Random acts of elasticity: MoE, G and EN408

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ABSTRACT

The paper examines the variation in modulus of elasticity (MoE) and shear modulus (G) within sawn timber and the implications for testing timber in accordance with EN408:2003. A combination of mechanical testing and simple analytical modelling is used to examine the differences between global and local MoE, and the extent to which these differences can be explained by the inhomogeneity of stiffness within a batten. In the light of these findings, the methods prescribed for determining G from flexural tests are called into question.

INTRODUCTION

EN408 (CEN 2003) provides two options for measuring flexural modulus of elasticity (MoE) from a four-point bending test. Local MoE is based on deflection of the central 'pure bending' region of the specimen while global MoE is determined from a deflection measurement made across the whole test span. It is not usual for laboratories to measure both local and global MoE simultaneously. Instead they use the method that is deemed most convenient, or, perhaps, the method that is expected to give the most favourable result.

Global MoE was introduced to EN408 in the 2003 revision, and the values obtained can differ considerably from the local MoE measurement. This has been studied by several researchers (e.g. Boström, 1999; Holland, 2000; Solli, 2000), and it is commonly assumed that this difference is due to shear deformation in the shear spans or an underlying systematic effect dependent on species, grade or specimen dimension. This paper is concerned with the extent to which this difference is due to the inhomogeneity of stiffness within the length of a specimen. It is, however, recognised that the following also have effects that differ between laboratories:

- Difficulty measuring local MoE accurately, especially when the specimen has twist
- Embedment at the support and load points
- Flexibility of the testing apparatus (especially the supporting beam)
- Reference points for deflection measurements (i.e. neutral axis, top, or bottom surface)

METHOD

Twenty-four Sitka spruce (*Picea sitchensis*) specimens were used for this study. The specimens were nominally 3.6 m long, 100 mm deep and 50 mm wide. The specimens were divided up into five sections of 600 mm length and elastically tested in four-point bending about the major axis in accordance with EN408 to obtain simultaneous values of local MoE and global MoE (Figure 1). The arrangement was such that the global MoE of section 3 was made on a span comprising sections 2, 3 and 4 for which local MoEs could be obtained.

The specimens were also elastically tested in torsion to obtain measurements of shear modulus (G) for the five sections (see Khokar *et al* 2008 for further details of the torsion testing procedure).

For the bending test the instrument location and the points of load application with respect to the specimen depth is likely to make a difference to the result because of the short span of pure bending relative to the depth (six times). For these tests the specimen was supported on the bottom surface and loaded on the top surface. Local MoE was measured by means of a cradle that measured deflection of the top surface of the specimen using a displacement transducer on each side of the specimen.

Laboratory test results were compared through application of a numerical ‘beam model’ based on Euler-Bernoulli beam theory with simple shear deformation superimposed. This numerical model represented the bending test by means of 36 short sections with linearly varying MoE and G values (See http://cte.napier.ac.uk/teo/moe/ for an interactive version of the model)

DISCUSSION

The ratio of global and local MoE obtained (Figure 2) was typical but did not compare well to best fit relationships obtained by Boström (1999), Holland (2000) or Solli (2000) although the two measurements were correlated to a degree ($R^2 \approx 0.5$). Overall, local MoE was higher than global MoE.

Shear deformation is a common explanation for the difference between global and local MoE even though, in theory, it should represent only about 0.5% to 4% of the central deflection. Weak correlation ($R^2 \approx 0.2$) was observed between the ratio of global MoE to local MoE for section 3 and the measured G values for sections 2 and 3. This suggests that shear deformation is not the main cause of the difference between global and local MoE.

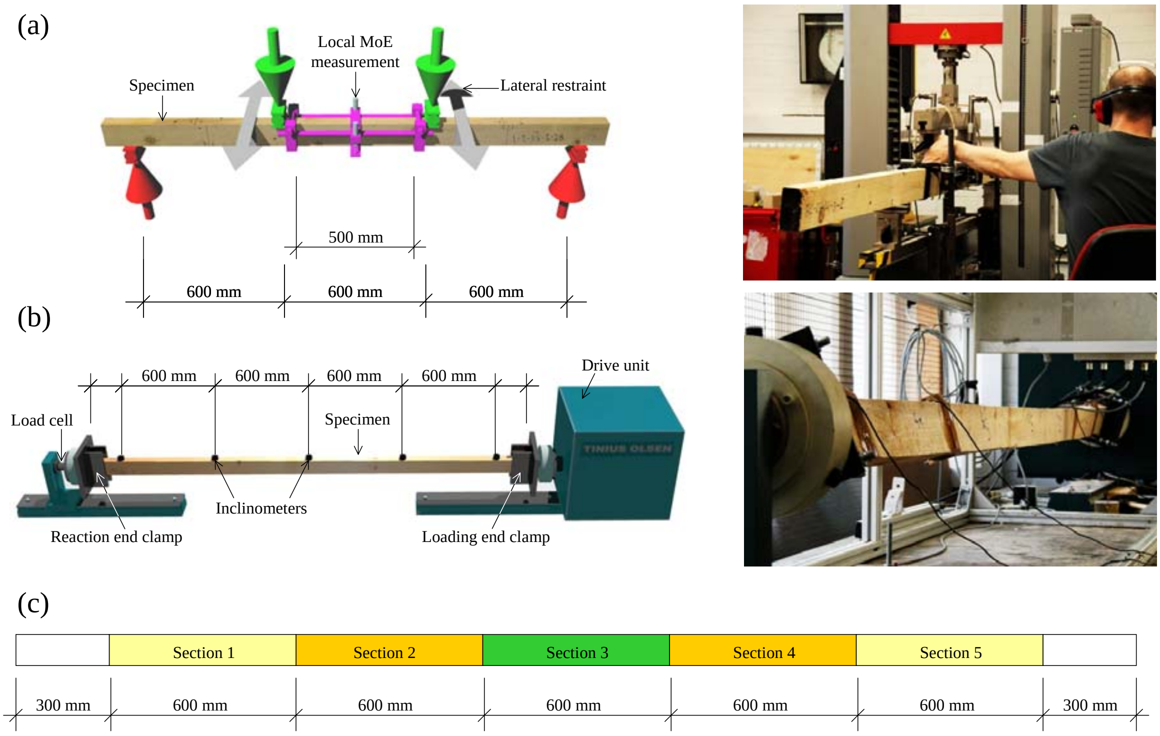

Figure 1: General arrangements for (a) bending and (b) torsion tests. (c) Sections of 3.6 m specimens

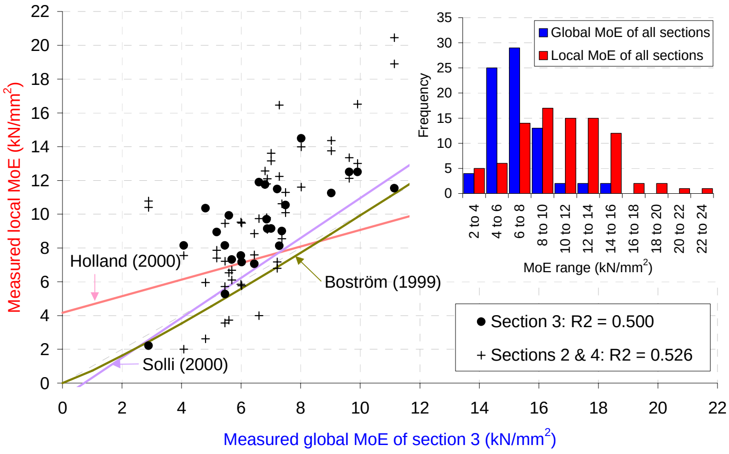

Figure 2: Measured local MoEs compared to the global MoE of section 3.

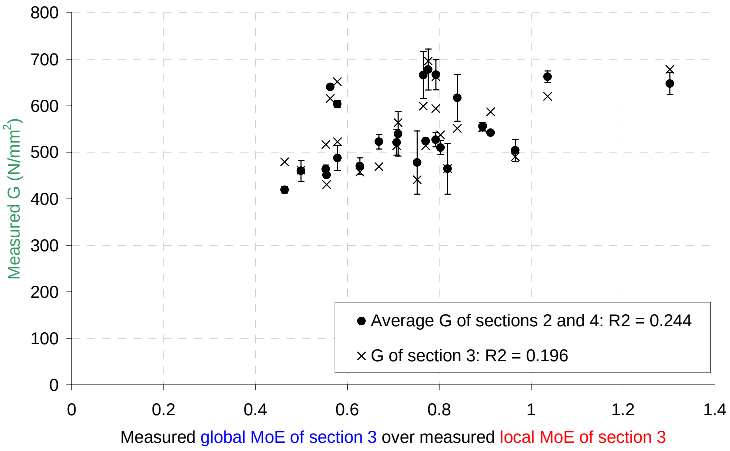

Figure 3: Ratio of global MoE to local MoE for section 3 compared to G for sections 2 and 4.

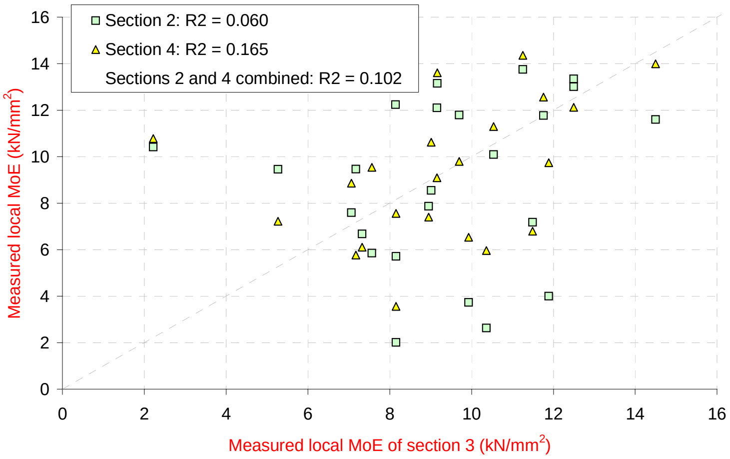

Figure 4: Correlation between local MoE of section 2 and those of sections 3 and 4.

Local MoE within a specimen was seen to vary considerably. Values for section 3 do not correlate ($R^2 \approx 0.1$) with values for sections 2 or 4 on the same specimens (Figure 4). This is partly due to the high sensitivity of local MoE measurement to the exact location of low stiffness zones (see below).

On the contrary, values of global MoE for sections 2 and 4 correlate well ($R^2 \approx 0.8$) with values for section 3 (Figure 5). This is because the measurements cover a larger region of the specimen, which overlaps for the measurements of the different sections.

The apparent lack of correlation between G and the ratio of global and local MoE is not due to a strong correlation between G and MoE because the two are not correlated at all (Figure 6, see also Khokar *et al* 2008). The values of G within a specimen are, however, well correlated to each other (Figure 7, $R^2 \approx 0.7$).

These results show that global MoE is largely driven by the local MoEs of all three sections. The beam model confirms this (Figure 8) as predicted values of global MoE based on the assumption of uniform local MoE within the three sections correlate well with the experimental results ($R^2 \approx 0.8$) and do so better than the measured local MoE of section 3 ($R^2 \approx 0.5$).

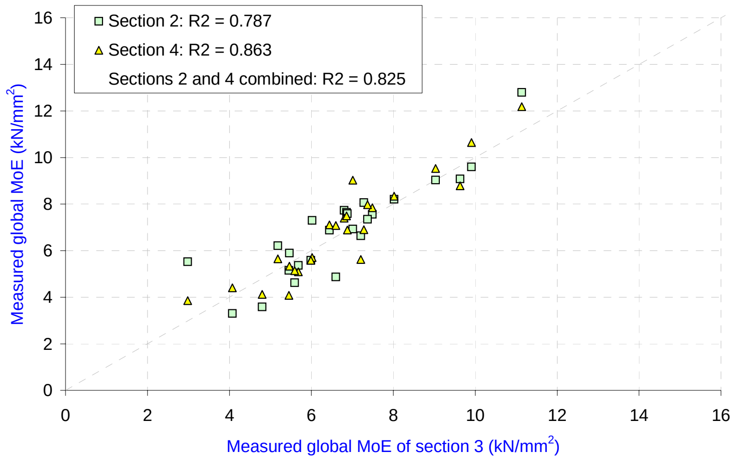

Figure 5: Correlation between global MoE of section 2 and those of sections 3 and 4.

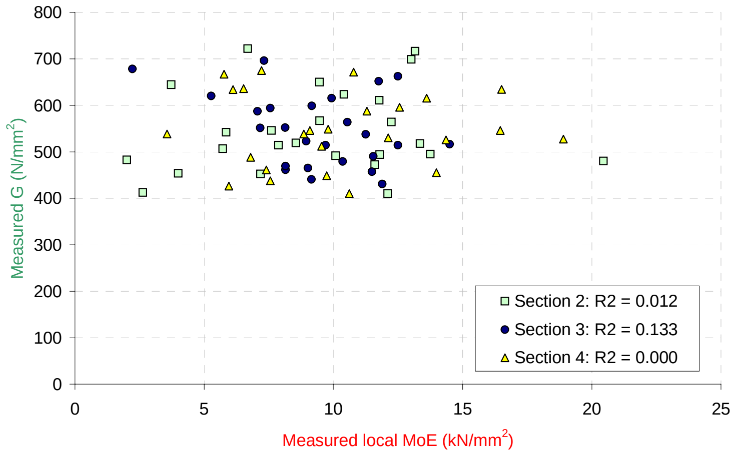

Figure 6: Measured G and local MoE of sections 2, 3 and 4.

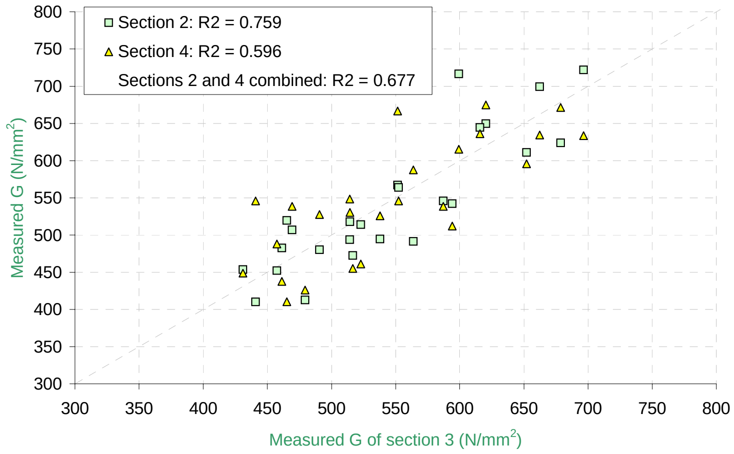

Figure 7: Correlation between G of section 2 and G of sections 3 and 4.

The inclusion of shear deformation into the beam model using measured values of G improves the model only slightly (Figure 8).

However, there is a discrepancy as the predicted values of global MoE are about 30% higher than the measured values. This is consistent with values of local MoE being higher than global MoE.

This could be due to systematic experimental error in the measurement of either local or global MoE, but when the apparatus was verified with a tubular steel specimen, the measurements of local and global MoE were indeed the same.

It is more likely that the difference is due to the relatively short central span resulting in a more complicated stress distribution than that assumed for the equations in EN408. The consequences of this differ depending on exactly how deflection is measured and so different laboratories will observe slightly different results.

The beam model can also be used to see the effect of the exact location of a zone of low stiffness. Figure 9 shows the situation for a low stiffness defect of length equal to the depth of the beam in which MoE varies linearly from 100% down to 50% and back to 100%.

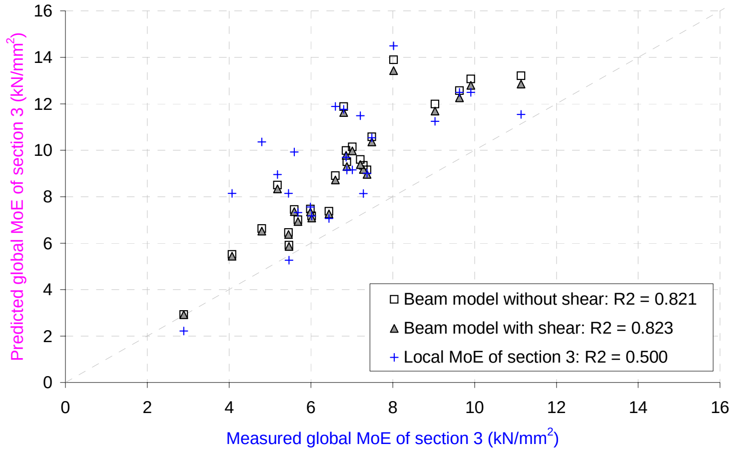

Figure 8: Measured global MoE of section 3 compared to beam model predictions.

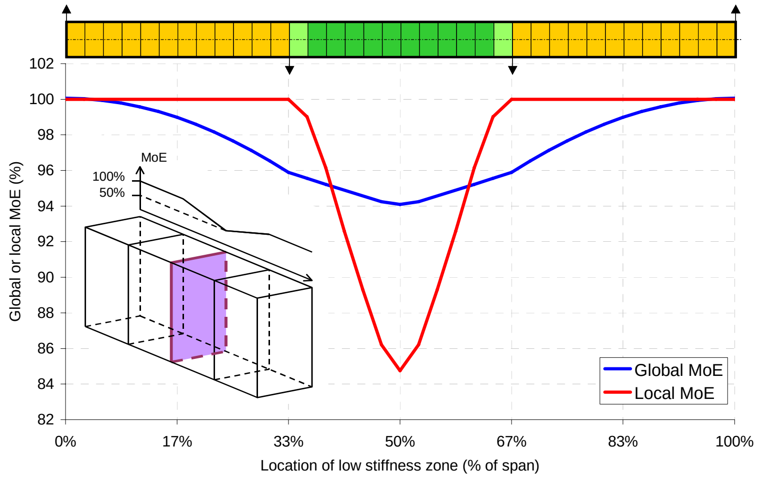

Figure 9: Influence lines for global and local MoE measurement for a single low stiffness zone.

CONCLUSIONS

The main reason for the difference between global and local MoE is not shear, but the variation of MoE within a specimen. This raises doubts as to whether the methods provided in EN408 for estimating G from bending tests are valid, supporting the changes in 09/30159969 DC.

The practice of correcting global MoE based on an assumed value of G that is proportional to MoE would not work for individual specimens because MoE and G are not correlated.

The exact location of low stiffness defects within the central span has a large influence on the value of local MoE obtained. Global MoE is less sensitive and, perhaps counter intuitively, may be a better measure of wood stiffness. The scale of such a defect relative to the span explains how species and size influence MoE comparisons

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