BOUNDED RATIONALITY IN HYPERPATH ASSIGNMENT: THE LOCALLY RATIONAL TRAVELLER MODEL

Achille Fonzone (corresponding author)
Centre for Transport Studies, Imperial College London
South Kensington Campus, London SW7 2AZ, UK
Telephone: +44 (0)20 7594 5977, Fax: +44 (0)20 7594 6102
a.fonzone@imperial.ac.uk

Michael G. H. Bell
Centre for Transport Studies, Imperial College London
South Kensington Campus, London SW7 2AZ, UK
Telephone: +44 (0)20 7594 6091 Fax: +44 (0)20 7594 6102
m.g.h.bell@imperial.ac.uk

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ABSTRACT
The concepts of optimal strategy and hyperpath were introduced in the transport field under the paradigms of equilibrium and expected utility theory, which assume perfect rationality. The initial scope for hyperpaths was frequency-based transit assignment, but increasingly they are used also for other transport problems, such as scheduled-based transit assignment and vehicle routing. Extensions to the original formulation have been developed to take into account congestion, the availability of information, and time-dependent networks.

Psychological research has shown that human rationality is bounded by limitations on computational ability. Frequently the issue has been neglected assuming that travellers can learn from experience but recent literature shows that this cannot be taken for granted. In particular, humans can evaluate only a reduced number of alternatives, so hypothesizing that travellers are able to think in terms of hyperpaths whatever their extension may be can give rise to unrealistic assignments.

This paper presents a behavioural model in which “myopic” travellers can determine hyperpaths only if their extension is under a certain threshold. At each node they reroute choosing stochastically a local intermediate node according to the cost of the hyperpath to this local destination and an *a priori* estimate of the cost of the remaining trip to the final destination.

Implementation on a test network demonstrates that the model can generate results that differ considerably from those derived from the perfect rationality assumption, and so further work is needed to improve its realism and to test it against real world problems.

HYPERPATH-BASED ROUTE CHOICE AND ASSIGNMENT

Most of route choice models, both in traffic and transit assignment, have been derived from Wardrop's first principle [1] under the expected utility paradigm [2]. Deterministic Traveller Equilibrium arises when travellers have perfect and identical knowledge of costs or, equivalently, models have no specification and measurement errors. Relaxing these hypotheses gives rise to Stochastic Traveller Equilibrium [3, 4]. The concept of equilibrium, often taken for granted, has many drawbacks [5].

Just postulating expected utility maximization and equilibrium, Spiess and Florian [6] (referred to as "S&F" in the following) show that, when routing problems involve common lines and waiting at nodes, travellers can reduce expected travel time considering rules like "take whatever attractive line comes first", which take account of en-route events. In [6] the problem of finding the optimal strategy is formulated as a linear problem and an elegant Dijkstra-type solution algorithm is given. [7] defines a graph-theoretic framework for the S&F problem, introducing the concept of hyperpath and showing that S&F solution is equivalent to an all-or-nothing hyperpath assignment. As in [8], a "strategy" can be thought as a mapping which associates each node of a network with a set of successors. A "hyperpath" is the combination of a strategy and the probability that each link is traversed once the traveller reaches its tail, provided that the strategy induces an acyclic subnetwork and there is a single origin-destination pair. [9] gives a sound game-theoretic foundation to S&F approach, proving its equivalence to the mixed strategy Nash equilibrium of a game between a transit traveller and multiple local demons that can impose the maximum delay on just one of the links exiting each node.

Hyperpaths have proved to be useful not only in their original context – frequency-based transit assignment models (see [10] for a review of the scope) – but also in schedule-based transit assignment [11] and traffic models [12, 13].

Many extensions of the original S&F model can be found in literature. A first line of research relaxes the infinite capacity assumption: [14-16] deal with congestion problems substituting effective for nominal frequencies, while [17] and [10] introduce explicitly the fail-to-board probability.

S&F uses exponential distributions of headways – which makes the mathematical formulation very simple – and no information at stops. [18] supports the use of the Erlang function for the headway distribution and makes the optimal strategy dynamic, considering that travellers at least know elapsed time since they arrived at a stop. [19] develops a model in which en-route information on arrival times is available to travellers through countdowns at stops. Authors highlight that headway can have a distribution which does not permit a closed form solution for the expected waiting times, making them difficult to calculate. [20] points out that alighting strategies – and therefore information available onboard – are as influential as boarding strategies on route choice.

Time adaptive route choice introduced by [21] to minimize expected travel time in Stochastic, Time-Dependent (STD) networks makes use of particular strategies, in which the next link to traverse from a given node depends on the arrival time at the node itself. This kind of strategies, or route policies, can be seen as a dynamic version of hyperpaths. [22, 23] make explicitly use of hyperpaths to cope with the STD shortest path problem. [24] presents a framework with route policies considering information. [25] proposes a quasi-dynamic solution for the assignment of time-dependent flows in capacitated networks.

BOUNDED RATIONALITY IN HYPERPATH APPROACH

A major shortcoming in literature on hyperpaths is the lack of models relaxing the assumption of perfect rationality which underpins the approach.

Utility theory, although largely plausible and simple from a modelling point of view, gives rise to models which are more normative than descriptive because it does not take into account important psychological factors affecting actual human behaviour [26]. Since the mid 50s, research has been carried out with the aim of building more realistic models of decision-making, considering not only the final output but also the reasoning process [27].

In particular, Simon [28] explains that finite computational ability sets bounds on the rationality of intelligent systems. Agents with bounded rationality rely on approximate methods for problem solving and "satisfice" instead of searching for optimal solutions. Describing and/or predicting the behaviour of a complex adaptive decision maker – such as that usually involved in assignment problems – requires modelling both substantive and procedural rationality.

Rationality in the transport field is a strong assumption, often justified as a consequence of a process of learning through repetition (known as "reinforcement") which should teach travellers their best option. This hypothesis is contradicted, on the one hand, by [29] which demonstrates that travellers become neither rational nor homogeneous as a consequence of learning, and on the other hand, by [30]. This emphasizes 1) that not only implicit (reinforcement) but also explicit (belief based) learning is important in route choice, especially when travel information plays a role, and 2) that memory, fundamental in learning, decays over time, so recent experiences are more influential than old ones.

If consideration of bounded rationality is a main requirement for every kind of reasonable transport model, it is even more necessary when hyperpaths are involved. In fact, in S&F and its extensions travellers are assumed to be endowed with a sort of hyper-rationality, which is unrealistic given 1) the increase in the number of options to be compared when each node can have more the one successor, 2) the complexity of calculation to define the expected waiting times corresponding to a given set of lines, and 3) the amount of information to which they should have access.

Actually humans are rather limited as information processors. A seminal paper by the cognitive psychologist Miller [31] shows that for absolute judgment of unidimensional stimuli, the channel capacity – *"the upper limit on the extent to which the observer can match his responses to the stimuli we give him"* – ranges between 1.6 and 3.9 with a mean of 2.6 bits of information, equivalent respectively to 3, 15 and 6.1 (reported as 6.5 in the paper) different categories. The conclusion refers to discrimination of non-numerical stimuli and to the limits of short-term memory (for critics of the results concerning memory, see [32]). Nevertheless this strengthens the idea that it is unrealistic to consider travellers able to calculate hyperpaths whatever their size may be.

The issue of the depth of analysis in a multistage decision task is dealt with in [33] which finds the length of the planning horizon being approximately 2 stages. The numerical value is related to the assumptions made in evaluating the results of experiments, nevertheless the paper presents evidence supporting the need for models in which travellers have a bounded capability of evaluating alternatives.

Bounded rationality in route choice is considered in [34], which decomposes decision-making process in three stages: Firstly, a mental map of the network is created selecting links on the basis of their probability of being recognized. Then alternatives not fitting travellers' preferences are eliminated. Finally, the actual path is selected by comparing the utilities of the remaining routes. The model does not take into account the existence of the channel capacity and requires path enumeration. Recognizing the complexity of the path choice process, [35] presents a sequential approach in which users reach the destination choosing links one after the after rather than choosing them jointly. In the network representation, nodes are anchor-points so at a given node users take into account only the immediate successors. The paper does not consider hyperpaths.

In the following, an assignment method is presented based on a behavioural model called the Locally Rational Traveller (LRT) which, considering bounded rationality due to limits of computational capacity and/or to incompleteness of information, is conjectured to generate more realistic outputs. The LRT can conceive hyperpaths but has a limited planning horizon and so cannot consider complex strategies. The level of rationality is defined by computational ability, knowledge of network costs and randomness of perception. Myopic rationality imposes a step-by-step route choice process.

The remainder of the paper is organised as follow. First LRT behaviour is described. Then an assignment problem for LRT is formulated and LRT reasoning shown by an example. Subsequently a solution algorithm is presented and applied to a test network. Conclusions with indications of future research directions close the paper.

LOCALLY RATIONAL TRAVELLER

Perfect rationality usually assumed in route choice can be relaxed in different ways: for instance, it could be called into question whether travellers evaluate options according to their expected utilities, or their attitude to think in terms of probabilities (see [36] for a general introduction to these topics).

Bounded rationality as used here can be thought as a kind of myopia. A short-sighted person cannot see distinctly beyond a certain distance and therefore if he needs to go somewhere, he has to rely on exact knowledge about what is within the horizon of his distinct vision and on more or less vague information – coming from experience or guesswork – on what is further ahead. As the trip unfolds, he can update his route choice.

Analogously, at a certain node of his route, the LRT may be not able to define completely the hyperpath (i.e., to define optimal strategies and related probabilities) to his final destination because his mental representation capacity is limited (he cannot make the calculations concerning a path with more than a given number of changes or he considers only few stages in his decision making) and/or he can access accurate information on links only when he gets close to the links themselves (e.g., he can know a given bus schedule only when he arrives at a stop or, in STD networks, travel times are so variable that trying to anticipate them is worthless when one is far from the links). It seems reasonable to hypothesize that, as a consequence, the LRT strategy is to move towards an intermediate destination closer to the final one and belonging to the boundary of his planning horizon. The intermediate destination is chosen combining a precisely calculated cost from the current position to the intermediate destination itself and an estimate of the cost of the remaining part of the trip. Of course the chosen intermediate destination can be sub-optimal with respect to that of a traveller with unbounded rationality. When he reaches the next node, the horizon of nodes and links he can "see distinctly" changes and he can amend his route.

To summarize, LRT considers incomplete strategies of the kind "I need to go from A to B. I'll go towards C, which is on my way, and then I'll decide how to continue".

The rationality of a traveller is defined by the assumptions concerning the criteria according to which he evaluates alternative routes, his computational ability, the information he has at his disposal and his tendency to stick to the best alternative or, equivalently, to the comprehensiveness of the utility model specification. In the LRT model:

- Expected disutility minimization is assumed, with disutility equal to the sum of the expected waiting time at nodes and expected travel time on links. Note that using only expected values in the cost function means disregarding travellers' risk attitude [37]. In the following, "cost" and "expected time" are used equivalently.
- Let the extension of a hyperpath be the maximum number of links which can actually be traversed travelling on it. Note that equivalently extension can be defined as the maximum number of nodes which can be used going from the origin (included) to the destination (excluded). Travellers are assumed able to determine only S&F hyperpaths whose extension is at most equal to a given number of links *ME* (Maximum Extension), i.e. LRTs cannot "see distinctly" nodes and links further than *ME* links, which define his planning horizon. When origin and destination are far away in terms of number of links, this could imply he is not able to compute the whole hyperpath from the origin to destination.
- Travellers know the overall network topology and the expected waiting and travelling times of each link which can be included in the hyperpath he is able to determine.. Time-expanded networks and mental maps extracted from actual networks can be assigned adopting the LRT approach. Link waiting times can be interpreted as proper waiting times, related to service frequencies as in S&F original formulation, but also as link delays in case of traffic assignment or vehicle navigation problems [13]. In the following a generic network is considered. Furthermore the LRT has got also information about the impedance of each node, i.e. the cost of the path from the node to the destination. The impedance is supposed to be known *a priori* and, in general, is not related to the actual link costs. An *a priori* knowledge of impedances is not such a strong assumption as it may appear at a first glance. In fact the impedance could even be simply the number of links in the path from a given node to the destination, information which can be easily obtained by maps, although it would have to be expressed in time units.
- Finally, given the approximations in the calculation and the vagueness of part of the input data, it does not seem sensible to assume deterministic behaviour in local destination choice. In the following, a logit model will be used despite its well-known shortcomings [38] because the different levels of stochasticity in LRT choices, with whose effects the present paper deals, can be captured by the dispersion parameter. Alternatives to logit which take into account the correlations arising from path overlaps [39] could be considered but are beyond the scope of this paper. Finally, note that the random term in the evaluation of the cost of the journey through each local destination can be regarded as deriving from impedances, which, being known *a priori*, in general can be not linked or only loosely linked to actual paths to the destination (see, for instance, *blind* impedance below), reducing the significance of actual path overlaps.

PROBLEM FORMULATION

Notation

Given a transportation network represented as a directed graph *G*, consisting of a set of nodes *I* and a set of links *A*, define

tt_a	Expected travel time of $a \in A$
wt_a	Expected waiting times for $a \in A$
o, d	Origin, final destination
i	Current node
A_i^+	Set of links departing from i
A_i^-	Set of links leading to i

ME	Extension of the hyperpaths LRT is able to determine
LD_i	Set of local destinations for node i i.e. nodes for which the shortest path from i is made up of ME links and from which at least one path to d exists
$hc^{i\to j}$	Cost of the S&F hyperpath from i to $j\in LD_i$
ic_j^d	Cost of travelling from $j\in LD_i$ to d (impedance) as known *a priori* by the LRT
$tc_j^{i,d}$	Total expected cost of travelling to d through $j\in LD_i$ for the LRT at node i
ε_j^d	Error term in the perception of the expected cost of travelling to d through $j\in LD_i$
α	Dispersion parameter of the logit model
$wt^{i,d}$	Expected waiting time at node i for the LRT heading to d
$wt^{i\to j}$	Expected waiting time at node i for LRT going to $j\in LD_i$ i.e. waiting time at i in S&F hyperpath from i to j
$TC(o, d)$	Total expected cost of the trip from o to d
$pr_j^{i,d}$	Probability of choosing $j\in LD_i$ at node i for the LRT travelling to d
$pr_a^{i\to j}$	Probability of using link $a\in A_i^+$ for the LRT choosing $j\in LD_i$ i.e. link probability of a in S&F hyperpath from i to j
$V(o, d)$	Flow from o to d
$v^i(o, d)$	Flow from o to d passing through node i
$v_a(o, d)$	Flow from o to d using link $a\in A_i^+$

For the sake of simplicity in the following the case of a single *od* pair is considered and references to o and d are omitted in the notation.

Route choice

At node i the LRT:

- Identifies all the local destinations
- For each local destination, calculates the cost of trip up to the final destination passing through it. This cost is equal to the cost of the hyperpath from i to the intermediate destination plus the impedance of the intermediate destination
- Chooses according to a perceived cost, including error terms supposed i.i.d. with Gumbel distribution

This behaviour is equivalent to the optimization problem

$$\text{Argmax}_{j\in LD_i} \Pr\left(tc_j^i + \varepsilon_j^i \leq tc_k^i + \varepsilon_k^i \quad \forall k\in LD_i\right)$$

Which, given the assumption about ε_j, implies

$$pr_j^i = \frac{e^{-\alpha\cdot tc_j^i}}{\sum_{k\in LD_i} e^{-\alpha\cdot tc_k^i}} \qquad j\in LD_i$$

Where

$$tc_k^i = hc^{i\to k} + ic_k \qquad k\in LD_i$$

Assignment

The LRT moves towards a local destination j just following the related S&F hyperpath because it minimises expected cost and he can “see” it clearly. This means that LRTs leaving from the current node i to the local destination j split among links going out of i according to the link probabilities in the S&F hyperpath from i to j (e.g., in transit assignment once LRT has chosen the stop towards which he heads, he waits for the first service in the set of attractive lines to arrive at that stop). Each link can be used to reach more than one local destination; the flow on a link is the sum of the flows generated on that link by each local destination.

The flow passing from a node is the sum of the flows using the links entering the node. Note that, as an effect of rerouting at each node, if j is more than one link far from i, not all LRTs going from i to j necessarily arrive to j (as explained in the example below for the case of passengers heading to m).

In formulas,

$$v_a = \sum_{j \in LD_i} v^i \cdot pr_j^i \cdot pr_a^{i \to j} \qquad a \in A_i^+$$

$$v^i = \begin{cases} \sum_{a \in A_i^-} v_a & i \in I \setminus \{o\} \\ V & i = o \end{cases}$$

Cost
Travellers heading for different local destinations experience different waiting times at node i; the waiting time for each local destination is equal to the expected waiting time in the corresponding S&F hyperpath. The waiting time at node i, different from d, is the weighted average of the waiting times of the different local destinations with weights equal to the probabilities of each local destination to be chosen; the waiting time corresponding to the final destination is 0.

$$wt^i = \begin{cases} \sum_{j \in LD_i} pr_j^i \cdot wt^{i \to j} & i \in I \setminus \{d\} \\ 0 & i = d \end{cases}$$

The expected total cost is the sum of the expected waiting times at nodes and the expected travelling times on links. The former kind of cost is the sum of the waiting times at each node weighted by the flows passing through each node. The expected travelling time is calculated analogously. At the end

$$\text{TC} = \sum_{i \in I} v^i \cdot wt^i + \sum_{a \in A} v_a \cdot tt_a$$

Example of LRT reasoning
The following example can help understanding the behaviour of a LRT. Suppose that the traveller is at node i of the network in FIGURE 1(a) and $ME = 2$.

The nodes which can be reached from i with a shortest (single) path of 2 links are k, m, n and p. As there is no path from p to d, LRT does not include it among local destinations. Therefore $\text{LD}_i = \{k, m, n\}$. If ME was greater than the number of links needed to reach the furthest node(s) from i, LD_i would be made up of these furthest node(s).

LRT can determine S&F hyperpaths from i to each node in LD_i, which can be, for instance, those represented by thick dark red lines in FIGURE 1(b-d). These can actually include more than one path as in (b) or degenerate in single path as in (c) and (d). Note that n can be reached from i also following the paths $i \to j \to k \to n$ and $i \to l \to k \to n$ including more than 2 links (FIGURE 1(d)) but LRT cannot calculate hyperpaths of such an extension so he does not take into account the 3 links routes. To represent this exclusion in the algorithm, the hyperpath to a given local destination is determined considering a fictitious network in which links leading into other local destinations – e.g. (j,k) and (l,k) in FIGURE 1(d) – are eliminated.

To choose the local destination towards which he moves, the LRT takes into account the sum of the cost of the hyperpath to a given local destination plus the cost, known *a priori*, of the journey from the local destination to d (shaded red lines in figure). Note that, in general, having made no assumption regarding the latter kind of information, it is not assured that it is related to the optimal solution. In particular Bellman's principle of optimality can be violated, e.g it could happen that $ic_k < ic_n$.

Passengers leading to a given local destination split between links going out from i according to the link probabilities in the S&F hyperpath, e.g. in FIGURE 1 all passengers moving from i towards m will use link (i,j) whereas those going towards k split among (i,j) and (i,l) with a probability depending on link waiting times. Link (i,j) is used in more than one hyperpath so the flow on it will be equal to the sum of the flows deriving from each of them.

LRT reroutes at each node because the horizon of his distinct vision changes: this implies that, for instance, people who in i decide to move in the direction of m, once they arrived in j, choose among q¸ r, s and t (local destinations from j) and only those who head for q will actually reach m.

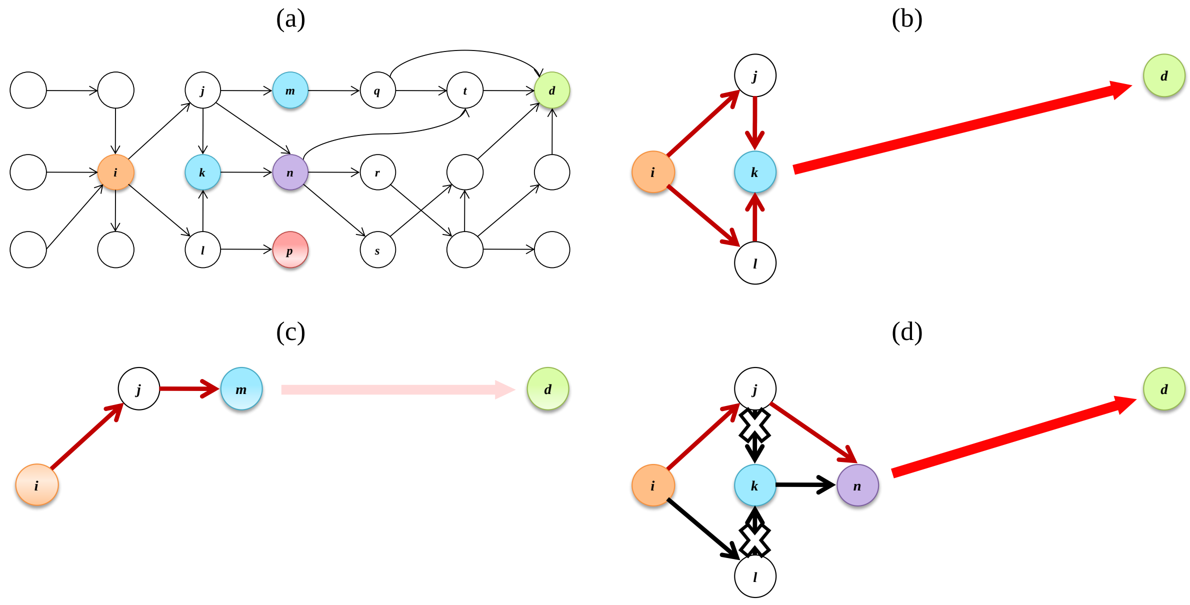


FIGURE 1 Example of LRT reasoning at a generic node.

ALGORITHM

The following algorithm performs an assignment consistent with LRT's behaviour.

Step	*Function*	*Operations*
0	Problem definition	Input ▪ Network: $G(I,A)$, tt_a, wt_a ▪ Trip: o, d ▪ User: ME, ic_j^d,α
1	Initialization	▪ $I \leftarrow I \backslash \{k \in I$ for which there is no path to the final destination$\}$ ▪ $I_{ME} = \{k \in I$ at least ME links afar from the final destination$\}$ ▪ $I_R = \{k \in I$ reached by the LRT$\} \leftarrow \{o\}$
2	Horizon definition	If $I_{ME} \cap I_R = \emptyset$ $ME \leftarrow ME$-1 If ME = 0 Go to step 4 Otherwise Update I_{ME} Go to step 2 Otherwise Go to step 3
3	Route choice	Select $i \in I_{ME} \cap I_R$ ▪ Determine the set of local destinations [LD_i] ▪ For each local destination $j \in LD_i$ - Build the fictitious network $G_j \subseteq G$ eliminating from G all links leading into other local destinations of i - Determine the hyperpath $H^{i \to j} \subseteq G_j$ from i to j [$hc^{i \to j}$ and $pr_a^{i \to j}$] - Calculate the total cost of reaching the final destination from i passing through j [tc_j^i]

Step	*Function*	*Operations*
		▪ Distribute travellers among $j\in LD_i$ following a logit model with $-tc_j^i$ as utility $[pr_j^i]$ ▪ Add US = {successors of i included in at a least one hyperpath $\mathcal{H}^{i\to j}$ } to the set of reached nodes: $I_R \leftarrow I_R \cup US$ ▪ $I_R \leftarrow I_R \backslash \{i\}$ ▪ Go to step 2
4	Assignment	Calculate link flows $[v_a]$ using a Markov chain approach with a transition probability matrix made up of pr_j^i
5	Cost calculation	Calculate the total cost of the trip $[TC]$

TEST NETWORK

Network

In order to explore the sensitivity of LRT assignment to assumptions about rationality and to evaluate the differences with S&F approach, a 9x9 grid network with unidirectional links between nodes has been analysed. The network has 170 links and from each node it is possible to move only towards east and south (FIGURE 2). Expected travel and waiting times (namely tt_a and wt_a) on each link have been randomly extracted from uniform distributions with supports respectively[5·k; 20·k] and [2.5·k; 10·k] minutes, where k is the number of links which make up the longest path (in terms of number of links) between the head and the tail nodes of the link (e.g., the supports for the extraction of travel and waiting times for link (1,19) have been respectively [10,40] e [5,20] because the longest path between nodes 1 and 19 consists of 2 links, (1,10) and (10,19)). The top-left and the bottom-right corner nodes have been assumed respectively as the origin and the destination of a demand equal to 1.

The S&F algorithm finds a hyperpath made up of 51 nodes and 76 links, with an expected cost of 221.82 min, whereas the single shortest path is made up of 14 nodes and links and it has an expected cost of 235.35 min.

(a) (b)

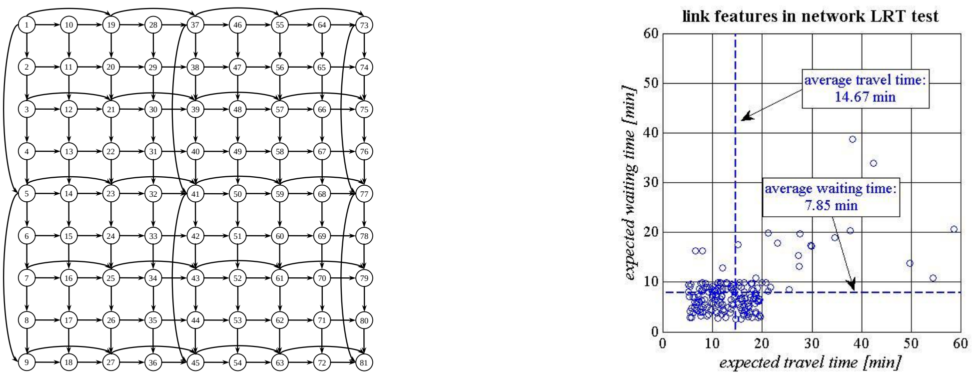


FIGURE 2 Test network – structure (a) and link features (b).

LRT characteristics

The test has been carried out implementing LRT model with

- *ME* ranging from 1 to 10 (the minimum number of links for which the destination enters the set of the local destinations of every other node).
- Three different types of impedance with different contents of information
  - *Hyperpath*: impedance equal to the cost of the hyperpath from the local to the final destination

- *Average*: impedance equal to the cost of the shortest path from the local to the final destination, calculated by a traveller who does not know the actual travel and waiting expected times of each link but only their average value over the whole network, namely 14.67 and 7.85 min.
- *Blind*, impedance equal to 0, i.e. the LRT doesn't have clues on the network beyond his distinct vision and so he chooses by taking into consideration only the costs of the hyperpaths from the current node to its local destinations.

▪ Three values for the dispersion parameter α of the logit model: 0.01, 0.1 and 1. In the case the dispersion of LRTs among different local destinations is assumed linked to different perceptions of impedances, α can be interpreted as an indicator of the reliance of LRT on information about impedances: the higher α, the more LRT considers reliable the information he knows about impedance, the less he tends to choose local destination different from the optimal one. When *blind* impedance is assumed, this interpretation becomes meaningfulness – because LRT has no information at all – and the randomness of choices is to be considered due to the incompleteness of the utility specification.

Results

Indicators

The results of the assignment with different values of model parameters have been evaluated and compared using the following indicators

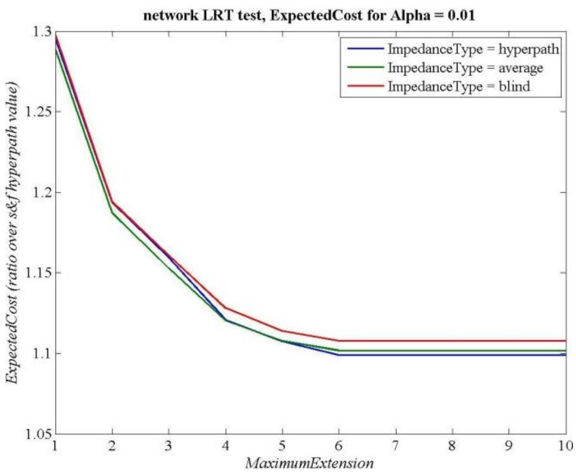
DiffLink	Links used in S&F but not in LRT and vice versa/total number of links
EuDist	Euclidean distance between S&F and LRT vectors of flows
ExpectedCost	Total expected cost of the trip (*TC*)
MaxLinDif	Highest (in absolute value) difference between flows on the same link in S&F and LRT solutions

Costs

Be $\overline{ME}$ the value of *ME* for which the final destination belongs to the set of local destinations of the origin; in the network under analysis, $\overline{ME} = 6$. As a general trend (FIGURE 3(a)), for $ME \leq \overline{ME}$, as *ME* increases, *ExpectedCost* of the trip gets closer to S&F one. *ExpectedCost* is constant for all $ME > \overline{ME}$. In the present case, LRT and S&F costs are never identical but other tests have shown that they can actually be undistinguishable even for $ME < \overline{ME}$.

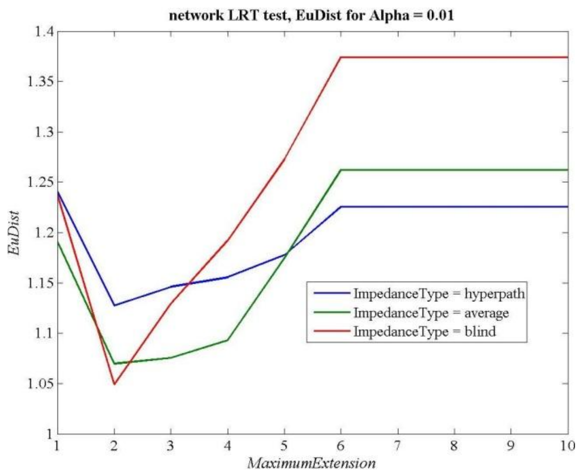
The effect of the content of information of impedance depends on randomness of choices: when choice is highly random (α = 0.01), the kind of impedance does not really matter; relative differences increase (i.e., the lines representing different cost types tend to be separated) with increasing α. Interestingly for α=1 and ME=2, LRT with blind information has an expected cost lower than that of LRT who knows average impedances.

(a1)

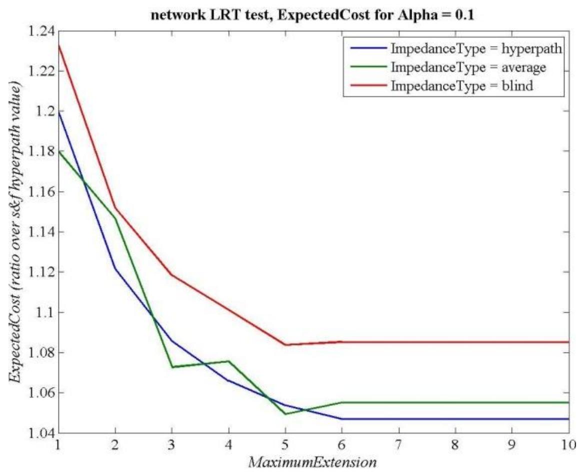


(a2)

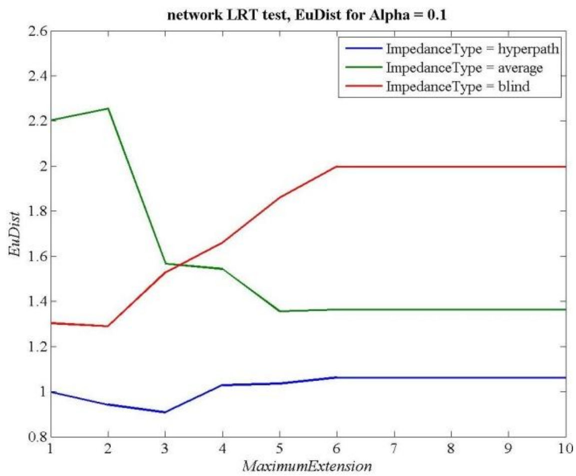
(b1)



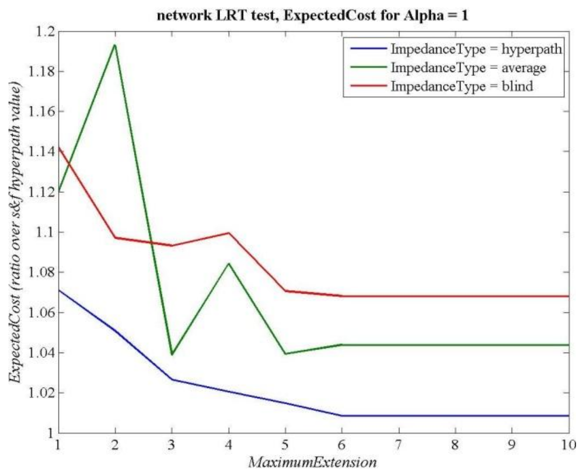
(b2)



(a3)



(b3)



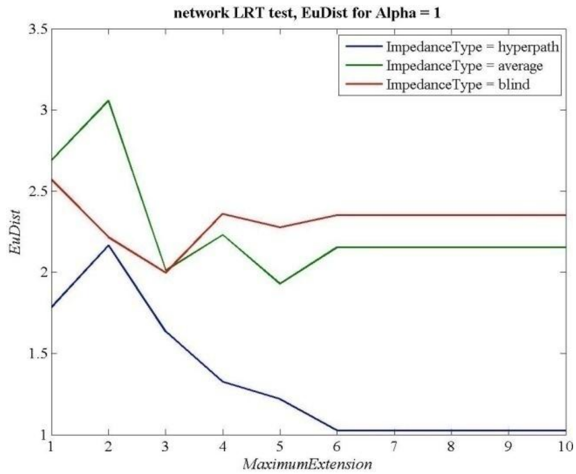


FIGURE 3 Expected costs (a) and distance between LRT and S&F solutions (b).

Link flows
Within the same network, *EuDist* is as an indicator of the global dissimilarity between the solutions provided by the different models. It's not possible to recognize a common pattern in the variation of *EuDist* with *ME* (FIGURE 3(b)). In general the difference between LRT and S&F assignments increases with α because LRTs spread over fewer links. For low values of *ME*, *blind* impedance solution is more similar to S&F than *average* impedance one.

As a general trend, when *ME* increases, LRT tends to use the same links as S&F, even if *average* impedances show some exceptions. For α<1, *DiffLink* never falls under 40% and the differences between different kinds of impedance are small, if any (FIGURE 4(a)).

It's noteworthy that the maximum difference between flows assigned to the same link by LRT and S&F can be even equal to 1, i.e. there are links used by the entire demand in one model and not used at all in the other (FIGURE 4(b)).

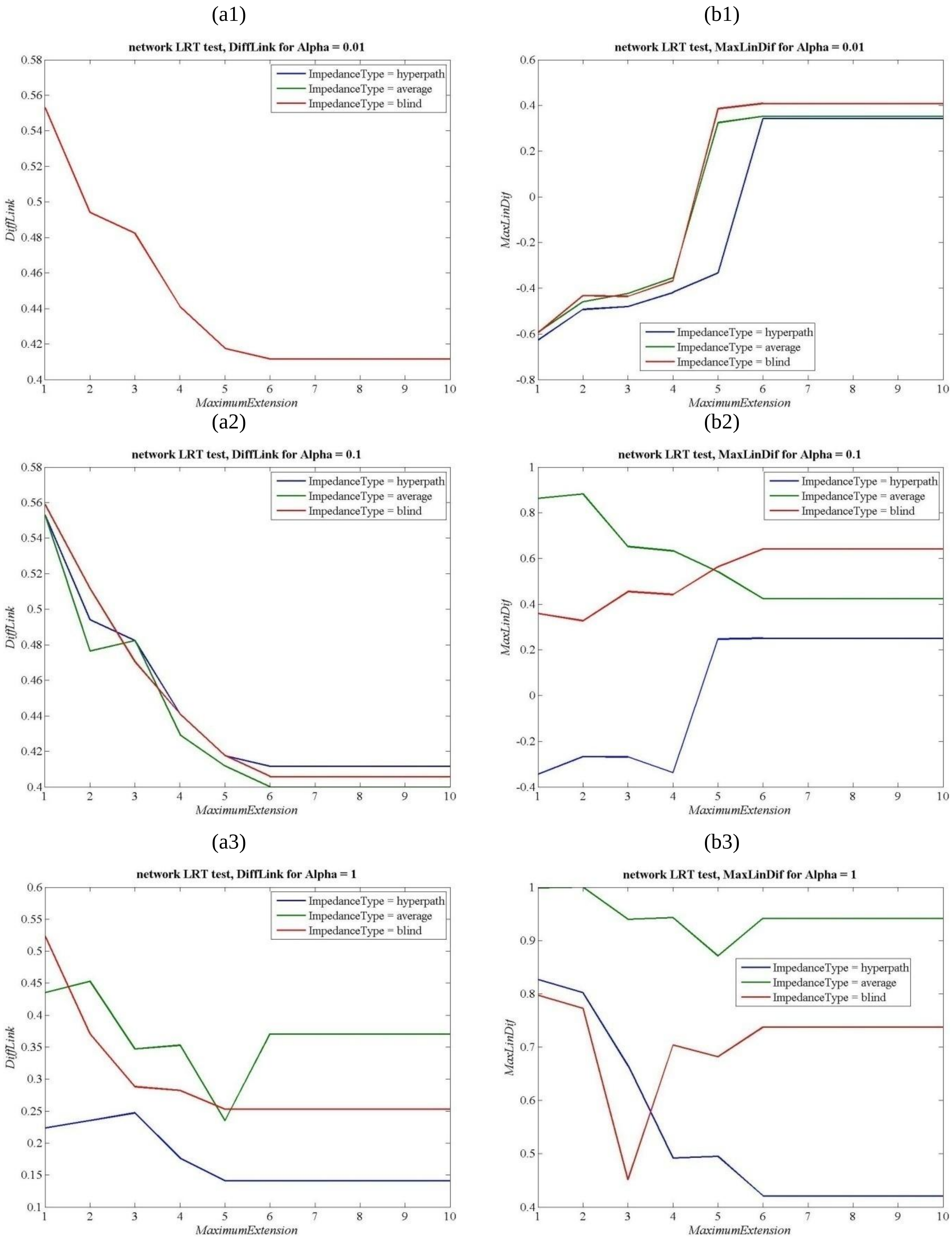


FIGURE 4 Link usage – links used in only one of the two models (a) and maximum flow difference on the same link (b).

CONCLUSIONS

Given the limitations human beings have as information processors, considering "hyper-rational" travellers can bring about unrealistic assignments, at least when travellers' choices are not assisted by ITS tools. A behavioural model has been developed and tested of "myopic" travellers, whose local rationality is defined by the maximum extension of hyperpaths they are able to determine (*ME*), by the knowledge they have on node impedances and by the level of stochasticity (α) of their choices. In route choice a stochastic distribution is pursued among a set of local destinations rather than among hyperpaths leading to the final destination and rerouting at each node is considered. The latter feature is reasonable in case of traffic assignment. It could be not always appropriate in a static transit assignment but it is justified in a dynamic one where new information can change travellers' perception of travel and waiting time of links and/or of nodes ahead.

Sensitivity of results to the degree of rationality and differences with S&F assignment have been tested with a 9x9 grid network, with an average number of exiting links per node equal to 2.1 and a single OD pair.

As to costs, the more travellers become rational – higher computational ability, better information on local destination impedances, smaller stochasticity of choices – the more LRT and S&F assignments tend to become similar. The loss caused by a lack of rationality, measured as the percentage difference between S&F and LRT cost, generally decreases with *ME* and the influence of the kind of impedance increases with α. When α is ≥ 0.1, the loss is always <25% and tends to drop under 10% for low-medium values of *ME*. This can be interpreted as a lack of incentives for travellers to enhance their hyperpath

The distance between S&F and LRT assignments in terms of link flows doesn't show a regular trend when *ME* increases, whereas it increases with α. In some cases the impedance type has an unexpected effect, LRT solution being closer to S&F's when travellers have no idea on the cost of the journey from local destinations to the final one (*blind* impedances) than when they rely on average costs (*average* impedances). It is noteworthy that some links which are crucial – because they are used by the whole demand – in the solution given by one model of rationality (S&F or LRT) are not used at all when the other approach is applied.

To summarize, from the experiment it turns out that, depending on the assumed level of rationality, S&F and LRT can have similar solution costs. In these cases travellers are not offered incentives towards full rationality. In the same cases, flows brought about by the two approaches can be significantly different and so, if it turns out that current models actually overestimated rationality, we risk having results which do not actually fit reality.

This conclusion confirms the importance of exploring the effects of local rationality in hyperpath assignment to see whether and in what cases it can be more realistic than perfect rationality. Three main research directions can be identified:

- Route choice model: The substitution of expected time minimization with more realistic decision criteria has to be evaluated, also to consider risk attitude. The issue of the stochastic model according to which travellers split among local destinations deserves particular attention.
- Behavioural characteristics: Behavioural as well as psychological research is needed to verify the assumptions about LRT way of reasoning – in other words, whether routing can be represented as a multistage decision process – and to determine the values of factors defining the level of rationality.
- Implementation: LRT assignment has to be tested with different trial networks to explore its sensitivity to topology. Finally, LRT has to be compared to other hyperpath assignment models applying them to real world networks.

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