

# Building low CO<sub>2</sub> solutions to the Vehicle Routing Problem with Time Windows using an Evolutionary Algorithm

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**Abstract**—An evolutionary Multi-Objective Algorithm (MOA) is used to investigate the trade-off between CO<sub>2</sub> savings, distance and number of vehicles used in a typical vehicle routing problem with Time Windows (VRPTW). A problem set is derived containing three problems based on accurate geographical data which encapsulates the topology of streets as well as layouts and characteristics of junctions. This is combined with realistic speed-flow data associated with road-classes and a power-based instantaneous fuel consumption model to calculate CO<sub>2</sub> emissions, taking account of drive-cycles. Results obtained using a well-known MOA with twin objectives show that it is possible to save up to 10% CO<sub>2</sub>, depending on the problem instance and ranking criterion used.

## I. INTRODUCTION AND MOTIVATION

Vehicle Routing problems (VRP) are well known optimisation problems that arise in the transport and logistics sector. Typically they involve despatching vehicles from centralised location such that all customers are visited. An extension of this problem is the VRPTW in which time-windows define a period of time in which each customer must be visited. The VRPTW problem is known to be NP-Hard. VRPTW problems are well studied from the optimisation perspective—they typically involve attempting to minimize the distance travelled and number of vehicles required to deliver all goods while ensuring that all customers are satisfied within the expected time-frame. However, with increasing legislation coming into force, companies in the real-world are increasingly under pressure to reduce the levels of CO<sub>2</sub> emissions associated with their operations, adding another objective to an already difficult problem. In this paper we use a multi-objective evolutionary algorithm to examine the trade-offs between solutions found using different combinations of objectives; thus we examine the solutions found using dual objectives of first distance and vehicles and then emissions and vehicles as objectives of the MOA. In each case we calculate the CO<sub>2</sub> cost of the solutions found and determine the potential savings that can be made.

The multi-objective algorithm used to find solutions is the based on that described by [4] for solving standard VRPTW problems in which the goal is to minimise the number of vehicles and the cost (i.e. distance travelled) of routes. Their algorithm evolves groupings of customers,

such that each group is assigned to one vehicle without violating either capacity or time constraints. Within each group, route costs are calculated using Euclidean distance. As we are concerned with real-world data based on an actual road network, we utilise an A\* algorithm [16] to construct paths on the road network between each pair of customers rather than a simplified Euclidean distance. Although there are alternative routing algorithms such as Dijkstra [17] for constructing routes between two points, they are computationally expensive. A\* was chosen as it gives an acceptable balance between cost and quality. The CO<sub>2</sub> cost of any solution is determined retrospectively by applying the emissions metric to the route determined by the A\* algorithm. Euclidean distance is used as the heuristic within the A\* algorithm, at present there is no equivalent emissions based measure.

## II. PREVIOUS WORK

There is a wealth of existing literature on evolutionary approaches to solving VRPTW problems, using a variety of different approaches. An overview of earlier work may be found in [12], and the reader is referred to a recent survey by [1] for examples of the state-of-the art. Much prior research has concentrated on the problem instances first presented by Solomon [14]. These instances are not based on realistic underlying data, but have supported the benchmarking of many VRPTW solvers.

A number of nature inspired approaches to solving the VRPTW have been investigated, including Ant Colony Optimisation [11] cellular genetic algorithms [8], simulated annealing [10] and genetic algorithms incorporating a messy representation [7].

The approach of using of Pareto ranking in multi-objective GAs such as [13] has been applied to a number of vehicle routing problems [5]. As the focus of the paper is investigating the impact of including CO<sub>2</sub> emissions as one of the objectives in a MOA, rather than presenting a novel MOA, we choose to employ a recent MOA by [4]. Ombuki's MOA represents a mature approach to the VRPTW that has achieved significant published results on benchmark solutions. The original MOA developed in [4] has been applied to the problem of scheduling garbage collection in [3].

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OSM Category	Speed (kph)
default	32
unclassified	32
secondary	36
residential	36
primary	38
tertiary	38
trunk	64
motorway-link	80
motorway	112

TABLE I

AVERAGE SPEEDS ALLOCATED TO OSM LINK CLASSES. FOR ANY LINK CLASS NOT IN THE LIST, THE DEFAULT VALUE (32KPH) IS USED.

### III. PROBLEM DESCRIPTION

#### A. The Geographical Data Source

Accurate geographical data is now readily available online for many regions. Such data encapsulates the topology of streets as well as the layout and characteristics of junctions. In this paper, we use data from the Open Street Map (OSM) project [18].

We focus on OSM Data representing the road network within the City of Edinburgh, Scotland. The entire road network for the city is stored; this consists of the length and class of every road section as shown in figure 1.

Using data presented in [6] we allocate a realistic average free-flow speed to each class of road (for example, see table I). For each junction on the map, we store data regarding the appropriate intersecting street details, alongside any other relevant features such as the existence of traffic signals or a roundabout. Data regarding the gradient of any road is not available, therefore this attribute of roads in the network is not taken into account in the emissions model employed. Similarly, we do not have accurate data available regarding waiting-time at junctions, which further simplifies the emissions calculation. A more complex model will be developed, but it is useful to understand the effects of routing based on emissions calculated using free flow speeds and times.

#### B. Emissions Calculations using a drive cycle and fuel consumption model

The accurate estimation of vehicle emissions is a non-trivial exercise for which a number of models have been proposed ([15],[9]). The emissions characteristics of vehicles will differ depending on a range of factors, such as engine size, fuel type and mass. The driving activity and locality will also influence emissions through variables such as speed, acceleration and gradient. In this study, we utilise a power based instantaneous fuel consumption model for road vehicles, proposed in [15]. This model allows the estimation of fuel used over a given interval (e.g. 1 second), this maps onto the concept of a drive cycle which will typically calculate speeds at similar intervals. Once a route has been constructed between points on the map it has to be converted into a series of interconnected drive cycles. First, an average free-flow speed for a street section is established (see table I). Based

on the length of the street and speed, a set of speeds are created to represent the vehicles speed at 1 second intervals. Such speeds are not constant, deviations above and below the average are added based on data in [6]. Additional data points to represent changes in speed at junctions are added, based on the attributes of each individual junction and the classification of the incoming/outgoing roads. For example, at roundabouts and traffic lights, vehicles are assumed to stop then restart. Other junction types may simply require a change in speed. Once this has been calculated, appropriate data points can be added to the drive cycle using acceleration/deceleration curves from the TRL data [6].

The fuel consumption model may now be applied sequentially to each of the speed values calculated for the street section. The model may be described as follows:

$$dF = \alpha dt + \beta_1 R_1 dx + [\beta_2 a R_1]_{a>0} \quad \text{for } R_T > 0 \quad (1)$$

$$= \alpha dt \quad \text{for } R_T \leq 0 \quad (2)$$

where

$dF$  = fuel (mL) consumed over distance  $dx$  (metres) during time  $dt$ (s)

$\alpha$  = idle fuel rate (mL/s)

$\beta_1$  = fuel consumption per unit of energy

$\beta_2$  = fuel consumption during positive acceleration

$a$  = acceleration (m/s), negative when slowing down

$R_t$  = total force required to drive the vehicle (kN) expressed as follows:

$$R_t = R_D + R_l + R_G,$$

$$R_D = b_1 + b_2 v^2,$$

$$R_l = M a / 1000,$$

$$R_G = 9.81 M (G / 100) / 1000,$$
(3)

where

$v$  = speed ( $dx/dt$ ) m/s

$G$  = gradient (%) +ve or -ve

$M$  = vehicle mass (kg)

$b_1, b_2$  = drag force function parameters

Appropriate values are provided in [15] to allow the model to be calibrated with respect to an instrumented vehicle. This model allows an estimate of fuel consumed to be derived for any given route. The litres fuel consumed are converted to Kg of CO<sub>2</sub> by multiplying by a conversion factor of 2.317 as specified in [2]. Within this paper emissions are measured in Kg of CO<sub>2</sub>.

### IV. PROBLEM INSTANCES

Using the OSM data for the city of Edinburgh, we generated three problem instances, each using a set of randomly selected delivery points located within the City of Edinburgh. The problems generated contained 20, 40, 60, 80 and 100 delivery points; the entire set of delivery points is shown in



Fig. 1. A fully rendered version of the dataset used. All 100 customers are highlighted.

figure 1). We utilised a delivery window size of 5 minutes for each customer, all time windows starting and ending within a 12 hour period.

## V. THE EVOLUTIONARY ALGORITHM

As previously stated, we use the multi-objective evolutionary algorithm described by [4]. The algorithm utilises an indirect representation; each chromosome defines an ordering of customers which need to be allocated to routes (vehicles). To construct a solution, each customer is considered in the order defined by the chromosome. The customer is added to the current route if this does not cause any violations of time constraints. If the customer cannot be added, the current route is considered complete and a new route is initiated.

The evolutionary algorithm uses a steady state model with a population size of 300, a crossover rate of 0.8 and a mutation rate of 0.1, these parameters and the associated operators are based upon [4]. For the experiments being conducted here we execute the algorithm for 10,000 generations. A two point cross over operator as used in [4] is used to construct new individuals, this incorporates a repair function to ensure that each child is a valid permutation. Parents are selected based on tournament selection (using a tournament size of 2), the resulting child has a random swap mutation operator applied before being replacing the loser of a tournament (size =2) in the main population.

The algorithm uses *pareto-ranking* to compare the quality of solutions, where the ranks are sequential integer values that represent the layers of stratification in the population obtained via dominance testing [4]. In the experiments

presented in this paper a third objective is added, that of minimising the CO<sub>2</sub> produced by the solution.

## VI. EXPERIMENTAL RESULTS

Experiments were performed with the EA described above using the following ranking criteria:

- minimise vehicles and minimise distance (D+V)
- minimise vehicles and minimise CO<sub>2</sub> (EM+V)

For each criterion, 10 runs of the algorithm were performed on each of the five data sets (giving 100 runs in total). For each solution produced the qty of vehicles, distance and emissions were recorded. Given the nature of the MOA algorithm, more than one non-dominated solution may be produced at the end of a run.

The solutions produced by these runs are plotted in figures 2 to 6. In each figure the left hand and right hand graphs show the same set of results plotted by vehicles and distance, and vehicles and emissions. Where the run resulted in the production of more than one solution, the multiple solutions are plotted linked by a line. The plots represented by diamonds are those results produced when using the D+V ranking. The plots represented by squares are those results produced when using EM+V ranking.

Figure 2 shows the results obtained on the 100 customer problem. When the results are plotted by distance versus vehicles, there appears to be no division between the D+V and the EM+V results. However when the results are plotted by emissions and vehicles, it may be observed that the EM+V results mostly show a decrease in emissions compared to the D+V results.

The solutions produced by the 80 customer data set are shown in figure 3. When plotting by distance and vehicles it may be noted that there appears to be no distinction between results produced using D+V and EM+V ranking. When the results are plotted by emissions and vehicles we note that the EM+V results, in general, show a decrease in emissions compared to those obtained using D+V ranking.

The results in figure 4 suggest that for this dataset it is more difficult to find low CO<sub>2</sub> solutions. When plotting the solutions by distance and vehicles we may note that the solutions produced using EM+V mostly require a longer distance and more vehicles than those produced using D+V. When plotting by emissions and vehicles we note no significant differentiation between the two sets of results. The solutions shown in figure 5 suggest that as with the 60 customer dataset it is difficult to find low CO<sub>2</sub> solutions for the 40 customer dataset. No significant trends differentiating the D+V and EM+V results are apparent. No significant difference in results produced using EM+V and D+V may be seen in figure 6, showing the results obtained with the 20 customer problem.

A summary of results is shown in tables II and III, note that the distance and emissions values are averaged over all of the results produced for that data set. On average the number of vehicles required for the EM+V solution rises slightly, but not significantly. Given that the MOA produced multiple solutions in some instances not all of the averages are calculated over the same number of solutions.

## VII. CONCLUSIONS AND FUTURE WORK

Using a standard MOA, we have investigated the routes obtained in a VRPTW problem based on realistic data from the perspective of the amount of CO<sub>2</sub> emitted, by varying the objectives used to compare solutions.

Of the five datasets investigated it has been possible in three cases (100,80 and 40 customers) to use the EM+V criterion to find solutions with lower CO<sub>2</sub> characteristics than those solutions found using D+V as the ranking criterion (see table II). On average a 10% decrease in emissions is noted when using EM+V rather than D+V on the 100 customer data set, this drops to 5% and 5% respectively on the 80 and 40 customer datasets. Note the increases in the average distance of between 5% and 9% within the same results on the 20-80 customer data sets. On the 100 customer data set a small decrease in distance (approx. 1%) is noted.

It should be noted that although an MOA is utilised, only in very few cases was more than one non-dominated solution found by the algorithm. Previous work with ranking-based MOAs has found the production of multiple non-dominated solutions, forming a Pareto front to be common. Of the 100 runs of the MOA undertaken 76% returned only one solution, of the remainder 23% returned 2 solutions and only one run returned 3 solutions. It might be surmised that the relationship between the emissions produced by a solution and its length may be encouraging this phenomena of producing only one solution.

To facilitate further analysis of the MOA performance, it is proposed to solve the problem instances presented in this paper using a differing algorithmic approach to the multiple objectives. The underlying data used is based upon free-flow speeds and without gradients. The effect of adding congestion data which will vary link speeds depending on the time of day has still to be observed. Other areas to be investigated are modifying the emissions model to allow multiple vehicles with differing characteristics to be included.

Future work to be carried out includes the investigation of other time window constraints to observe whether increasing the length of the time windows improves the abilities to find low CO<sub>2</sub> solutions. It is also proposed to investigate ranking on three criterion, vehicles used, distance and emissions.

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Data set	Avg dist:EM+V (KM)	Avg dist: D+V (KM)	Avg emissions:EM+V (Kg/CO <sub>2</sub> )	Avg emissions: D+V (Kg/CO <sub>2</sub> )
100	784.2	796	35465.8	39392.8
80	662	629.4	29196.3	30803.7
60	541.8	497.4	24722	24181.7
40	383.2	372.7	17368	18605.5
20	202.9	197.7	10366.5	9970.4

TABLE II  
RESULTS SUMMARY

Data set	Avg vehicles:EM+V	Avg vehicles: D+V
100	13.2	12.3
80	10.6	10.2
60	9.2	7.9
40	6.5	5.8
20	3.2	2.7

TABLE III  
SUMMARY OF VEHICLES USED

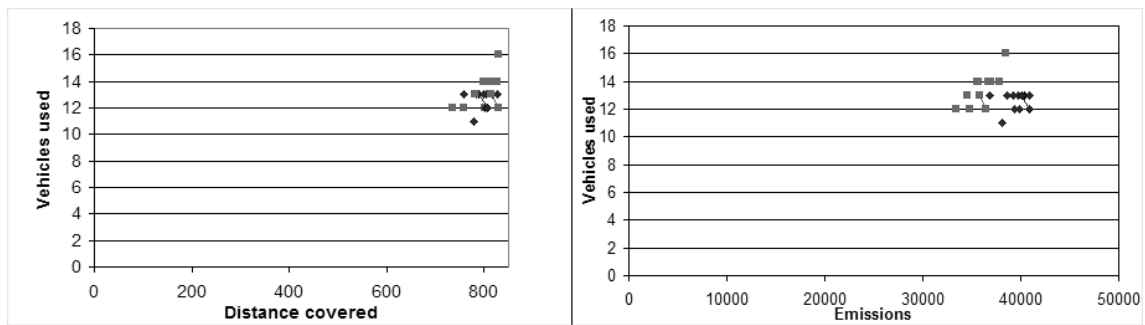


Fig. 2. Results for the 100 customer data set. The left-hand plot shows the solutions by vehicles and distance, the right hand shows the same results plotted by vehicles and emissions. The diamond plots represent solutions created using the D+V ranking and the squares those created using EM+V ranking.

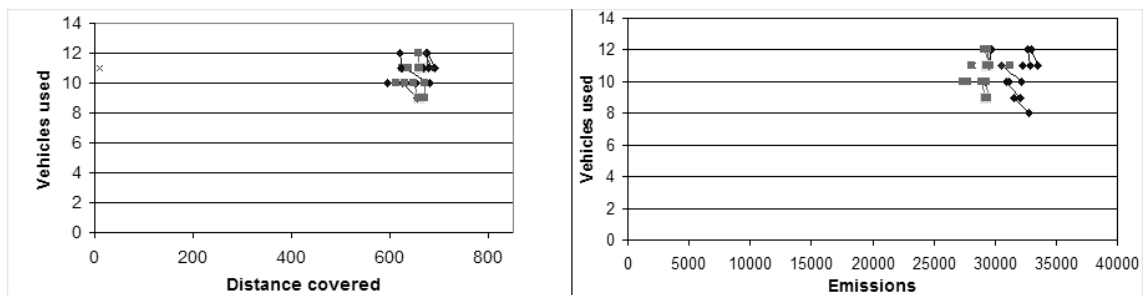


Fig. 3. Results for the 80 customer data set. The left hand plot shows the solutions by vehicles and distance, the right hand shows the same results plotted by vehicles and emissions. The diamond plots represent solutions created using the D+V ranking and the squares those created using EM+V ranking.

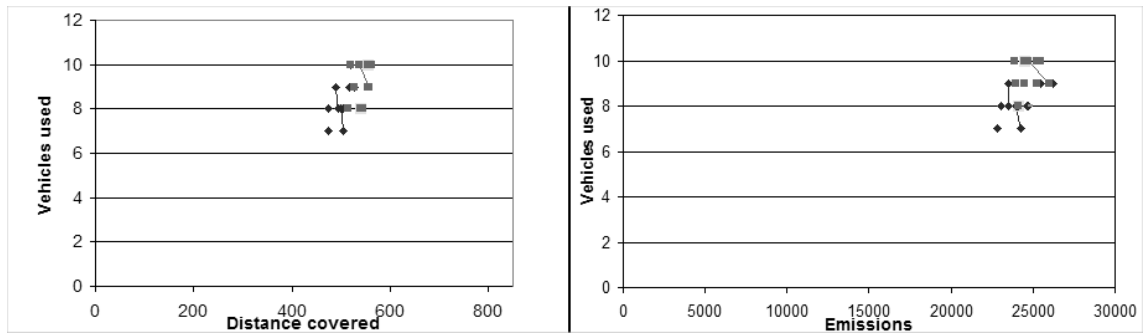


Fig. 4. Results for the 60 customer data set. The left hand plot shows the solutions by vehicles and distance, the right hand shows the same results plotted by vehicles and emissions. The diamond plots represent solutions created using the D+V ranking and the squares those created using EM+V ranking.

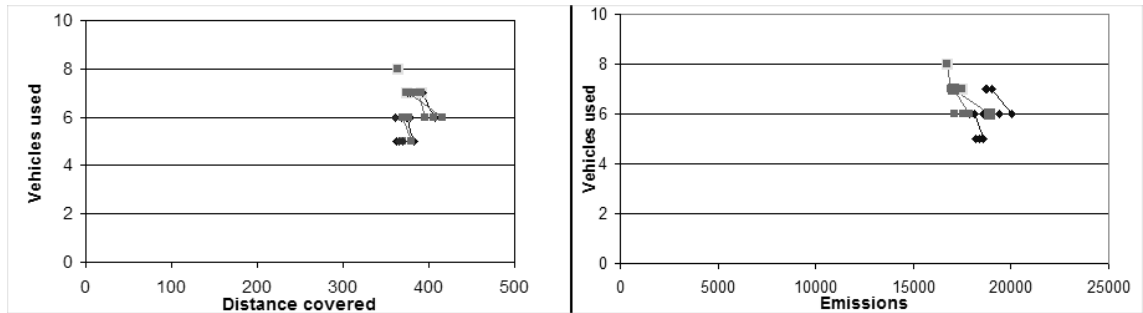


Fig. 5. Results for the 40 customer data set. The left hand plot shows the solutions by vehicles and distance, the right hand shows the same results plotted by vehicles and emissions. The diamond plots represent solutions created using the D+V ranking and the squares those created using EM+V ranking.

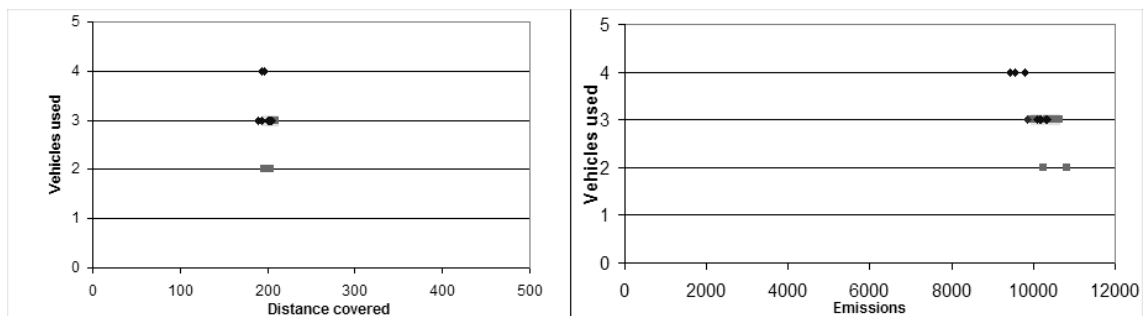


Fig. 6. Results for the 20 customer data set. The left hand plot shows the solutions by vehicles and distance, the right hand shows the same results plotted by vehicles and emissions. The diamond plots represent solutions created using the D+V ranking and the squares those created using EM+V ranking.