacting the overall turbulent
172 burning velocity as will be further investigated later. In the following subsections, we will
173 discuss the turbulent burning characteristics in global and local perspectives for different
174 Karlovitz number conditions.

175 *3.1. Turbulent flame speed*

176 Turbulent flame speed as the global burning rate, S_T , is defined by the consumption rate
 177 of the fuel species as

$$178 \quad S_T = \frac{1}{\rho_u Y_{u,F} A_L} \int_V \dot{\omega}_F dV, \quad (2)$$

179 where ρ_u and $Y_{u,F}$ are the mixture density and fuel mass fraction, respectively, at the un-
 180 burned gas side, A_L is the projected flame area onto the direction of the flame propagation,
 181 $\dot{\omega}_F$ is the production rate of the fuel species, and V is the volume of the computational
 182 domain.

183 Figure 3 displays the temporal evolution of the turbulent flame speed and flame surface
 184 area for all the cases. The flame surface area was extracted from the iso-surface of the
 185 temperature-based progress variable, $c_T \approx 0.6$, at which the rate of reaction (R1) $H + O_2 \leftrightarrow$
 186 $O + OH$ becomes the maximum in the reference one-dimensional laminar flame. For such a
 187 wide range of Karlovitz number, Fig. 3 shows that S_T/S_L and A_T/A_L are closely correlated
 188 for all cases, implying that the Damköhler's first hypothesis ($S_T/S_L \sim A_T/A_L$) is valid even
 189 at Ka conditions that are expected to fall into distributed combustion regime. Moreover,
 190 although F1 has a lower turbulence intensity ratio, $u'/S_L = 5$, compared to F2 (35) and F4'
 191 (18.3), the larger integral length scale of F1 ($l_T/\delta_L = 5.6$) generates a larger scale wrinkling
 192 and larger flame surface area, which in turn enhances the overall turbulent flame speed at a
 193 much larger level than that of F2 and F4'.

194 For the case F4' (Fig. 3(d)) with an extremely small integral length scale, the stretch
 195 factor $I_0 = (S_T/S_L)/(A_T/A_L)$ approaches unity. This may be attributed to the reasons
 196 discussed for the case F4 in the early part of the Section 3: the small size of the energy-
 197 containing turbulent eddies, large turbulent dissipation rate, ($\epsilon \approx u'^3/l_T$), and the constraints
 198 of the transversal dimensions of the computational domain. Considering that the cases F1
 199 and F3 have the same Ka (20) but present significantly different responses of S_T to A_T , these
 200 differences are attributed to the different turbulent Reynolds numbers of 700 (F1) and 50
 201 (F3). Furthermore, the larger length scale ratio of 6.57, in contrast to the RMS velocity ratio
 202 of 1.91, between cases F1 and F3 suggests that the integral length scale has a stronger impact
 203 on the global turbulent flame speed. Last, by examining the three cases with nearly the same

204 integral length scale but different turbulence intensity (cases F2, F3 and F5), regardless of
 205 the Ka (1126, 22 and 60, respectively) and Re (700, 55 and 101, respectively), the turbulent
 206 flame speed and surface area are increased by similar factors (around two) as compared to
 207 the laminar counterparts.

208 Although the last result may seem counter-intuitive, it is consistent with the theoretical
 209 work of Peters [38], who extended the level-set method to the thin reaction zone regime and
 210 showed that as l_T decreases, S_T becomes less sensitive to u' , particularly as l_T/δ_L approaches
 211 one. Considering that for cases F2, F3 and F5, the ratio of length scales $l_T/\delta_L < 1$, it is
 212 reasonable to expect a reduced sensitivity of S_T to u' . Moreover, according to the technique
 213 of Intermittent Turbulent Net Flame Stretch (ITNFS) [52] based on vortex-flame interac-
 214 tion, stretch effects are found to be negligible at small integral length scales and almost
 215 independent of u' . This indicates that the efficiency for small scale vortices to wrinkle the
 216 flame front is substantially small, thereby the stretch acting on the flame becomes smaller
 217 as integral length scales becomes smaller. Overall, these results indicate that the integral
 218 length scale is a key parameter dictating the flame surface and global flame speed behavior.

219 Figure 4 displays the temporally averaged turbulent flame speed and surface area for the
 220 time range indicated in Fig. 3 by the horizontal lines, together with the mean stretch factor,
 221 $\bar{I}_0 = (\bar{S}_T/S_L)/(\bar{A}_T/A_L)$. In contrast to a large number of studies in the literature, where
 222 S_T/S_L has been sought to correlate with either u'/S_L or non-dimensional numbers including
 223 Re, Da, or Ka, the current result indicates that S_T/S_L and A_T/A_L strongly correlate and
 224 proportionally increase with the integral length scale. This is consistent with the previous
 225 work by Luca et al. [20] and Attili et al. [21] for a slot jet flame configuration in that S_T/S_L
 226 is proportional to l_T/δ_L . In fact, Luca et al. [20] reported that the stretch factor was nearly
 227 unity in a global sense, while from a local perspective, Attili et al. [21] suggested that the
 228 slight increase of the local I_0 with local l_T/δ_L involves an extra factor in enhancing the
 229 turbulent flame speed, which is the inner layer thickening. A stretch factor of nearly unity
 230 was also reported by Lapointe [42] based on DNS data of a turbulence-in-a-box configuration
 231 and by Kulkarni et al. [41] for spherically expanding flames with different integral length
 232 scales.

233 Considering that the mean stretch rate (\bar{I}_0) is nearly unity for all cases, the flame surface

Table 2: Values of the area ratio (A_T/A_L), cutoff ratio (ϵ_o/ϵ_i), outer cutoff ($\epsilon_o = l_T$), inner cutoff ($\epsilon_i = \nu/S_L$), and fractal dimension (D_f).

Case	A_T/A_L	ϵ_o/ϵ_i	ϵ_o [m]	ϵ_i [m]	D_f
F1	5.855	137.316	2.001×10^{-3}	1.457×10^{-5}	2.359
F2	1.863	19.929	2.904×10^{-4}	1.457×10^{-5}	2.208
F3	1.821	20.901	3.045×10^{-4}	1.457×10^{-5}	2.197
F4	1.002	2.916	4.249×10^{-5}	1.457×10^{-5}	2.002
F5	1.982	20.172	2.939×10^{-4}	1.457×10^{-5}	2.228
F3'	3.330	50.527	7.362×10^{-4}	1.457×10^{-5}	2.307
F4'	1.041	7.145	1.041×10^{-4}	1.457×10^{-5}	2.021

area, and thereby S_T , can be estimated as represented by the fractal theory [53, 54]: $A_T/A_L = (\epsilon_o/\epsilon_i)^{D_f-2}$, where ϵ_o and ϵ_i are the outer and inner cutoff scales, respectively, and D_f is the fractal dimension. In the current study, D_f is found to be between 2 and 2.36 with the outer and inner cutoff scales taken as the integral length scale and nominal laminar flame thickness ($l_f = \nu/S_L$), respectively. In Table 2, the values of the area ratio, cutoff ratio, outer cutoff, inner cutoff, and fractal dimension are all listed. Except for the cases of F4 and F4', which have extremely small integral length scales and are deemed unimportant for practical conditions, D_f is between 2.20 and 2.36, which is consistent with the previous studies summarized in [5, 53, 54]. Through the fractal theory relation for A_T/A_L with $\epsilon_o = l_T$, the predominant role of the integral length scale in the generation of flame surface and magnitude of the turbulent flame speed is further substantiated.

3.2. Displacement speed

In this section, more detailed local turbulence-chemistry interaction characteristics are analyzed in terms of the flame displacement speed, which is defined as the local flame front speed relative to the flow velocity, evaluated based on a species k . In the density-weighted form [55], it is expressed as

$$S_d^* = \frac{\rho S_d}{\rho_u} = \frac{1}{\rho_u |\nabla Y_k|} [\dot{\omega}_k - \nabla \cdot \mathbf{J}_k], \quad (3)$$

251 where ρ , $|\nabla Y_k|$, $\dot{\omega}_k$, and $\nabla \cdot \mathbf{J}_k$ are the mixture density, absolute value of the mass fraction
 252 gradient, net production rate, and diffusion rate of a species k , respectively. The subscript u
 253 refers to the unburned state. The diffusive term has the form $\nabla \cdot \mathbf{J}_k = -\nabla \cdot (\rho D_k \nabla Y)$, with
 254 D_k being the mixture-averaged diffusion coefficient of the species k . The density-weighting
 255 eliminates the inherent effect of the velocity acceleration through the flame due to thermal
 256 expansion. In the current study, S_d^* is computed based on the fuel species (H_2).

257 To illustrate the variation of S_d^* and other key solution variables across the flame front,
 258 two-dimensional (2D) cuts are presented in Fig. 5, including the heat release rate (HRR),
 259 temperature (T), mass fraction of H_2 (Y_{H_2}), mass fraction of H_2O_2 ($Y_{H_2O_2}$), and mass fraction
 260 of OH (Y_{OH}) at the time instant shown in Fig. 2. Note that case F4' was excluded from
 261 Fig. 5 due to exhibiting relatively minor wrinkling. The gray lines indicate the isocontours
 262 of the progress variable (c_T) from 0.1 to 0.7 with an increment of 0.2 for F1 and 0.1 for the
 263 rest. The complex structures in the rectangular regions in F1 are magnified in the inset.

264 Comparing the cases on the same scale of the domain indicates that the averaged normal
 265 distance between $c_T = 0.1$ and 0.7 is comparable for F1, F3, and F5 while it becomes more
 266 blurred and thickened for F2, which is mainly due to the increased level of corrugations in
 267 the upstream, for example, isolevels $0.1 < c_T < 0.4$. Comparing F1 with F2, case F1 involves
 268 larger Kolmogorov scale eddies which cannot perturb the flame, while in F2, substantially
 269 smaller turbulent eddies disrupt the flame structure, as seen by the large variations in the
 270 temperature layer thickness. For the larger turbulence scales of F1, the flame front exhibits a
 271 higher level of wrinkling, which is attributed not only to a wider range of eddies interacting
 272 with flame front, but also to a stronger level of thermodiffusive imbalance, as discussed
 273 earlier in [56–58], in which the large corrugation of the flame front was only observed when
 274 the thermodiffusive instability was pronounced. Comparing F2, F3, and F5 which have the
 275 same integral length scale, the level of wrinkling shown in HRR and Y_{OH} looks similar for
 276 these cases. However, turbulence in F2 with a much higher u' than that in F3 and F5
 277 interacts with the flame strongly as evidenced by T and $Y_{H_2O_2}$, particularly in the upstream
 278 region. Finally, although not shown here, but implied in Fig. 2(d'), F4' with a substantially
 279 small integral length scale exhibits a less wrinkled flame, despite Ka being 712. Again,
 280 this suggests that the integral length scale plays a key role in dictating the turbulent flame

281 topology, while Ka based on the Kolmogorov eddy scale is limited to the highly localized
282 flame characteristics.

283 For S_d^* distribution shown on the top row in Fig. 5, conditioned for $0.01 < c_T < 0.99$
284 and set to zero outside where the flame is undefined, F1, F3, and F5 show more uniform
285 distributions along the wrinkled flame fronts as compared to F2. In F2, S_d^* fluctuates largely
286 due to stronger and more localized turbulence-chemistry interaction. The high turbulence
287 intensity in F2 results in a significantly larger distortion of the upstream layers of the flame
288 front, but still the reaction zone remains intact. This is evident by the progress variable
289 isocontours greater than 0.3 that are nearly parallel with one another. The temperature
290 starts to rise in the more upstream region than the actual region of heat release due to
291 the disruption of the flame front by turbulence, which, at the same time, enhances the
292 dissipation by viscous effects (see the profiles of HRR and T in Fig. 5). Moreover, this
293 intense turbulence-chemistry interaction and front disruption are clearly manifested in the
294 broadened $Y_{H_2O_2}$ distribution that spreads upstream, yet the distribution of Y_{OH} nearly
295 coincides with that of the heat release rate. In summary, F2 at $Ka > 1,000$ clearly shows
296 different flame structures with much broader and disrupted transport zones. Even so, due
297 to the thermal expansion in the region, the small scale eddies are dissipated by the time
298 they reach the reaction zones, and hence the theoretical “distributed combustion” regime
299 is not realized. This finding is consistent with previous experimental and computational
300 studies [11, 13–15, 30].

301 As a statistical analysis, the probability density functions (PDF) of S_d^* are computed
302 and displayed in Fig. 6 where (a) to (e) correspond to cases F1 to F5, respectively. S_d^* was
303 computed by collecting data over multiple time steps after the turbulent flame was fully
304 developed. Hence, 17, 33, 54, 65, and 79 data sets were utilized for the cases F1 to F5,
305 respectively, corresponding to all eddy turnover times within the horizontal lines in Fig. 3.
306 Two isocontour values, $c_T = 0.2$ (blue) and 0.6 (red), are chosen to represent the preheat
307 and reaction zones, respectively. These values are selected because in the reference laminar
308 premixed flame $c_T = 0.2$ is where $\dot{\omega}_{H_2O_2}$ crosses from negative to positive, and $c_T = 0.6$ is the
309 point at which the reaction rate of $R1=H+O_2 \leftrightarrow O+OH$ reaches its maximum. The dashed
310 lines are associated with the S_d^* values obtained from the reference laminar premixed flame

311 solution for each selected c_T value.

312 As seen from Fig. 6, the highest probability of S_d^* is found to closely match the S_d^* value
313 of the reference laminar flame. This implies that for all Ka conditions considered in the
314 current study, the majority of the flame components burn as the laminar flame. Moreover,
315 although the global turbulent flame speed is higher in F1 (see Fig. 3) due to the growth of
316 the flame surface area, the local front speed does not follow this trend. As expected from
317 Fig. 5, the distribution of S_d^* for F2 (Fig. 6(b)) spans most widely to both negative and
318 positive sides, followed by the F4', F5, and F3 (\sim F1) - with the order of Ka. Although F5 in
319 Fig. 6(e) shows a relatively strong motion of turbulent eddies at the preheat region evident
320 by the fact that a negative distribution of S_d^* exists, turbulence is attenuated downstream
321 and the PDF shifts to positive values of S_d^* (red line). This behavior is more pronounced in
322 the cases F1 and F3 where practically no negative distribution of S_d^* exists. For F4' with the
323 same u' as F4 but larger l_T , unlike F4 in which S_d^* is strongly localized at the laminar flame
324 speed, local stretch effects are evident, showing a similar distribution to F2.

325 The trend of the front speed as a function of the turbulent intensity is consistent with
326 the previous work by Nivarti and Cant in a turbulence-in-a-box configuration [39] in that
327 at a relatively smaller turbulence level, the distribution of the PDF is narrower and more
328 localized, but as the turbulent intensity increases, the distribution of the PDF is extended to
329 the negative side. The most probable front speed reported in [39] was far from S_L , which is
330 expected considering that it was density unweighted. In summary, the results of front speed
331 confirm that the global flame speed may be largely different from the local speed which is
332 strongly affected by the local stretch effects.

333 Figure 7 shows the variance and skewness of S_d^* based on data collected for the time
334 period indicated by the horizontal lines in Fig. 3. The same color convention is applied: blue
335 (diamond symbol) and red (triangle symbol) colors indicate the upstream and downstream
336 regions, defined by the progress variable $c_T = 0.2$ and 0.6 , respectively. Figs. 7(a) and
337 (b) present the variance and skewness plotted against l_T/δ_L , while in (c) and (d) they are
338 plotted against u'/S_L . Although a clear trend was not observed for both parameters, the
339 variance appears to be rather correlated with u' than with l_T , whereas the skewness is rather
340 correlated with l_T than with u' . Moreover, the variance becomes smaller downstream due

341 to the decay of turbulence, but the skewness becomes larger downstream, indicating that a
342 faster speed is more probable there. A noticeable observation is the large variance of cases
343 F2 and F4' due to the strong motion of turbulence in the upstream region, which clearly
344 shows a small value of skewness (i.e., nearly symmetric) for F4' and even a negative value of
345 skewness for F2.

346 *3.3. Turbulent flame structure*

347 In the following, we discuss the global flame structure from two different perspectives:
348 one via cross-sectional averaging and the other via conditional averaging. While the cross-
349 sectional averaging would be more realistic since it accounts for situations of unburned
350 pockets moving through the burned gas, it may not be entirely intuitive as will be discussed
351 below. On the other hand, by conditionally averaging, all mass fractions and temperature
352 are conditioned on each “bin” of the progress variable, hence the structures are expected to
353 be more similar among the cases.

354 *3.3.1. Cross-sectional averaged structure*

355 Figures 8 and 9 respectively show the temporal average of the cross-section means of
356 temperature along with major species, and intermediate species for cases F1–F5. To facilitate
357 the comparison among all the turbulent cases and contrast them with the laminar one, the
358 profiles are plotted as a function of the progress variable $c_{\bar{T}}$, which is computed using the
359 temporally and spatially averaged temperature field for each turbulent case. Data from the
360 DNS are represented by the solid lines (mean) and shaded regions (standard deviation about
361 the mean), while the laminar counterparts are marked by dashed lines. All species mass
362 fractions and the temperature of DNS data are normalized by the maximum value of the
363 corresponding variable in the reference laminar premixed flame solution.

364 Starting from F4' in Fig. 8(d), the mean values of all major species and temperature
365 profiles closely follow the laminar ones, despite exhibiting fluctuations caused by the effects
366 of turbulence, even for the sufficiently large $Ka > 700$. As discussed earlier, this laminar-like
367 flame structure is attributed to the large dissipation rate of turbulent kinetic energy as well
368 as the small size of the energy containing eddies, which are dissipated rapidly. On the other
369 hand, for all the other cases with larger integral length scales, deviations from the laminar

370 profiles are observed in all major species and temperature, which are also accompanied by
371 significant levels of fluctuations, as indicated by the wider shaded areas. This is especially
372 true for the fuel species H_2 , whose profiles substantially differ from the laminar one, while
373 T and all other species of O_2 , and H_2O are found to be closer to their laminar counterparts.

374 The deviation of the turbulent flame structure from the laminar one at different integral
375 length scale is more pronounced in the profiles of the intermediate species. For example, in
376 Fig. 9(a), corresponding to F1 with the largest integral length scale, not only are the peaks
377 of H , O , HO_2 , and H_2O_2 much smaller than the laminar counterparts but also the location of
378 the peak is shifted for HO_2 and H_2O_2 to further downstream. For F2, F3, and F5 having the
379 same integral length scale, the mean distributions and peak locations of H , O , and OH are
380 closer to the laminar ones, and the peaks of HO_2 and H_2O_2 are not as much shifted as in F1,
381 yet their values are still significantly smaller. As expected, for F4' with further diminished
382 integral length scale but a large enough $Ka > 700$, the HO_2 and H_2O_2 profiles noticeably
383 approach the laminar ones and, despite some deviations, other intermediate species also
384 follow the laminar ones. It is clear that the turbulent flame structures in terms of all major
385 and minor species concentrations are also largely affected by the integral length scales.

386 3.3.2. *Conditionally averaged structure*

387 Figures 10 and 11 display the conditionally averaged temperature and major species,
388 and intermediate species, respectively. The notation is the same as in the cross-sectional
389 averaging: solid lines and shaded regions are the mean and standard deviation, and the
390 dashed lines are quantities obtained from the reference laminar flame. As shown in Fig. 10,
391 in addition to F4' in Fig. 10(d) having the small integral length scale and its mean mass
392 fractions and temperature matching well with the corresponding laminar's counterparts in
393 the spatial averaging, all other cases exhibit a marked resemblance between laminar and
394 DNS values for most quantities, except for the mass fraction of H_2 . The trend of the mass
395 fraction of H_2 deviating more from the laminar profile is attributed to preferential diffusion,
396 which is affected by the local level of curvature as well. It is also noted that the deviations
397 in H_2 from the laminar profile are more pronounced for F2, F3 and F5, which correspond to
398 $Da < 1$, i.e., when turbulent transport is faster than chemistry.

399 Figure 11 shows the conditional average of the intermediate species of DNS together with
400 the reference laminar flame. It also indicates clearly that the conditionally averaged mass
401 fractions follow closer the laminar counterparts, compared to the spatially averaged ones.
402 Moreover, there are noticeable features among the cases. First, as expected by the 2D slice
403 figures (Fig. 5) for the case F2, the spreading of $Y_{\text{H}_2\text{O}_2}$ is significant, which may result in
404 displacement of the peaks of Y_{HO_2} and $Y_{\text{H}_2\text{O}_2}$. In addition, the conversion between the two
405 species is not as efficient as in other cases, which is evident by the much lower peak of $Y_{\text{H}_2\text{O}_2}$
406 compared to that in the other cases. Second, Y_{H} of F1 develops much earlier than in other
407 cases, and its peak is found further upstream. This is associated with preferential diffusion,
408 which is a characteristic feature of the light species H_2 and H and its effects are manifested
409 in the thermodiffusive instability.

410 So far we have demonstrated that the integral length scale is a key factor that determines
411 the global behavior of turbulent premixed flames including the turbulent flame speed and
412 turbulent flame structure. To further examine the effect of the integral length scale on the
413 statistical structure of the fuel consumption rate, the cross-sectional and conditional averaged
414 distribution of the reaction rate of H_2 for F1–F5 is compared against that of the reference
415 laminar flame, as shown in Fig. 12. Note that the curves for different cases are not to scale
416 with one another because the net flame brush thickness is different. For the cross-sectional
417 averaging in Fig. 12(a), qualitatively, F4' follows the laminar flame profile closely because of
418 the small integral length scale, while all other cases show the parabolic shape that is typical
419 of a turbulent flame brush consisting of highly corrugated laminar flamelets. The fact that
420 all cases F1-F3 and F5, except for F4', show the parabolic behavior regardless of the Ka
421 condition, further confirms that the turbulent flame topology and burning characteristics
422 are highly dictated by the integral length scale rather than the smallest turbulent eddies.
423 For the conditional averaging, all the cases qualitatively follow the laminar profile, differing
424 quantitatively in the location and magnitude of their peaks, with the latter being larger.
425 Although the case F4' is the closest to the laminar one, it is again emphasized that this
426 case is subject to limitations imposed by the small domain size ($L_y/\delta_L \approx 1.5$) and should be
427 interpreted with caution.

428 In the flamelet modeling framework, the chemical composition is typically determined

429 via the tabulation of chemistry, which is essentially the chemical state conditioned on the
430 progress variable. Recently, Lipatnikov et al. [59] assessed the flamelet prediction capability
431 of major and intermediate species, reaction rates, and heat release rate for turbulent hydrogen
432 flames located in different turbulent combustion regimes. The prediction with PDF suggested
433 that the major species were well predicted even for their highest Karlovitz number ($Ka =$
434 126), while the predictions of intermediate species and reaction rate of H_2 were less accurate.
435 Consistent with [59], in the current study the conditional averages on the progress variable c
436 suggest that the flamelet assumption for tabulation of major species may still be applicable
437 for $Ka > 100$, while limited accuracy may exist for intermediate species and the reaction rate
438 of H_2 . Since H_2 exhibits preferential diffusion, a H_2 -based progress variable is recommended.

439 4. Conclusions

440 Direct numerical simulations of turbulent hydrogen-air premixed flames in a periodic box
441 configuration were used to investigate the turbulent flame speed and statistical characteris-
442 tics of various flamelet quantities at a wide range of Karlovitz numbers. While the number
443 of cases is limited, the seven cases under consideration systematically cover the parametric
444 conditions to explore the effects at a fixed Ka , integral length scale, and turbulence intensity.
445 Whereas the conceptual understanding of high Ka involves more turbulence-chemistry inter-
446 action, the statistical analyses of the displacement speed demonstrate that the overall flame
447 behavior at Ka as large as 1,000 still shows remarkable resemblance to that of a laminar
448 flame. Moreover, instead of the non-dimensional numbers such as Re or Ka , the integral
449 length scale plays a vital role in dictating the global characteristics of turbulent premixed
450 flames. Some key findings are summarized as:

- 451 1. The turbulent flame speed is closely linked to the integral length scale.
- 452 2. The most probable S_d^* coincides with the value computed at the 1D reference laminar
453 condition.
- 454 3. Even at high Ka well above 1,000, the reaction zone stays intact due to the decay of
455 turbulence, and only the preheat zone is strongly affected by turbulence.

- 456 4. The cross-sectional averages of the temperature and major species appear to be within
457 an acceptable range in comparison with those of the laminar flame, suggesting that the
458 laminar flamelet assumption may be applicable for the cases considered in the current
459 study. However, the distributions of intermediate species and fuel reaction rate are
460 more sensitive to variations in the integral length scale: larger (smaller) integral length
461 scales resulted in more (less) deviation from the 1D reference flame.
- 462 5. The conditional averages indicate that the deviation of the flame structures and reac-
463 tion rate of the fuel species between DNS and the laminar flame appears to be small.
464 Nevertheless, the averaged structure of H_2 is found to be least matched with that of
465 the laminar flame due to different levels of thermodiffusive effects depending on the
466 case.

467 The results suggest that the formation of different levels of the turbulent flame brush is
468 largely determined by the integral eddy scale relative to the flame thickness combined with a
469 large spectrum of scale, not solely by the Karlovitz number. Although similar findings have
470 been reported in the past, the effect of the integral length scale relative to the flame thickness
471 has been largely overlooked in previous studies, mainly due to the prohibitive computational
472 cost, and how the present findings extend at flames at much larger physical scales demands
473 further investigation.

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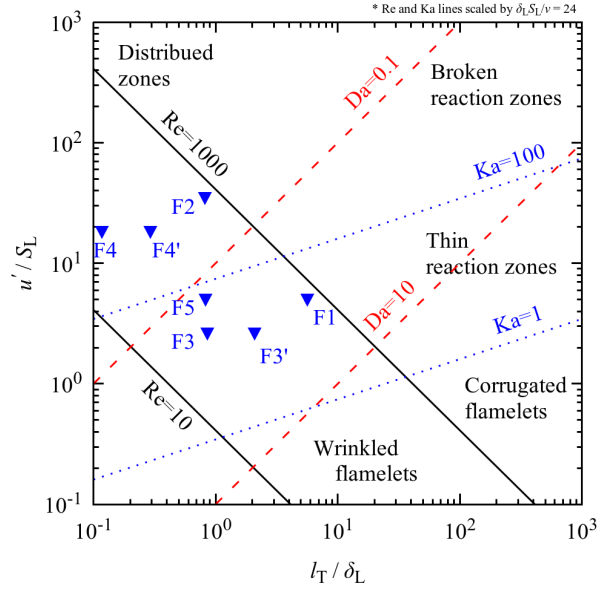


Figure 1: Simulation conditions on the Borghi-Peters diagram. F1 to F4 are in the parallelogram, while F5 has the same l_T as F2 and F3 and the same u' as F1. F3' and F4' denote the same u' as F3 and F4, respectively, but larger l_T .

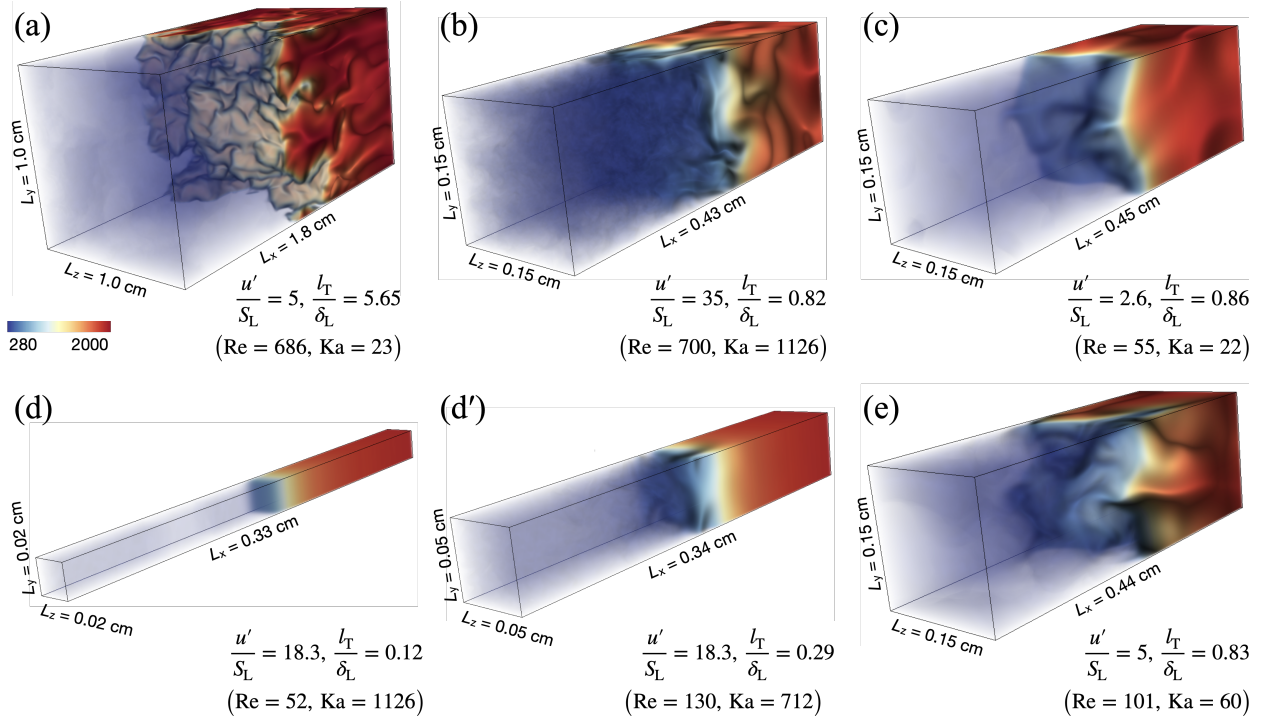


Figure 2: Configurations of the computational domain for flames (a) F1, (b) F2, (c) F3, (d) F4, (d') F4', and (e) F5. Shown figures are the temperature at the time step $t/\tau_{\text{eddy}} =$ (a) 19, (b) 53, (c) 65, (d) 182, (d') 65, and (e) 95.

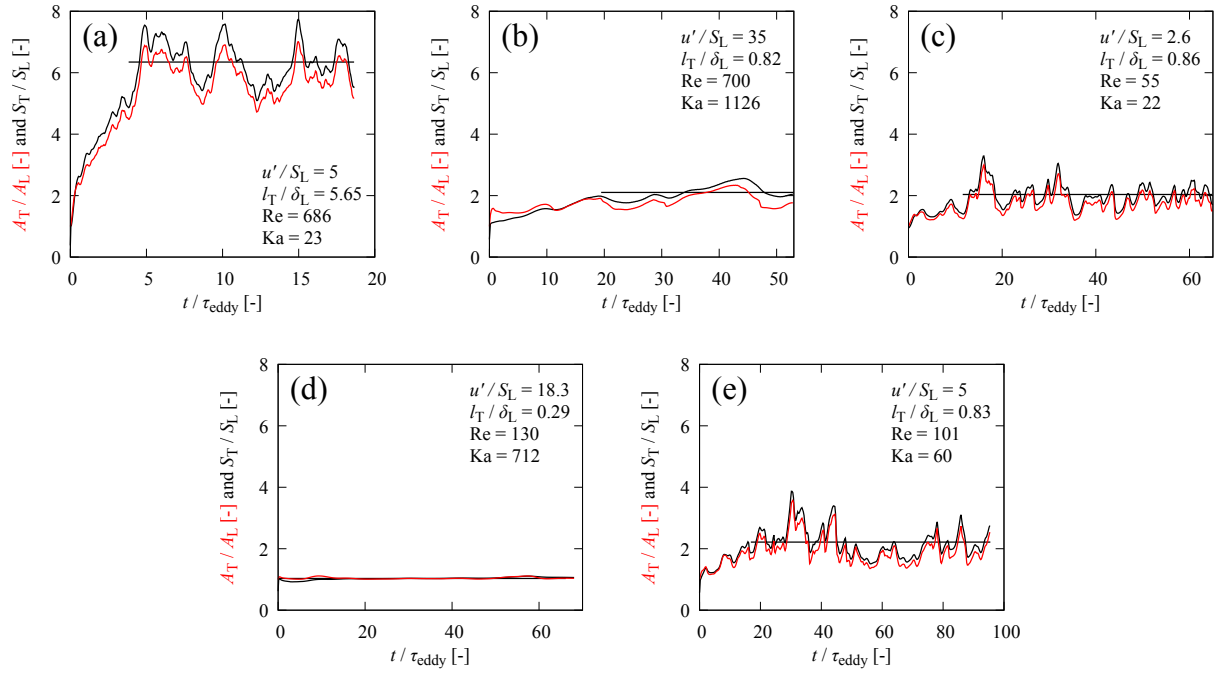


Figure 3: Temporal evolution of the turbulent flame speed and the flame surface area divided by the corresponding laminar quantity for (a) F1, (b) F2, (c) F3, (d) F4', and (e) F5. The horizontal lines denote the mean turbulent flame speed computed for the shown period of time, truncating the initial transient period.

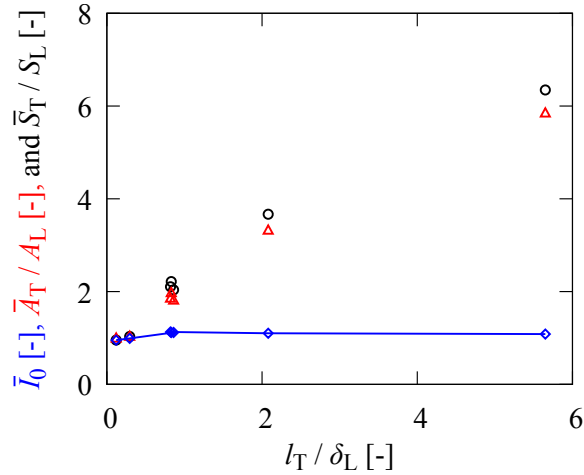


Figure 4: Scaling of the mean turbulent flame speed and mean flame surface area against the integral length scale.

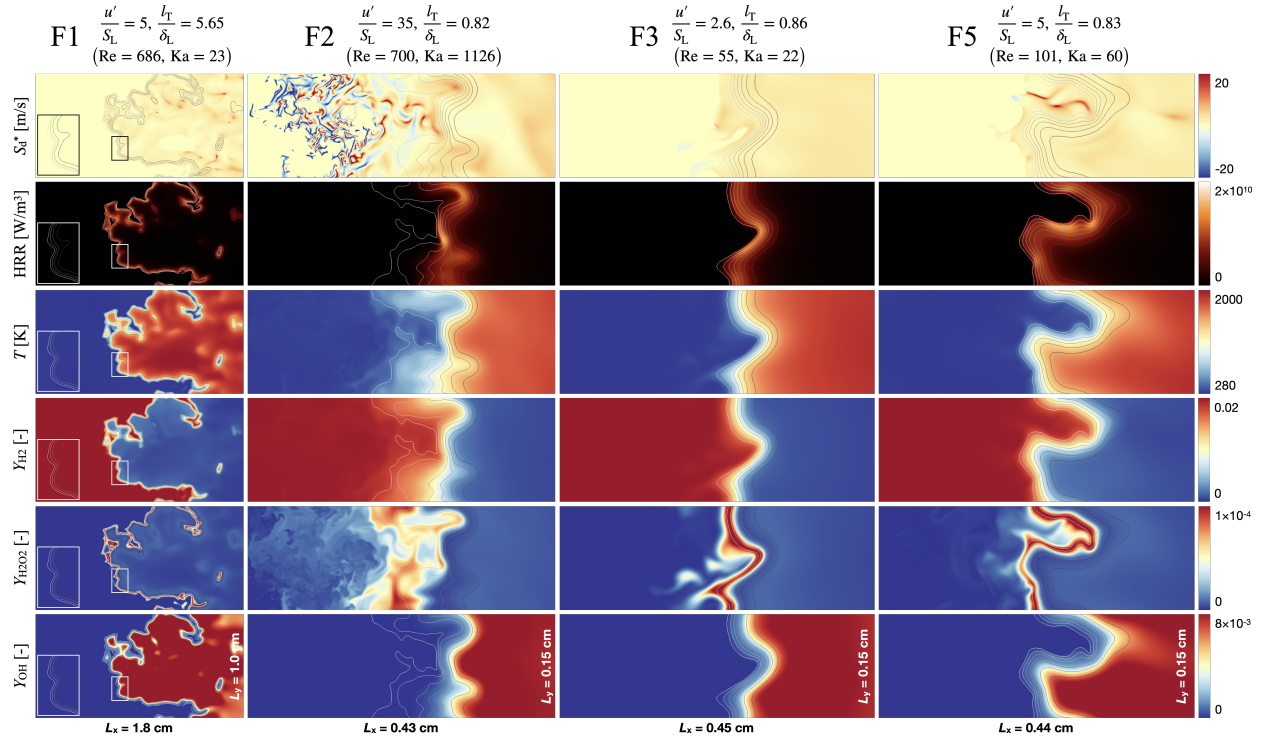


Figure 5: Two-dimensional slices of representative solution variables: the density-weighted displacement speed (S_d^*), heat release rate (HRR), temperature (T), mass fraction of H_2 (Y_{H_2}), mass fraction of H_2O_2 ($Y_{H_2O_2}$), and mass fraction of OH (Y_{OH}) from top to bottom at $t/\tau_{eddy} = 19$ (F1), 53 (F2), 65 (F3), and 182 (F5).

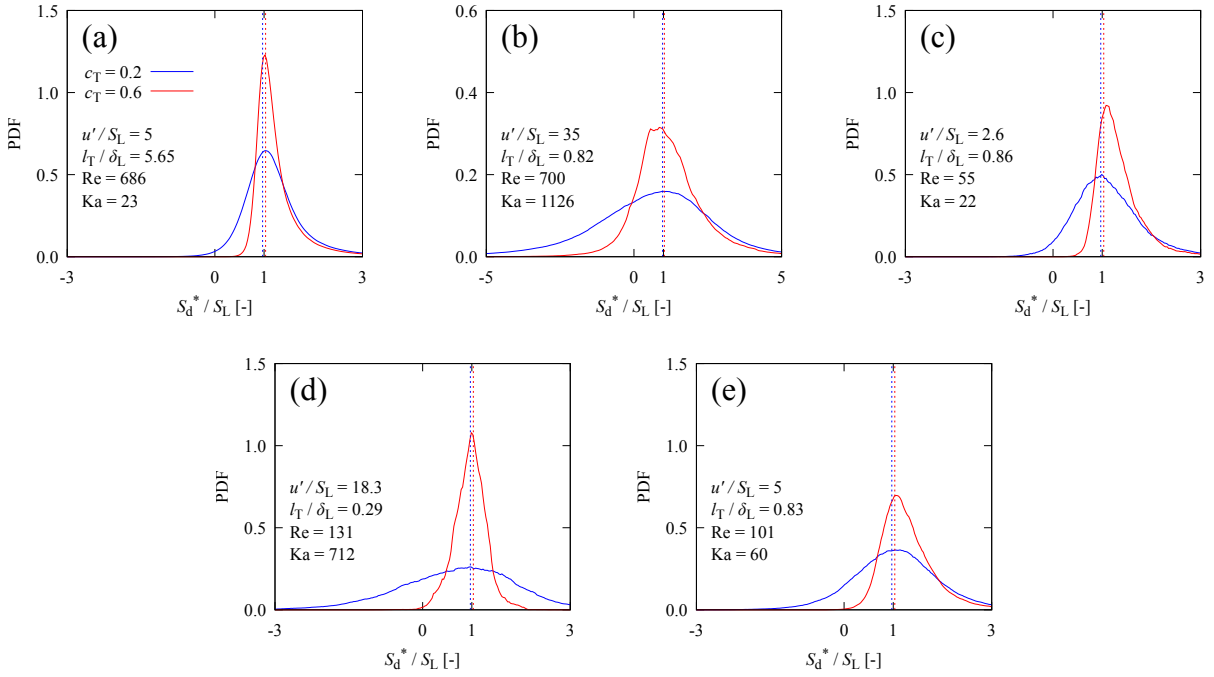


Figure 6: Probability density function (PDF) of the density-weighted displacement speed normalized by the laminar quantity, associated with the progress variable $c_T = 0.2$ (blue) and $c_T = 0.6$ (red) for (a) F1, (b) F2, (c) F3, (d) F4', and (e) F5. Solid lines are PDF from DNS and dashed lines are the laminar quantities.

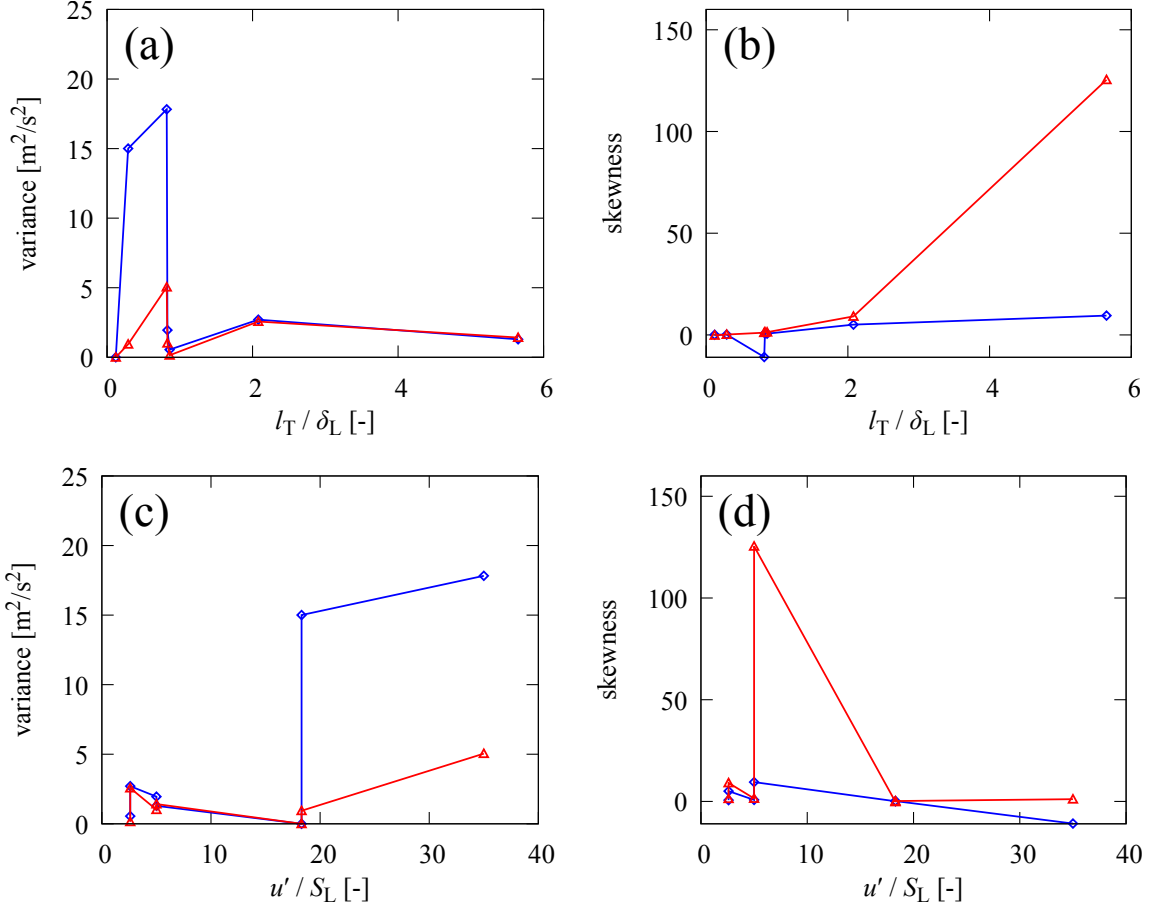


Figure 7: Variance and skewness of S_d^* based on data collected for the time period indicated by the horizontal lines in Fig. 3. (a) and (b) are represented against l_T / δ_L , while (c) and (d) are represented by u' / S_L . Blue lines (diamond symbols) are for $c_T = 0.2$ and red lines (triangle symbols) are for $c_T = 0.6$.

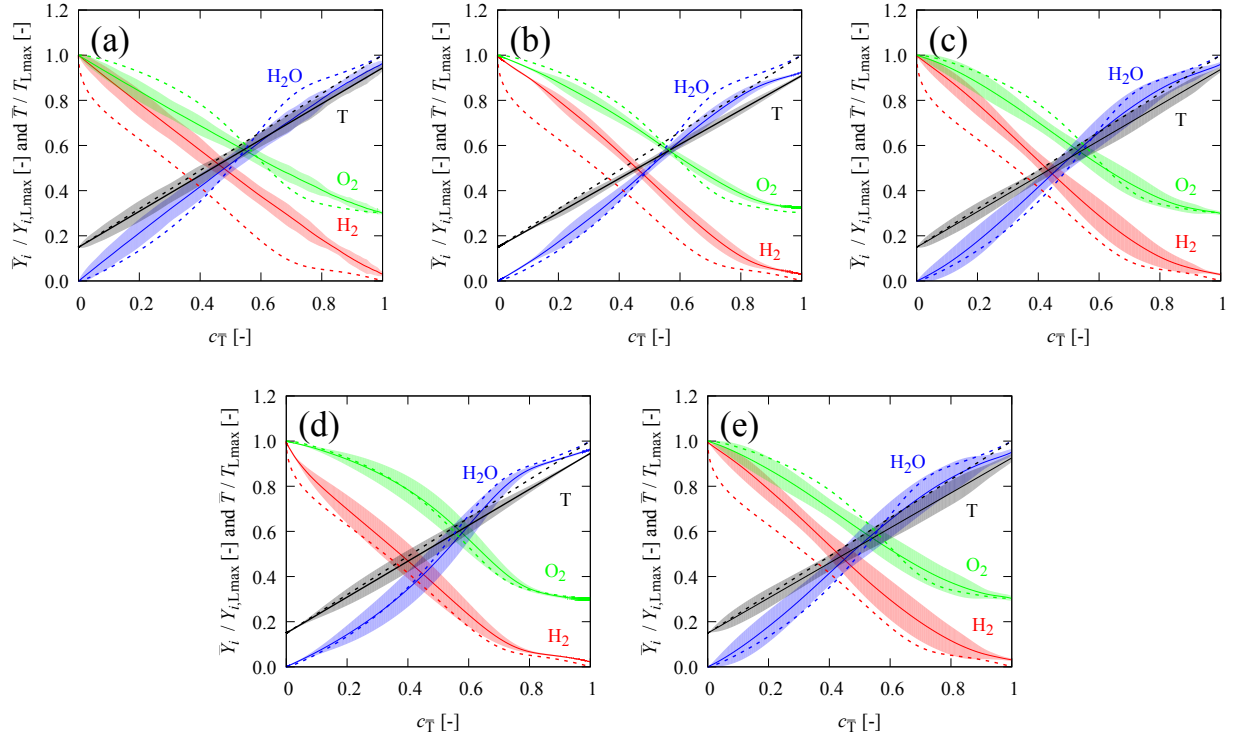


Figure 8: Cross-section average of the temperature and mass fraction of major species for (a) F1, (b) F2, (c) F3, (d) F4', and (e) F5. Solid lines are mean values for all eddy turnover times, shaded regions are the standard deviation to the mean, and the dashed lines are the laminar quantities.

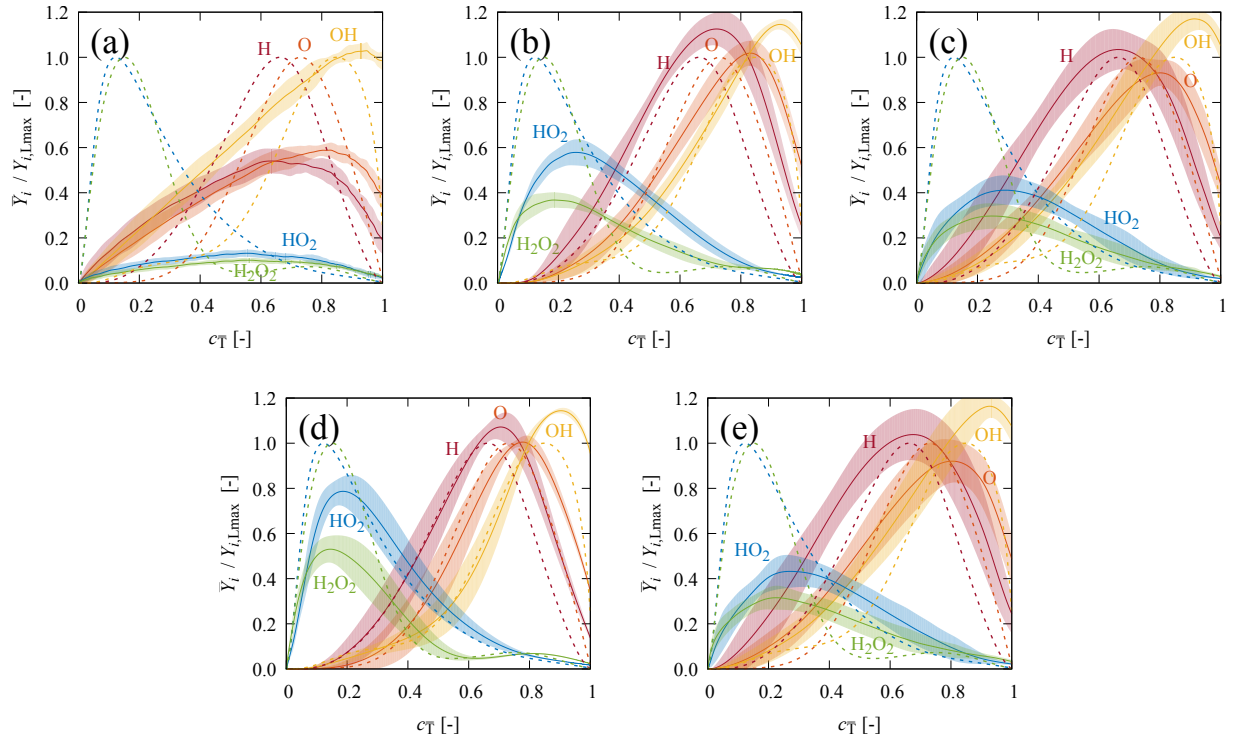


Figure 9: Cross-section average of the mass fractions of intermediate species for (a) F1, (b) F2, (c) F3, (d) F4', and (e) F5. Solid lines are mean values for all eddy turnover times, shaded regions are the standard deviation to the mean, and the dashed lines are the laminar quantities.

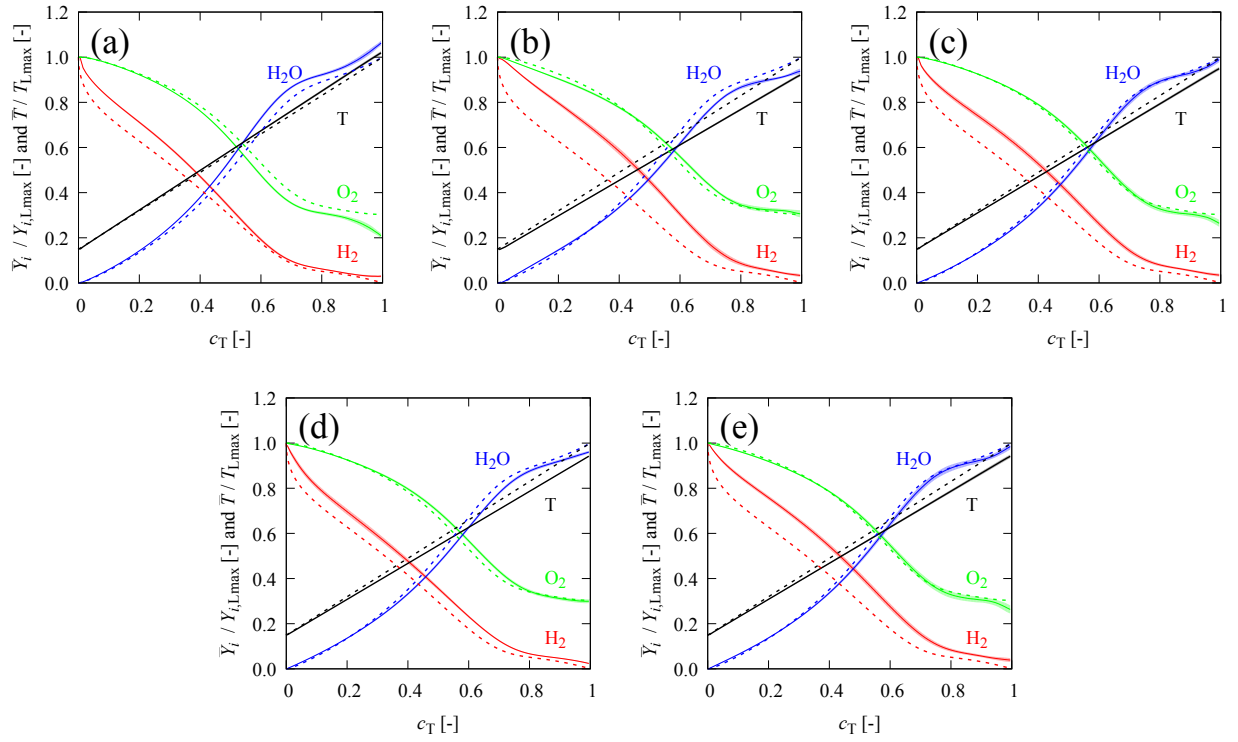


Figure 10: Conditional average of the temperature and mass fraction of major species for (a) F1, (b) F2, (c) F3, (d) F4', and (e) F5. Solid lines are mean values for all eddy turnover times, shaded regions are the standard deviation to the mean, and the dashed lines are the laminar quantities.

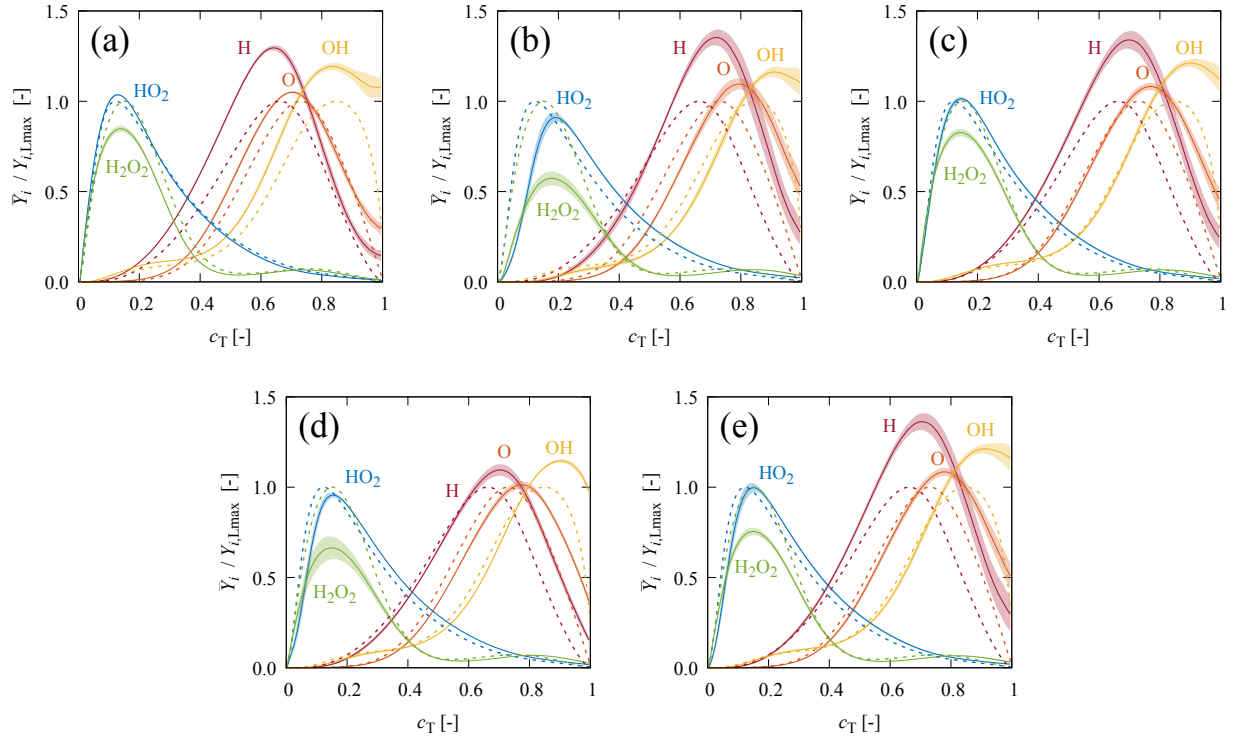


Figure 11: Conditional average of the mass fractions of intermediate species for (a) F1, (b) F2, (c) F3, (d) F4', and (e) F5. Solid lines are mean values for all eddy turnover times, shaded regions are the standard deviation to the mean, and the dashed lines are the laminar quantities.

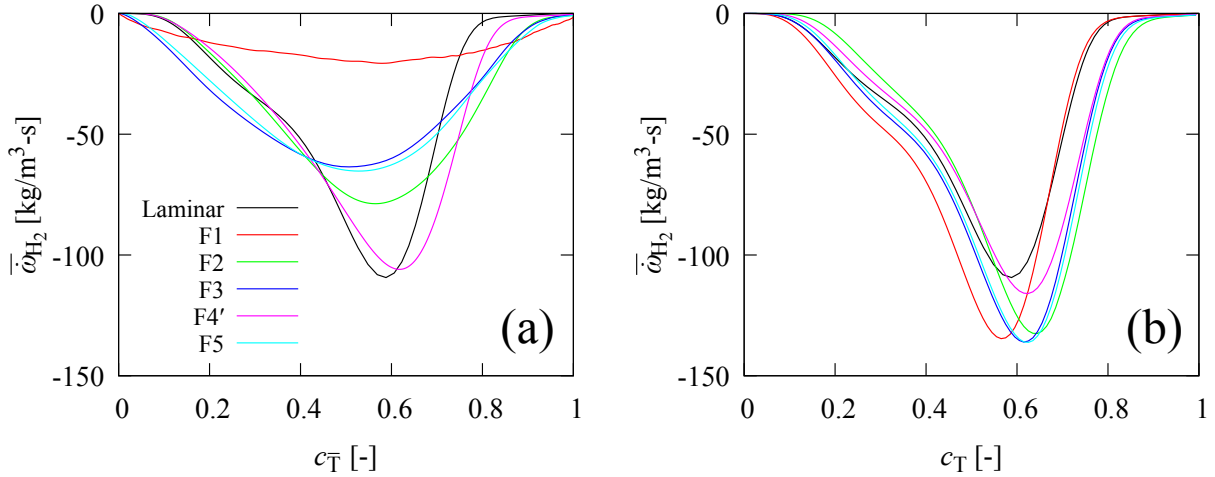


Figure 12: Cross-sectional (a) and conditional (b) average of the consumption rate of the fuel species in comparison with the laminar counterpart.