Gokula Vasantha
School of Engineering and the Built Environment,
Edinburgh Napier University,
10 Colinton Road, Merchiston Campus,
Edinburgh, EH10 5DT, Scotland
e-mail: G.Vasantha@napier.ac.uk

Jonathan Corney[1]
Department of Design, Manufacture and
Engineering Management,
University of Strathclyde,
75 Montrose Street,
Glasgow, G1 1XJ, Scotland
e-mail: jonathan.corney@strath.ac.uk

Struan Stuart
Department of Design, Manufacture and
Engineering Management,
University of Strathclyde,
75 Montrose Street,
Glasgow, G1 1XJ, Scotland
e-mail: struan.stuart.2014@uni.strath.ac.uk

Andrew Sherlock
School of Engineering,
University of Edinburgh,
Robert Stevenson Road, The King's Buildings,
Edinburgh, EH9 3FB, Scotland;
ShapeSpace Ltd,
Edinburgh, Scotland
e-mail: a.sherlock@ed.ac.uk

John Quigley
Department of Management Science,
University of Strathclyde,
199 Cathedral Street,
Glasgow, G4 0QU, Scotland
e-mail: j.quigley@strath.ac.uk

David Purves
Department of Management Science,
University of Strathclyde,
199 Cathedral Street,
Glasgow, G4 0QU, Scotland
e-mail: david.purves@strath.ac.uk

A Probabilistic Design Reuse Index for Engineering Designs

Many companies offer a range of related products that are constructed using similar components and processes. This enables them to meet customer expectations of product variety while minimizing the overheads (e.g., development and manufacturing costs). To support the management of product variety several indices have been proposed in the literature that measure the degree to which component use is standardized across products within the same product family. However, the derivation of some of these statistics can be laborious to calculate due to the effort required to assemble the necessary information. In this paper, we develop an index more suited to the automated data-mining of a company's product portfolio, which is derived from the Kullback–Leibler divergence. The new measure provides an easily computed probabilistic measure that can be used to characterize the degree of component reuse within a single product, across a family of products, and at the individual component family level. To illustrate their applications, the indices and several existing measures are calculated for two contrasting product types; using the non-differentiating components of two flat-pack furniture ranges and the components of a range of bicycles. [DOI: 10.1115/1.4046435]

Keywords: component reuse, commonality, product family design

1 Introduction

Product design manufacturers experience efficiency gains through the production of families of related products which share many components and design processes but offer enough variety to meet customer needs [1]. Instead of producing an inventory of highly differentiated products, a collection of related yet distinct products are constructed using similar components [2]. The benefits of creating a family of product variations is widely recognized [3], with the technique leading to a more efficient design process [2], lowering design complexity with reduced component inventories, and with new products introduced more quickly [4,5]. Various commonality indices have been proposed in the literature to quantify the extent to which companies share components and processes across the product families. These indices provide a measure of component reuse [6,7], and extend to including costs [5], materials and processes [8,9], and the diversity in the end product [1,4], however, they have not been widely adopted in the industry [3]. There is a requirement for providing important, relevant, and relatable information for effective reuse and metrics usage [10], and details on stock inventory, material costs, and inputs on manufacturing designs and processes all may be required to calculate several of these indices. Additionally, there may be difficulties in leveraging the results—the value of an index can give an indication that an improvement in design reuse is possible, but it may not be specific on which facet of the design could be improved.

This paper extends the Probabilistic Design Reuse Index (PDRI) that was introduced in Ref. [3], which provides a measure of reuse within product designs. It can quantify the extent to which component reuse across a family of engineering products diverges against using a random design strategy, where it is assumed that equivalent components are selected for use uniformly from each component family. The further a product design configuration is from a design created using randomly selected components, the better the

[1]Corresponding author.
Contributed by Design Theory and Methodology Committee of ASME for publication in the JOURNAL OF MECHANICAL DESIGN. Manuscript received February 4, 2019; final manuscript received January 29, 2020; published online February 22, 2020. Assoc. Editor: Christopher Mattson.

degree of product reuse. The index provides a computationally tractable measure of reuse, which is sensitive to design complexity, and that can be used to quantify the amount of component reuse at the individual component family level or aggregated over the complete design. This supports the identification of the best and worst designs in terms of component reuse within a product range, providing managerial utility for design process improvement, which is illustrated via a case study.

In Sec. 2 we briefly review the literature describing existing commonality indices before we introduce the PDRI in Sec. 3. The PDRI is then demonstrated and compared against other reuse measures using a range of flat-pack furniture designs and bicycle ranges in Sec. 4. Finally, the results and implications of the index on management of the design process are discussed.

2 Related Research

The literature focuses on evaluating commonality within a product family, for example, the percentage of components that are common across a product family; greater commonality reflects an increase in standardization of components and processes across the different products. We adopt the definition of "Commonality" proposed by Boas et al. as the "sharing of components, manufacturing processes, architectures, interfaces, and infrastructure across the members of a product family" [11]. A commonality index is a metric that is used to assess the degree of commonality within a product family based on different parameters such as the number of common components, the component costs, the manufacturing processes, and so on. Table 8 of the Appendix summarizes the existing commonality measures proposed in the literature.

2.1 Existing Reuse Measures. The terminology used in the literature varies significantly from paper to paper and creates difficulties in the interpretation of the different contributions (see Table 8 and references therein). We have adopted the following terminology to enable a consistent presentation:

- *Item:* Any product, assembly or component.
- *Component:* An item that does not further divide into sub-assemblies or components.
- *Assembly:* A collection of sub-assemblies and/or components.
- *Product:* An assembly usually for sale as a line item to a customer.
- *Type (component type, assembly type):* An item "type" (e.g., M8 nut or USB port assembly) refers to all items of the same type, with, for example, same geometry, material, tolerances, etc.
- *Family (component family, assembly family, product family):* A group of functionally or structurally similar items.
- *Presence:* Records the presence of an item in a product (e.g., whether the product has that component?).
- *Occurrence:* An occurrence records the cumulative presence of an item type in a product. This defines how many times the product has that component (e.g., eight bolts, three USB ports).
- *Product structure:* A graph-like structure recording the presence of sub-assembly and/or component occurrences in products.
- *Option (component, assembly, product):* Alternative items available within each family.

2.2 Balancing Variety and Commonality. Researchers have sought to understand how levels of reuse vary with different business models and market strategies. For example, Simpson [12] proposes the commonality-variety benchmarking chart generated by integrating commonality and variety indices to compare competing product families and their platform elements, with an example as shown in Fig. 1. The chart plots normalized Generational Variety Index (GVI) [13] against Product Line Commonality Index (PCI) values [8]. The GVI measures the amount of redesign effort required for future designs of the product and assesses the necessary component variety in customer needs and requirements through the quality function deployment approach. The PCI measures commonality at the component/module level as well as at the product family level. The GVI and PCI scores for each component can be associated with the following classifications: costly components and unvalued uniqueness, which indicate when variety is not required for the design but components are not made common thus impacting costs, properly platformed, when variety and commonality are appropriately matched, and market mismatch and confusing commonality, where there is a disparity on the desire for a unique product but a high level of commonality between components.

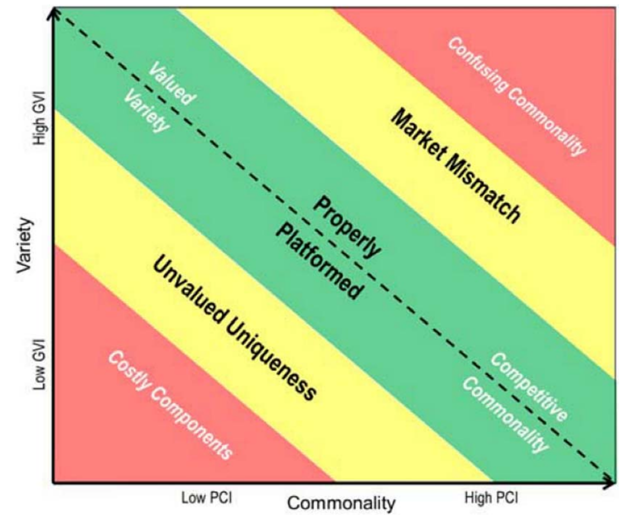

Fig. 1 Commonality-variety benchmarking chart using GVI and PCI indices [12]. All figures are best viewed on screen.

Another tool used to visualize the relationship between commonality and variety is the commonality/variety trade-off angle [14]. The angle value varies between 0 deg to 90 deg and is defined as a function of the weighted sum of the strategic factors' quantitative impact on commonality and variety in a product family. These factors cover five categories—market, product characteristics, life-cycle processes, government and industry regulations and/or standards, and organizational capabilities. The angle factor in the product family evaluation graph (PFEG) can be used to find best the product family design option among sets of alternatives based on their performance with respect to the ideal commonality/variety trade-off. So whereas Kota et al. [8] suggest typical amounts of commonality and variety, Ye et al. [14] support the benchmarking of ideal and actual commonality and diversity measure. However, the measures used are constructed using several subjective elements which could lead to uncertainty and impact the decision-making process.

2.3 Observation on Commonality Measures. The review of the reported commonality metrics in Table 8 in the Appendix gives rise to the following observations. The emphasis of much of the reported work has been to maintain metric consistency in the face of increasing/decreasing product and component types. However, the published measures are typically validated on small datasets and consequently their behavior when applied at scale to a product portfolio consisting of many hundreds or thousands of items is not well understood. Given that high value products are typically assemblies of many sub-assemblies of specialist components, it is important that measures are available to characterize commonality at different levels of product structure (e.g., component, assembly, function, within/between product platform, and product types) (e.g., %C, PCI, CDI, CMC). Additionally, many of the reported metrics are information intensive (e.g., $CI^{(C)}$, CDI, CMC) and require multiple factors to calculate the commonality (e.g., product structures, component costs, materials, manufacturing processes, assembly, end-item volumes, quantities per assembly, and so on). To operate at scale, the data required for the calculation should be both readily available and accurate. The commonality measure should be informative; a normalized commonality measure that defines levels of design reuse with a relative index between absolute boundaries (i.e., scale of 0–1), rather than open or moving boundaries (e.g., Degree of Commonality Index (DCI) measure), simplifies the comparison of designs between product ranges and revisions. Ideally, a reuse measure should highlight components whose redesign would have a significant impact on the product's commonality measure (e.g., expensive, high volume, multi-functional, and components used in important products). $CI^{(C)}$ and CMC measures take a component's cost and volume information into account. For example, it would be preferable to increase the use of high-volume components across a number of products rather than low-volume components. Given the above it is obvious that a commonality metric should factor component quantities (e.g., production volume) even if the usage of components is uniform across products. Metrics that distinguish between differentiating and non-differentiating components—those components which provide additional product functionality or variety and those which do not—can avoid penalizing commonality score when designers deliberately incorporate differentiating factors (e.g., PCI, CDI, TCI). Most of the metrics focused solely on identifying commonality at the different levels of product family structure. However, very few give insights into the implicit trade-offs that exists between commonality and variety across a product type (e.g., CDI). This line of research enquiry is important because commonality may have adverse impacts on design efficiency, for instance when extreme standardization results in excessive functionality [15].

Another major limitation of the literature metrics is the lack of a reference dataset to gauge results against. A relative scale of zero to one provides a benchmark against an ideal scenario, but this may not provide sufficient insights for increasing commonality. This research develops a commonality index that incorporates a benchmark mechanism. The proposed index can be used to identify component types that can be potentially reduced, and also highlight the opportunity to increase occurrences of other components. Section 3 introduces the proposed reuse index that addresses many of the issues discussed.

3 Probabilistic Design Reuse Index

The PDRI was introduced in Ref. [3] as a measure to quantify the degree of component reuse within a product design process. It is assumed that there are a number of component families that are used within an product, and that each of these component families can contain multiple non-differentiating component types, which are substitutable with each other. That is, each of the substitutable components serves the same function and does not impact the quality of the final product. In practice, care is required at this stage to ensure that the components are substitutable.

We can characterize the component reuse strategy by contrasting an efficient process that demonstrates a higher concentration of use on fewer components, against a purely random design mechanism (PRDM), where each component family member is equally likely to be selected for use. We can then model this difference in reuse performance by comparing the probability distributions under each strategy. The hypothesis motivating this comparison is that the best performing organizations concentrate all reuse on one member of each component family and the poorest makes equal use of all family members.

We do not directly observe an organization's probability distribution for component selection, only the frequency in which components have been selected. Assuming that component selection is independent of past product choices, and the probability of a component being selected is the same for all products then we have a multinomial distribution [16] to describe the randomness of the data. An implication of this assumption is the number of selections for each component is a sufficient summary statistic on performance compared with the PRDM. Dependency on past product choices could reflect a learning period and as such the analyst should carefully select the time horizon for the assessment of the comparison, as recent observations may provide a better assessment of current practice. Such a judgement should be based on shifts in product technology or designer experience (Table 1). Specifically, we use c to denote the number of component families within the comparator set and the total number of products is denoted as m. For each family of components i we denote with n_i the number of options, i.e., the number of members of the component family. We denote with p_{ij} the conditional probability that component j from family i is selected, given a component from component family i is selected for a particular product. We denote the observed number of selections of component j from family i with x_{ij}. P represents the matrix of probabilities p_{ij} and the likelihood function for the data, i.e. the probability of observing the data as a function of the conditional probabilities, is expressed in Eq. (1)

Table 1 Notation for PDRI derivation

c	number of component families
m	total number of products
n_i	the number of unique components within component family i
p_{ij}	probability that component j from family i is selected given a component from component family i is selected
x_{ij}	observed occurrence of component j from family i

$$L(P) \propto \prod_{i=1}^{c} \prod_{j=1}^{n_i} p_{ij}^{x_{ij}}$$
$$\text{s.t.:} \sum_{j=1}^{n_i} p_{ij} = 1, \quad \forall i \tag{1}$$

While we do not directly observe P, we can make inferences. The maximum likelihood estimate (MLE), i.e., the value of the conditional probabilities that would most likely result in the observed data, denoted by $\widehat{P}$ has elements expressed in Eq. (2)

$$\widehat{P}_{ij} = \frac{x_{ij}}{\sum_{j=1}^{n_i} x_{ij}} \tag{2}$$

The PRDM has $p_{ij}^{\text{PRDM}} = 1/n_i$ and as we increase the number of products, i.e., m, the estimates $\widehat{p_{ij}}$ will converge with the true underlying probabilities for the organization, i.e., p_{ij}. We seek a measure for the difference between $\widehat{p_{ij}}$ and p_{ij}^{PRDM} $\forall i, j$. The Kullback–Leibler divergence measure [17] provides a means to measure the difference between distributions and is expressed in Eq. (3) for family i

$$D_i = \sum_{j=1}^{n_i} \widehat{p_{ij}} \ln \left(\frac{\widehat{p_{ij}}}{p_{ij}^{\text{PRDM}}} \right) \tag{3}$$

If $\widehat{p_{ij}} = p_{ij}^{\text{PRDM}}$ for all options j then $D_i = 0$ and as the difference grows so does D_i. This measure can be re-expressed as a ratio of likelihood functions, so the difference between the observed frequencies and the PRDM is measured relative to the probability of having generated the observed data. If the observed frequencies are not consistent with PRDM, then this measure will be large. This is derived in the following Eq. (4)

$$D_i = \frac{1}{\sum_{j=i}^{n_i} x_{ij}} \ln \left(\prod_{j=1}^{n_i} \left(\frac{\widehat{p_{ij}}}{p_{ij}^{\text{PRDM}}} \right)^{x_{ij}} \right) \tag{4}$$

$$= \frac{1}{\sum_{j=i}^{n_i} x_{ij}} \ln \left(\frac{L(\widehat{p_{ij}})}{L(p_{ij}^{\text{PRDM}})} \right) \tag{5}$$

Asymptotically, when there is a large number of products, the value of this measure is characterized with the χ^2 distribution when the differences are due to sampling variation only, i.e., $p_{ij} = p_{ij}^{\mathrm{PRDM}}$. This result can be used to map the distance measure to a scale between zero and one and can facilitate interpretation. Specifically, as m tends to infinity, under the null hypothesis $p_{ij} = p_{ij}^{\mathrm{PRDM}}$ then

$$-2\ln\left(\frac{L(P)}{L(\widehat{P})}\right) = -2\sum_{i=1}^{c}\sum_{j=1}^{n_i} x_{ij}\left(\ln\left(\sum_{j=1}^{n_i} x_{ij}\right) - \ln(x_{ij}) - \ln(n_i)\right) \tag{6}$$

has a χ^2 distribution with $\sum_{i=1}^{c} n_i - c$ degrees of freedom [18]. As such we can evaluate the cumulative distribution function (CDF) of the χ^2 distribution at the value of the statistic of Eq. (6) to obtain a measure between zero and one; this PDRI measures the probability that such a χ^2 random variable would be less than the observed statistic, which provides a measure of how extreme the statistic is. When there is no difference between the PRDM and the organization, then the measure would be zero and as the observed difference grows, the measure approaches one.

3.1 Performance of PDRI With Few Designs. The use of the measure χ^2 distribution is justified using asymptotic theory for a large number of products. For situations with few products, this distributional assumption would not be appropriate, and a simulation exercise would be required. For small samples we would caution using this for strict hypothesis testing as the actual significance level attached to the test would be different from the value obtained from the χ^2 distribution. However, even with few products, the measure with the χ^2 distribution can still be used as a measure of distance with the PRDM; it will be a monotonic transformation of the true CDF value and as such if an organization were to score higher than another using the χ^2 distribution this ranking would be maintained with the true CDF evaluation.

We conducted a simulation study using R statistical software [19]. The following three parameters were controlled for in the study: the number of products, i.e., $m = 3100$, the number of component families, i.e., $c = 310$, and the number of component family members (options), i.e., $n = 310$. For each set of parameters, 200 simulations were realized and the empirical CDF (ECDF) was compared with the corresponding χ^2 CDF. That is, for one iteration of the simulation, c random draws of size m were taken from the multinomial distribution with n classes, each sampled with uniform probability $1/n$. This was repeated 200 times, and the χ^2 and ECDF calculated. Figure 2 illustrates a sample of the results of this simulation. As expected the χ^2 statistic quickly tends to the ECDF as the number of products increases, and additionally, it is clear that the performance is also a function of the number of component families and options; the χ^2 statistic is typically greater than the empirical CDF evaluated at the same value, particularly when there are more component options but component use is low or the number of products is sparse. The difference between the CDF's (empirical minus χ^2) was assessed and the following summary measures were produced: maximum difference, minimum difference, median difference, as well as Pearson's correlation between the two CDF's. A sample of these results is provided in Table 9 in the Appendix. From Table 9 it can again be observed that while the correlation coefficient is high (typically above 0.9) there can be aspects where the CDF's are far apart, but the difference generally decreases with more products.

We conducted a second simulation study to assess the 95th percentiles. Specifically, for situations where the χ^2 CDF was below or above its 95th percentile, we assessed whether the empirical distribution was above or below its 95th percentile. The results are provided in Table 2 and for this simulation 100,000 runs were performed for each parameter combination. It is observed that if the χ^2 CDF is below the 95th percentile, then the empirical was also. However, even with designs of 100, agreement above the 95th percentile is questionable. This would be relevant if the measure was to be used in the form of a hypothesis test, suggesting the χ^2 test is more accurate when accepting the null hypothesis than when rejecting it. In sum, if the χ^2 analysis show no significant difference then the true CDF evaluation which is more computationally expensive would conclude similarly, however if the χ^2 analysis suggests a significant difference, the true CDF evaluation may differ. This is only a problem if the analysis is being used as a strict hypothesis test rather than as a distance measure.

4 Application of PDRI to Case Study

In this section we introduce the datasets used in the case studies followed by the application of the existing and new commonality indices.

4.1 Description of Datasets. Several datasets describing various product lines were created from the manual inspection of either the assembly instructions of a flat-pack furniture company or using the bike model specification sheets (BMSS) available from a bike manufacturers' website. The bicycles were categorized as either road, mountain, or hybrid, consistent with the manufacturers' classifications and the furniture items by product use. We assume that each dataset comprises of one product family, although recognize that the products could be further categorized into sets of related product families. Component occurrence for each product was entered into a spreadsheet. Unique components were listed in the rows, unique products along the columns, and the occurrence for each component/product pairing entered into the relevant

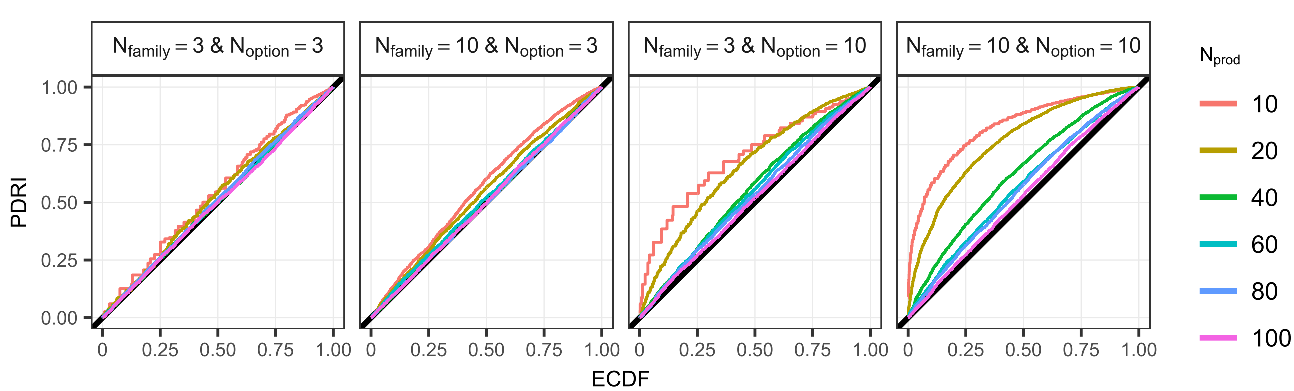

Fig. 2 Comparison of PDRI with empirical cumulative distribution function: the PDRI is on the *y*-axis, the ECDF on the *x*-axis, and the black diagonal gives the line of perfect agreement; where the PDRI is equal to the ECDF. The number of products in each simulation run is differentiated by color.

Table 2 Simulation to compare difference between the 95th percentile of the χ^2 statistic and the ECDF

Options	Families	Products	P (ECDF <0.95| χ^2 < 0.95)	P (ECDF >0.95| χ^2 > 0.95)
3	3	50	1.00	0.96
5	5	50	1.00	0.84
7	7	50	1.00	0.70
10	10	50	1.00	0.49
3	3	100	1.00	1.00
5	5	100	1.00	0.92
7	7	100	1.00	0.87
10	10	100	1.00	0.74

Table 3 Product descriptive summary

Product	N_{product}	N_{families}	$N_{\text{option}>1}$	Mean options
Chair	137	26	22	18.3
Wardrobe	209	27	23	13.1
Hybrid	10	20	20	5.2
Mountain	10	20	20	6.1
Road	10	20	20	5.5

field. This data format facilitates the calculation of the PDRI with standard spreadsheet tools. Subsequently, structurally similar components were organized into families (e.g., for the furniture set hinges, dowels, etc., and for the bike set frames, saddles, etc.) to allow application of the proposed PDRI measure. Although not all structurally similar components will be substitutable in practice, by assuming they could be, the PRDM can be used to focus design reviews that require differences to be justified.

Table 3 summarizes the number of products within each product family, the unique component families (e.g., hinges), how many of these families have more than one component option, and the average number of component options per family.[2] While there are many more wardrobe than chair products the number of component types is similar, and both lines have multiple options and few single component family choices indicating some potential for reuse improvement. The number of component families is the same across the bike models with all having multiple options; however, the Mountain bikes have a greater number of component options which reflect an increased use unique components; for example, each mountain bike model has a different fork type.

Table 4 provides some descriptive statistics of components across each of the datasets; the number of distinct components, and of these the number of unique components that are only used in one product, and the number of common components that are used in more than one product. The total component presence gives the sum of the parts present across the product range, and total occurrence gives the cumulative presence. The component range within products summarizes the range of parts that are used within a product (e.g., for presence, wardrobe-1 may have four different components present, whereas wardrobe-2 has 68), and between products summaries how the individual components are used across the products (e.g., one component may only be used in one wardrobe and another component in 194 different wardrobes). There is a wide range of how components are used in the chairs and wardrobes, with many components only used once, and others used multiple times; a small nail to secure the back panel is used 12,150 times across the range of wardrobes. In comparison to the wardrobes, the chairs have a greater number of component options which in part may indicate a need for a greater variety in these types of products but may also evidence that the chairs in our dataset form a more heterogeneous set of products; the line extends from plastic children's highchairs to armchairs. Interestingly the bike models have a similar proportion of components that are only used in one product when compared to the furniture items; a priori it may have been expected that the bike models would have a greater number of unique components to increase product variety. This can be seen in Fig. 3 which shows the distribution of how often a specific component is used across the products; the x-axis gives the number of products that have a component, and the y-axis the proportion of component types that are used. So for example, with the mountain bike data, ∼65% of components are used once while only 15% of components are used in more than two bike designs. In both product lines components are most frequently used only once or twice. A dataset and an example of the PDRI calculation are available.[3]

4.2 Assessment of Existing Reuse Measures. The focus of this case study is on the presence and occurrences of the component types in each of the datasets. As only assembly components were collected for the furniture datasets none were considered to be differentiating, and so no product variety, such as material color, was incorporated in the datasets. In contrast, different components provide variety to the bike models and so while differentiating, for the purpose of this study we assume that they are substitutable. Again color and size were not included.

We compare a range of commonality measures and the PRDI on the furniture datasets in Table 5; however, commonality measures requiring non-geometric information (such as component costs, materials, manufacturing processes, assembly, and quantities per assembly) are not incorporated in this work (e.g. Component Part Commonality Index $CI^{(C)}$, Commonality versus Diversity Index (CDI), Comprehensive Metric for Commonality (CMC) and Total Commonality Index (TCI)). *Correlation between component usage:* Spearman's rank correlation was used to measure any association between component presence and occurrence within and between products. To clarify, for "within product," the correlation between the distributions of component presence and component occurrence in each of the products was measured; for chairs, the component presence and occurrence are summed within each model giving 137 values of each, and the correlation between these measured. Similarly, 'between products' measures the correlations between presence and occurrence, which are summed across the products. These correlations demonstrate that the total component occurrences in each product increase with the addition of a new component, and together with Fig. 3 demonstrate that each company does not reuse components effectively across its products. *Degree of Commonality Index (DCI) and Total Constant Commonality Index (TCCI):* The product structure of each of the furniture datasets is flat as there are no sub-assemblies, and consequently every component is assumed to have a single parent in the product structure. As such, we calculate the DCI as a ratio between the total component occurrences in a product type and the total number of component types; it represents the average usage of a component type in each product. However, comparison of the DCI scores across the products is difficult as larger products are likely to use more components, leading to an increase in the DCI. The TCCI's are calculated using similar assumptions as that of the DCI. The TCCI scores across the products illustrate the scope for commonality improvement; however, it is debatable whether high TCCI values truly benchmark against the maximum level of commonality possible; larger products again score higher. Given the assumptions in our derivation, DCI and TCCI rank the products similarly in terms of commonality. *Commonality index (CI):* The CI

[2]It should be noted that only families with more than one option are considered in this reuse calculation; the reuse is already optimal if there is only one component type being used across the product family.

[3]https://doi.org/10.15129/f931bfd7-1090-408e-8823-e4604b485713

Table 4 Product component summary

Product	N_{comp}	N_{uniq}	N_{comm}	Total component presence	Total component occurrence	Component range within products		Component range between products	
						Presence	Occurrence	Presence	Occurrence
Chair	406	284	122	790	3118	1–24	1–137	1–36	1–216
Wardrobe	306	112	194	7274	48,769	4–68	19–1094	1–194	1–12,150
Hybrid	104	55	49	192	246	17–20	21–26	1–6	1–8
Mountain	122	79	43	186	232	17–20	20–26	1–5	1–6
Road	111	65	46	192	246	19–20	24–26	1–7	1–8

Component presence and occurrence are described and the range of components within and across products is reported. There is a wide range in the proportions of unique components between the related properties which is suggestive of contrasting requirements on the component variety.

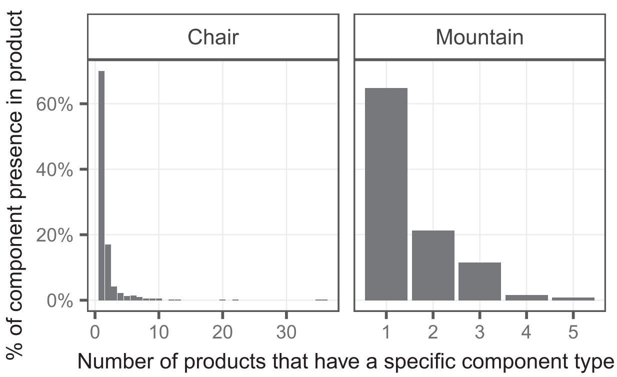

Fig. 3 Distribution of components across products: figures indicate that many components are only used once or twice

represents the ratio between the total number of component types to the component's presence; the higher the CI value, the lower this ratio, signifying greater commonality. The bike models are again similarly ranked, as are the furniture products, however in this case the chairs are scored similar to the bicycles due to a comparable ratio between the number of component types and the total number of component presence. *Percent Commonality (%C):* In this study, only the component commonality percentage is used within the Percent Commonality score (i.e., connection commonality and assembly commonality are not considered). The %C score provides a measure of how often components are used across product, and so whether an increase in common component use is indicated. The % C score is lower for the chairs compared to wardrobes confirming the previous summaries, that the chair product range uses more unique components. *Product Line Commonality Index (PCI):* Since size/shapes for each component type are considered identical in this study, the *f*1 factor is considered to be one. The other *f* factors (e.g., materials/manufacturing processes and assembly processes) are removed since they are out of scope for this work. The revised PCI formula used in this study is provided in Eq. (7)

$$\frac{\sum_{i=1}^{P} n_i - \sum_{i=1}^{P} 1/n_i^2}{P \times N \times \sum_{i=1}^{P} 1/n_i^2} \times 100 \tag{7}$$

where P denotes the number of component types that can potentially be standardized in a product type, N the number of products in a family, and n_i the number of products in a family that have component i. The PCI percentage for a product type quantifies the component types' presence across products with reference to the maximum possible component commonality index (i.e., $P*N$). There is a lot of variation in the PCI between the furniture types, and the low PCI percentages indicate the scope for improvement of the level of component presence across products. Particularly, reducing the component types used only once in a product will increase the PCI percentages. *Relative commonality (RC):* Figure 4 presents a plot of the RC score versus the cumulative percentage of component types. The RC of each component is ranked by magnitude, and plot after scaling the number of components within each dataset to the (0,100) range. The *x*-axis plots the RC and the *y*-axis gives proportion of component types. On average 60% of the component types received a zero RC score across the bike products providing another indicator of when a component is only present in one model. It is again seen that chair designs use many more unique components compared to wardrobes. RC has a strong linear association with log-transformed component presence, and a more dispersed linear association with log-transformed component occurrence. This indicates that the RC score gives some measure of priority to components used across multiple products rather than increasing component occurrences.

4.3 Assessment of PDRI. The χ^2 distribution value for both the furniture products is one.[4] The high score demonstrates the degree of differentiation in component usage between the furniture company and if the components were selected using a PRDM; the furniture company, at the aggregated level, used a higher concentration of fewer component options within the families. This also indicates one of the shortcomings of using the probability measure as a strict hypothesis test. As m increases, the test has increased power to identify any differences from the null hypothesis, and so the PDRI tends to, but never reaches one; it is through each independent choice that designers distinguish themselves from a purely random designer, which is measured in the χ^2 statistic. As such the χ^2 converges to one as the number of independent design choices increases. Thus, the test may return a high value even if there is little practical difference between a companies component selection and the PRDM. While we can evaluate the measure to more decimal places to make a rank comparison between design practices, the intention with this measure is not to discriminate between the performances of such high-performing designers; company databases of products will grow with time, the value of such a measure is in assessing current practice and as such only data that are representative of recent practice should be used within the study. Results for the bike models are a bit more interesting; the PDRI ranks the bike models similar to the previously calculated indices but additionally provides an indication that the levels of reuse within these products are low. If the bikes are grouped into one product family, so that there are 30 products, the PDRI tends to one, again illustrating a sensitivity to the number of products.

We further examined the PDRI at the component family level; the χ^2 value was calculated for each family to understand the variation individually with reference to PRDM, and Fig. 5 plots the results at the component type level for a sample of products. In terms of efficient reuse we would want component families to have few options

[4]These results was replicated through a simulation exercise.

Table 5 Commonality statistics: calculation of standard commonality measures and PDRI for a range of products (see text for details)

Product	Correlation		DCI	TCCI	CI	%C	PCI	PDRI	
	Within products	Between products						Stat	χ^2
Chair	0.85	0.60	7.68	0.87	0.50	30.00	0.88	3056.90	1.00
Wardrobe	0.90	0.85	159.40	0.99	0.97	63.40	11.20	13,7138.70	1.00
Hybrid	0.96	0.82	2.37	0.58	0.51	47.10	13.10	63.10	0.04
Mountain	0.94	0.80	1.90	0.48	0.39	35.20	8.72	46.90	<0.001
Road	0.79	0.80	2.22	0.55	0.47	41.40	11.40	65.60	0.02

and have a high PDRI score, which places them in the top left of the plot. The borders around the points represent groups of the component family within which it is most likely that component reuse improvements can be made: The *blue* border identifies emerging or rare families with very low χ^2 values. This zone has few options and occurrences in a family. Consider for example, a component type that is rarely used; it will not find itself with a high score as it has not demonstrated itself as being significantly different from the PRDM; therefore, this zone contains a significant number of families that behave similar to that of the PRDM. The *red* border identifies matured families with many available options. All families with a high number of options have greater than 0.95 χ^2 values. However, the families in this zone should be the focus of initiatives to reduce/eliminate the available options. The *orange* border identifies the high component occurrence families. Again all families with high component occurrences have greater than 0.95 χ^2 values. The focus in this region is to reduce the spread of component occurrences across available options. The *grey* border indicates component families that have reasonable reuse performance; with fewer component options and high PDRI.

4.4 Comparison of PDRI to Degree of Commonality Index. Given our assumptions, the PDRI, DCI, and TCCI are more sensitive to increasing product quantity and component occurrence compared to the other measures which are calculated using component presence; however, Sec. 4.2 illustrated that in ranking the different product families by the indices the measures produced similar results. Motivated by this we compared how PDRI ranks component families against the other measures; Fig. 6 plots the rank DCI against the rank PDRI for each component type. For both the chairs' dataset and the bikes, which were combined into one product family, the PDRI is strongly correlated; however, there are several outliers. The PDRI identifies the chair "support" components as being efficiently reused, whereas the DCI returns a moderate finding; the high PDRI is due to one component being used several times in multiple chairs compared to most of the other support components which are only used in one product, whereas the DCI is lower as it averages the high occurrence over the many support component options. Similarly, there is one "fork" component that is used in 12 out of the 30 bikes which generates the high PDRI, whereas the DCI captures that there are many other options that were only used once. Both of these cases again

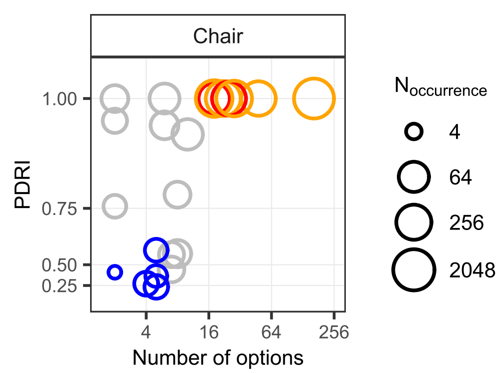

Fig. 5 Probability Design Reuse Index for chair component families: the points give the PDRI for each component family. The total number of occurrences is represented by the size of the points, and is the sum of all occurrences aggregated by component family, the *y*-axis represents the test statistic and the *x*-axis gives the number of different component types within a family. The colors indicate the varied scope for reuse improvement (details in text).

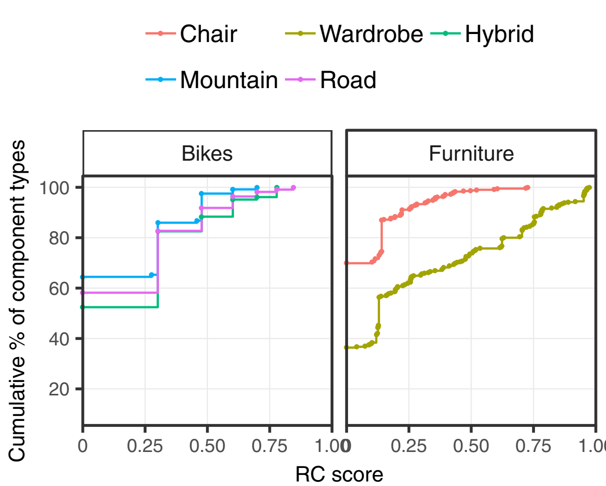

Fig. 4 Relative commonality score and cumulative percentage of component types: low RC values indicate more differentiable components and high RC values indicate components that are more frequently used

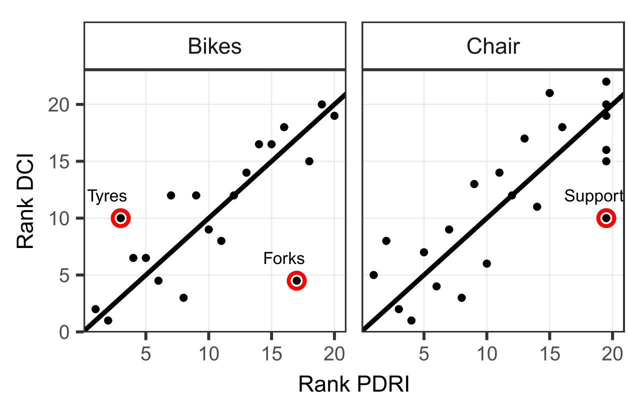

Fig. 6 Comparison of DCI against PDRI scores: ranked indices for each component type are plot with disparity for some components indicated

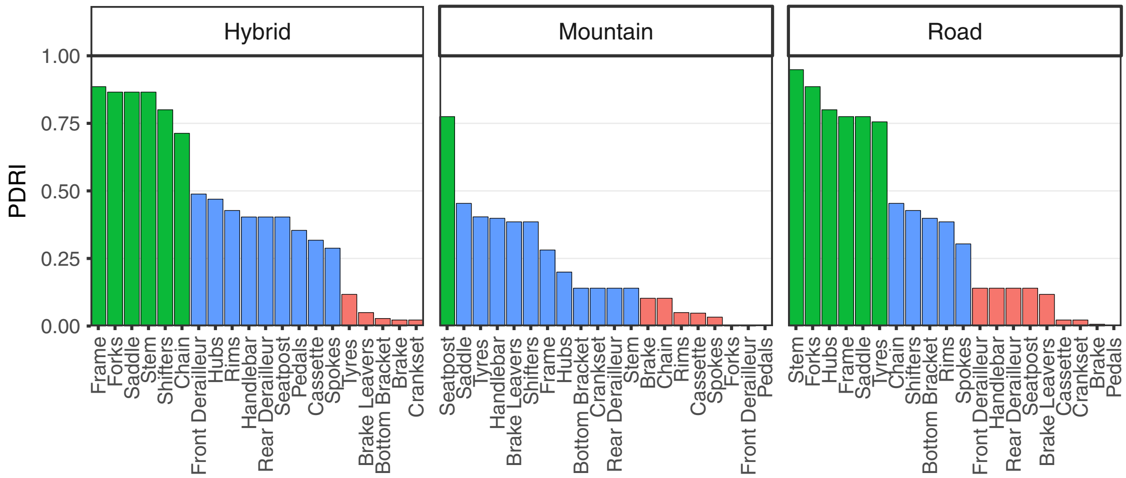

Fig. 7 Distribution of PDRI across bike range: the PDRI was calculated for each component type, and the subjective classification of low, medium, or high reuse levels is indicated by red, blue, and green colors, respectively

illustrate how the PDRI is sensitive to an increasing number of products. For "tires' the direction is reversed, with a low PDRI against a moderate DCI. The low PDRI is explained by the options being fairly evenly spread across the bike models; however, the DCI is larger compared to other component families as each bike model has two tires compared to many other components that occur only once. This draws the attention to the moving upper boundary of the DCI.

4.5 Comparison of PDRI to Expert View. Calculation of the component level PDRI for each bikes class gives rise to the observation that the levels of reuse in the component families can be categorized as either low (red), medium (blue), or high (green) in Fig. 7, although it is harder to distinguish between commonality levels for the mountain bike model, illustrated in the central panel of Fig. 7. We compared this reuse categorization to a subjective grouping performed by a product manager from the bike manufacturer who ranked the components reuse into the same three categories. A description of each level was communicated to the manager. We contrast the results from the PDRI against the bike manufacturers in Table 6. Twenty-seven of the classifications were in agreement with a further 30 one level different, which was somewhat anticipated due to the subjective classification boundaries. Interestingly, three components that the expert considered to have high reuse were classified as low by the PDRI; each of these components had six options across the ten bikes, and were fairly uniformly distributed. Identification of such items provides a system for a manager to evaluate component reuse.

5 Best and No Reuse Mechanisms

The PRDM is a concept developed to provide a benchmark for assessing the concentration with which components are reused within families. The PRDM selects components from a defined list and does so at random with components being equally likely to be selected. The χ^2 value provides a measure of the distance an organization is from the PRDM. However, the PRDM is not the most inefficient mechanism. The no-reuse mechanism (NRM) characterizes a designer that never visits the catalogue of historical products and consequently every new product they create is entirely original with no reuse.

Figure 8 illustrates the relationship between an organization (labeled X), the number of component types, and the total component presence in a product type. An efficient organization generates many products from few components and as such the steeper the slope, the more efficient the organization. There is a lower limit on the slope of this relationship; as NRM (labeled L) always creates new components for each new product then the NRM will always be on the 45 deg line. The angle α (known as the "healthier" angle) between the NRM line (L) and an organization's line (X) provides a measure of re-use (i.e., larger the value of α, the healthier to levels of reuse). To calculate the slope of the NRM line, the number of component types was calculated by multiplying an average number of component types in a product and the number of products in a product family. Whereas for the best reuse mechanism (BRM), the number of component types is equal to an average number of component types in a product. The improvement angle β is the difference between best the BRM line (B) and the current organization line (X). These reference angles for a single product type are illustrated in Fig. 8. The avoidance of subjective information in calculating is the primary advantage of these angles. The

Table 6 Comparison of PDRI to expert judgement: categorized PDRI contrasted against the subjective assessment of component reuse by the bicycle manufacturers' product manager

Component	Road	Mountain	Hybrid
Frame	***High***	*Medium*	***High***
Forks	***High***	*Low*	***High***
Handle Bar	Low	*Medium*	*Medium*
Stem	*High*	*Medium*	*High*
Seatpost	Low	***High***	**Medium**
Saddle	*High*	**Medium**	*High*
Pedals	*Low*	**Low**	**Medium**
Shifters	*Medium*	*Medium*	***High***
Front Derailleur	**Low**	*Low*	*Medium*
Rear Derailleur	**Low**	*Medium*	*Medium*
Brake	**Low**	**Low**	**Low**
Brake Levers	**Low**	*Medium*	**Low**
Cassette	**Low**	**Low**	*Medium*
Chain	*Medium*	**Low**	***High***
Crankset	**Low**	***Medium***	*Low*
Bottom Bracket	*Medium*	*Medium*	**Low**
Rims	*Medium*	*Low*	***Medium***
Hubs	***High***	***Medium***	*Medium*
Spokes	**Medium**	Low	**Medium**
Tyres	*High*	***Medium***	*Low*

The PDRI category is given in the text, and the different fonts represent the level of difference between the PDRI value and the expert. Italics indicates if the levels were the same, bold italics indicates if the PDRI level was one level greater than the expert, bold roman if the PDRI was one level lower, and roman indicates two levels of difference.

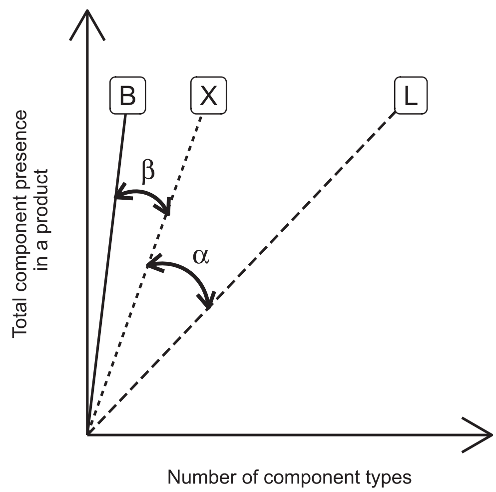

Fig. 8 Comparison of actual, minimal, and best reuse: component reuse is indicated by the number of component present against the number of product component types used. An organizations reuse within a product is labeled *X*, the potential best reuse mechanism labeled *B*, and the no-reuse mechanism labeled *L*. The angle α indicates current reuse levels and the angle β the scope for improvement.

Table 7 Best and no-reuse mechanisms angle calculation: lower values of the healthier angle α indicate more scope for reuse improvement

	Chair	Wardrobe	Road	Mountain	Hybrid
Number of products	137	209	10	10	10
Number of component types	406	190	111	123	104
Total component presence	790	5317	192	186	192
Organization reuse degree	63	88	60	57	62
Average number of component types	6	25	19	19	19
BRM reuse degree	90	90	84	84	84
Healthier angle (α)	18	43	15	12	17
Improvement angle (β)	27	2	24	27	22

PDRI values provide a method to benchmark component usage among each family. To benchmark an organization's reuse performance, the NRM and BRM were generated. Table 7 summarizes the so-called, healthier angle, (α) and the improvement angle (β) along with the calculated parameters for all products. The wardrobes have a better healthier angle (α) and low improvement angle (β) compared to the chairs, confirming the earlier findings of the commonality indices, and mountain bikes have the most scope for improvement which reflects the greater use of unique components.

6 Discussion

This paper describes the probabilistic design reuse index that can be used to benchmark reuse in an organization by comparing the distribution of usage within component families to that of a purely random design mechanism. The measure can support the design and development of both new and established families of products by providing a method of quantifying overall levels of reuse. Such a value will allow the impact of any proposed design changes to be easily assessed in a much broader context than would normally be the case. This motivation is similar to many of the proposed reuse measures documented in Sec. 2 of the paper. Its use, together with existing commonality indices, has been demonstrated across a range of flat-pack furniture designs. The existing measures provide information on which component type has a high probability of presence across products (RC), on average how many times has a component been used (DCI and TCCI), the ratio between component types and the total number of component presence in a product type (CI), and the percentage of common and unique components in a product (%C). Other than the RC score, these other measures provide different, aggregated interpretations of commonality score. In contrast, the new PDRI commonality measure presented here provides details on which component families deviate most from a PRDM, which are the best and worst performing component families in terms reuse, and it facilitates the categorization of component families into different reuse zones, which can be used to focus initiatives on design decisions; to prioritize an enterprise's engineering effort on individual component families for reuse improvements, rather than providing aggregated reuse value. The proposed measures objectively quantify levels of reuse and so aid understanding of component frequency distribution within a family. In a more recent work, Takai [20] has proposed an optimization routine that seeks to integrate commonality with both product design and costs. The results highlight that optimum commonality and product family design significantly differed when inventory decisions are incorporated in the design problem. Another recent commonality index reported in Ref. [21] again also seeks to integrate incorporate costs, from the design stage through to manufacturing. The proposed Probabilistic Design Reuse Index proposed here could be further developed to incorporate commercial data such as cost or other measures of value.

While it is clear that some variations in component type is both expected and necessary to meet customers' requirement, it is also apparent that a uniform distribution would suggest poor practice in component selection. The major advantage of this measure is that it helps to prioritize component families for review that offer the most scope for reduction in the number of component options and occurrences. It differs from the other commonality indices in that it does not explicitly measure the commonality across the products. Indeed, it is possible that the PDRI could return high values, indicating efficient reuse, even when each product uses its own unique set of components. Thus, it is more a index of component reuse rather than commonality. However, a measure for the commonality across the products can similarly be derived using the approach described for the PDRI, but making use of component presence instead of component occurrence in the calculation.

Additionally, the concepts of the healthier and improvement angles do not rely on subjective information and significantly reduce the calculation effort required to generate an informative measure of reuse. These probability measures along with the no-reuse and the best-reuse mechanisms provide clear benchmark references for organization to track improvement progression of component reuse.

Ultimately, the practicality and validation of the proposed reuse metrics will require their application in an industrial environment where the time and cost of the labor involved can be accurately quantified. Such an assessment in a commercial environment would also allow other quantities such as component costs, materials, and volumes to be incorporated into the metric and allow it to better reflect the context and implications of reuse levels in specific products.

Acknowledgment

We are grateful to Hannah McKeirnan and Scott Stevenson for their help in categorizing the components used in the case study. This work was supported by the Engineering and Physical Sciences Research Council, UK [grant reference EP/R004226/1]. A dataset supporting this research and the process to calculate the proposed probabilistic based measure is openly available.[5]

[5]https://doi.org/10.15129/f931bfd7-1090-408e-8823-e4604b485713

Appendix

Table 8 Reported commonality measures

Commonality indices and description	Formula
Relative commonality (R.C.) assesses the commonality of each component using an entropy-based measure [22]. p_{ij} denotes the quantity of component i required for product j divided by the sum of the quantities of i required for all products of the firm's line, n_i the number of products that use component i, N the number of products in the firm's line.	$\frac{-\sum_{j=1}^{n_i} p_{ij}\log_2 p_{ij}}{\log_2 N}$
Degree of Commonality Index (DCI) is a cardinal measure indicating the average number of parent items per average distinct component. It assigns a commonality level to the entire product family [6]. Φ denotes the number of immediate parents component j has over a set of end items or product structure level(s), d the total number of distinct components in the set of end items or product structure level(s), and i the total number of end items or the total number of highest level parent items for the product structure level(s).	$\frac{\sum_{j=i+1}^{i+d}\Phi_j}{d}$
Total Constant Commonality Index (TCCI) normalizes the DCI value to allow comparison of commonality at any level of the product structure. The authors also introduce measures for within-product constant commonality index, between-product constant commonality index, and incremental constant commonality index [7].	$1-\frac{d-1}{\sum_{j=1}^{d}\Phi_j-1}$
Commonality index (CI) indicates the ratio between the number of component types in a product family and the total number of component in the family [23]. u denotes the number of component types, p_j the number of components in model j, and v_n the final number of products offered	$1-\frac{u-maxp_j}{\sum_{j=1}^{v_n}p_j-maxp_j}$
Percent Commonality (%C) provides an overall measurement of commonality by weighting each index separately for measuring component commonality, connection commonality, and assembly commonality [9]. I_i denotes the Importance (weighting factors), $\sum I_i=1$, $C_i=\frac{100\ \times\ common\ components}{common+unique\ components}$, $C_n=\frac{100\ \times\ common\ connections}{common+unique\ connections}$, $C_l=\frac{100\ \times\ common\ assembly\ component\ loading}{common+unique\ assembly\ component\ loading}$, and $C_a=\frac{100\ \times\ commonassembly\ workstation}{common+unique\ assembly\ workstation}$	$\sum_{i=1}^{4} I_i\times c_i$
Product Line Commonality Index (PCI) illustrates the percentage of non-differentiating components that are shared across products in terms of their size/shapes, materials/ manufacturing processes, and assembly processes [8]. Nomenclature from Ref. [8].	$\frac{\sum_{i=1}^{P} n_i f_{1i} f_{2i} f_{3i}-\sum_{i=1}^{P}1/n_i^2}{PN-\sum_{i=1}^{P}1/n_i^2}$
Component Part Commonality Index ($\mathrm{CI}^{(C)}$) takes into account product volume, quantity per operation, and the cost of the component in addition to DCI parameters to find the commonality index [5]. Nomenclature from Ref. [5].	$\frac{\sum_{j=1}^{d}(P_j\sum_{i=1}^{m}\Phi_{ij}\sum_{i=1}^{m}V_iQ_{ij})}{\sum_{j=1}^{d}(P_j\sum_{i=1}^{m}V_iQ_{ij})}$
Commonality versus Diversity Index (CDI) assesses the commonality and diversity within a family of products or across families. The CDI score for a function computed by aggregating the CDI score for each sub-group of products that include all the components for that function. The CDI score of all the functions is aggregated to obtain the CDI score for the family [4]. Nomenclature from Ref. [4].	$\frac{1}{F}\sum_{i=1}^{F}\frac{1}{K_{ij}}\sum_{k=1}^{K_{ij}}\frac{1}{G_{ik}}\sum_{m=1}^{G_{ik}}(1-\frac{non\_allowed\_com\_div_{ikg_m}}{\max\div_{ikg_m}})$
Comprehensive Metric for Commonality (CMC) takes into account size, geometry, material, manufacturing process, assembly, cost, and production volume of components to generate a commonality metric [1]. Nomenclature from Ref. [1].	$\frac{\sum_{i=1}^{P}n_i(C_i^{\max}-C_i)\prod_{x=1}^{4}f_{xi}}{\sum_{i=1}^{P}n_i(C_i^{\max}-C_i^{\min})\prod_{x=1}^{4}f_{xi}}$
Total Commonalty Index (TCI) enables the evaluation of the overall commonality of a product family from the intermediate commonality metrics with respect to common components, must-generic items, and options. The TCI is the sum of two quantities: the commonality level with respect to common components and must-generic items, and commonality of the product family with respect to options [2].	$\left\{\frac{1}{n}\sum_{i=1}^{n}\prod_{k=1}^{n_{h_i}}(N_k)_{h_i}\sum_{j=1}^{n_i}\frac{w_{ij}^2}{n_i}\right\}+\left\{\frac{A}{m}\sum_{i=1}^{n}\alpha_i\prod_{k=1}^{n_{h_i}}(N_k)_{h_i}\sum_{j=1}^{m_i}\frac{w_{ij}^2}{m_i}\right\}$
Composite standardization index is calculated using the standardization index for each component and/or assembly and the commonality index for each assembly (TCCI), which are calculated using the absolute attribute-based standardization of each component of the system [24].	$I_c(a)@\frac{w_m\times I_m(a)+w_s\times I_s(a)}{w_m+w_s}$

Table 9 Simulation to compare χ^2 and ECDF

Options	Families	Products	Max	Min	Median	Correlation
3	3	3	0.02	−0.39	−0.12	0.94
3	3	4	0.05	−0.37	−0.19	0.93
3	3	5	0.02	−0.28	−0.15	0.96
3	3	6	0.01	−0.27	−0.12	0.97
3	3	7	0.05	−0.11	−0.03	0.99
3	3	8	0.01	−0.15	−0.08	0.99
3	3	9	0.02	−0.08	−0.04	1.00
3	3	10	0.03	−0.09	−0.04	1.00
5	5	3	0.04	−0.46	−0.23	0.96
5	5	4	0.01	−0.45	−0.26	0.93
5	5	5	0.01	−0.38	−0.25	0.95
5	5	6	0.00	−0.31	−0.23	0.97
5	5	7	0.00	−0.28	−0.19	0.97
5	5	8	0.00	−0.26	−0.20	0.97
5	5	9	0.00	−0.25	−0.16	0.98
5	5	10	0.01	−0.25	−0.15	0.97
7	7	3	0.14	−0.33	−0.09	0.97
7	7	4	0.05	−0.36	−0.16	0.98
7	7	5	0.02	−0.41	−0.23	0.96
7	7	6	0.00	−0.44	−0.30	0.94
7	7	7	0.00	−0.43	−0.28	0.94
7	7	8	0.00	−0.37	−0.27	0.96
7	7	9	0.00	−0.41	−0.31	0.94
7	7	10	0.01	−0.41	−0.28	0.92
10	10	3	0.52	−0.09	0.25	0.94
10	10	4	0.21	−0.22	0.03	0.98
10	10	5	0.07	−0.36	−0.16	0.99
10	10	6	0.02	−0.46	−0.28	0.95
10	10	7	0.00	−0.48	−0.30	0.94
10	10	8	0.00	−0.48	−0.32	0.95
10	10	9	0.00	−0.53	−0.34	0.90
10	10	10	0.00	−0.53	−0.38	0.92

References

[1] Thevenot, H. J., and Simpson, T. W., 2007, "A Comprehensive Metric for Evaluating Component Commonality in a Product Family," J. Eng. Design, **18**(6), pp. 577–598.
[2] Blecker, T., and Abdelkafi, N., 2007, "The Development of a Component Commonality Metric for Mass Customization," IEEE Trans. Eng. Manage., **54**(1), pp. 70–85.
[3] Vasantha, G., Sherlock, A., Corney, J., and Quigley, J., 2018, "A Probabilistic Design Reuse Index," ASME 2018 International Design Engineering Technical Conferences, Volume 2B: 44th Design Automation Conference; DETC2018-86155, Quebec City, Quebec, Canada, p. V02BT03A047.
[4] Alizon, F., Shooter, S. B., and Simpson, T. W., 2009, "Assessing and Improving Commonality and Diversity Within a Product Family," Res. Eng. Design, **20**(4), pp. 241–253.
[5] Jiao, J., and Tseng, M. M., 2000, "Understanding Product Family for Mass Customization by Developing Commonality Indices," J. Eng. Design, **11**(3), pp. 225–243.
[6] Collier, D. A., 1981, "The Measurement and Operating Benefits of Component Part Commonality," Decis. Sci., **12**(1), pp. 85–96.
[7] Wacker, J. G., and Treleven, M., 1986, "Component Part Standardization: An Analysis of Commonality Sources and Indices," J. Oper. Manage., **6**(2), pp. 219–244.
[8] Kota, S., Sethuraman, K., and Miller, R., 2000, "A Metric for Evaluating Design Commonality in Product Families," ASME J. Mech. Des., **122**(4), pp. 403–410.
[9] Siddique, Z., Rosen, D. W., and Wang, N., 1998, "On the Applicability of Product Variety Design Concepts to Automotive Platform Commonality," ASME Design Engineering Technical Conferences-Design Theory and Methodology 98-DETC/DTM-5661, Atlanta, GA, Sept. 13–16.
[10] Imoize, A. L., Idowu, D., and Bolaji, T., 2019, "A Brief Overview of Software Reuse and Metrics in Software Engineering," World Sci. News, **122**, pp. 56–70.
[11] Boas, R., Cameron, B. G., and Crawley, E. F., 2013, "Divergence and Lifecycle Offsets in Product Families with Commonality," Syst. Eng., **16**(2), pp. 175–192.
[12] Simpson, T. W., 2017, "Product Family and Product Platform Benchmarking With Commonality and Variety Indices," ASME 2017 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, Cleveland, OH, Aug. 6–9, American Society of Mechanical Engineers, New York, p. V02BT03A04.
[13] Martin, M. V., and Ishii, K., 2002, "Design for Variety: Developing Standardized and Modularized Product Platform Architectures," Res. Eng. Design, **13**(4), pp. 213–235.
[14] Ye, X., Alizon, F., Gershenson, J. K., Khadke, K., Simpson, T. W., and Shooter, S. B., 2009, "Using Product Family Evaluation Graphs in Product Family Design," Int. J. Prod. Res., **47**(13), pp. 3559–3585.
[15] Thomas, L. D., 1992, "Functional Implications of Component Commonality in Operational Systems," IEEE Trans. Syst., Man, Cybern., **22**(3), pp. 548–551.
[16] Johnson, N. L., Kotz, S., and Balakrishnan, N., 1997, *Discrete Multivariate Distributions*, Vol. 165, Wiley, Hoboken, NJ.
[17] Kullback, S., and Leibler, R. A., 1951, "On Information and Sufficiency," Ann. Math. Stat., **22**(1), pp. 79–86.
[18] Lawless, J. F., 2011, *Statistical Models and Methods for Lifetime Data*, Vol. 362, John Wiley & Sons, Hoboken, NJ.
[19] R Core Team, 2018, *R: A Language and Environment for Statistical Computing*, R Foundation for Statistical Computing, Vienna, Austria.
[20] Takai, S., 2019, "An Approach to Integrate Commonality and Product Family Design With Inventory Decisions," ASME J. Mech. Des., **141**(3), p. 031704.
[21] ZHANG, Y., Zheng, Y., Xianchao, M., Wenjun, D., Dayong, D., Zhaoguang, T., and ZHANG, S., 2019, "Exploration and Implementation of Commonality Valuation Method in Commercial Aircraft Family Design," Chinese J. Aeronaut., **32**(8), pp. 1828–1846.
[22] Moscato, D. R., 1976, "The Application of the Entropy Measure to the Analysis of Part Commonality in a Product Line," Int. J. Prod. Res., **14**(3), pp. 401–406.
[23] Martin, M. V., and Ishii, K., 1997, "Design for Variety: Development of Complexity Indices and Design Charts," ASME Design Engineering Technical Conferences, Sacramento, CA, Sept. 14–17.
[24] Sinigalias, P. C., and Dentsoras, A., 2015, "Index-based Metrics for the Evaluation of Effects of Custom Parts on the Standardization of Mechanical Systems," DS 80-7 Proceedings of the 20th International Conference on Engineering Design (ICED 15) Vol 7: Product Modularisation, Milan, Italy, July 27–30, pp. 145–154.