The efficacy of Euler diagrams and linear diagrams for visualizing set cardinality using proportions and numbers

Gem Stapleton^{1*}, Peter Chapman², Peter Rodgers³, Anestis Touloumis¹, Andrew Blake¹, Aidan Delaney⁴,

- 1 Centre for Secure, Intelligent and Usablity Systems, School of Computing, Engineering and Mathematics, University of Brighton, Brighton, UK
- 2 School of Computing, Edinburgh Napier University, Edinbugh, UK
- 3 School of Computing, University of Kent, Canterbury, UK
- 4 Bloomberg

This document is supplementary material for the associated paper submitted to PLOSOne. In contains the following information:

- Appendix A contains information on how the questions were generated for the tasks in the study.
- Appendix B presents the examples used for training participants.
- Appendix C presents the questions and associated diagrams used in the studies.
- Appendix D provides information on the statistical methods used to analyse the data.
- Appendices E-G present all of the statistical output generated from the methods employed for the three studies.

All of the diagrams included in this document are scaled to 50% of the actual size used in the study in order to fit within the page width.

A - Question Generation

For the S-type questions, a random permutation of the list of labels was generated using the website www.random.org. The first label in this permuted list then became the label which would appear in the question, and the next four labels in the permuted list were the labels which would appear as check-boxes, alongside the "None of the above" option. (Where there were only five sets in the diagram, the second step was redundant.)

For the I-type questions, a random permutation of the list containing those intersections containing at most three labels, was generated using the website www.random.org. The first intersection in this permuted list became the intersection whihe would appear in the question, and the next four intersections in the permuted list were the intersections which would appear as check-boxes, alongside the "None of the above" option.

^{*} g.e.stapleton@brighton.ac.uk

A check was performed on the number of check-boxes that appeared in the correct answer, which ranged from 0 (representing "None of the above") to 4. These numbers were:

• 0/"None of the above": 7 questions

• 1 check-box to be checked: 6 questions

• 2 check-boxes to be checked: 7 questions

• 3 check-boxes to be checked: 7 questions

• 4 check-boxes to be checked: 5 questions

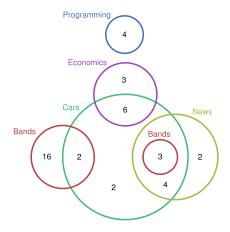
Thus, there was no systematic bias towards any number of check-boxes appearing in the correct answer.

B - Training for Studies A, B & C

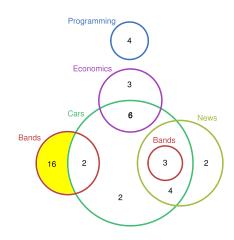
This section contains the data presented to participants for the purposes of training. After attempting each question, participants were given the correct answer, an explanation of the correct answer (with reference to the diagram) and a modified diagram with the regions of interest highlighted. An exception was for training question 4, as the correct answer was "None of the above", and thus no regions were highlighted. These modified diagrams for training questions 1 to 3 are presented alongside the respective question.

Question 1

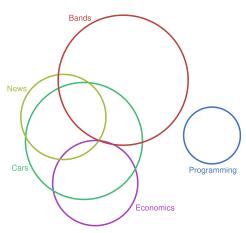
- Question: Tick the check boxes where more people have exactly that combination of interests than Cars and Economics only.
- Check boxes to be ticked: Bands
- Task Type: Intersection comparison More than.
- SNAP Data Set Used: 44130462
- ED-N



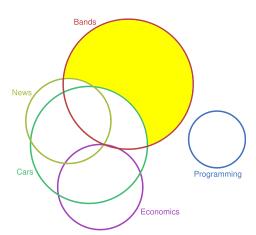
• ED-N:Explanation



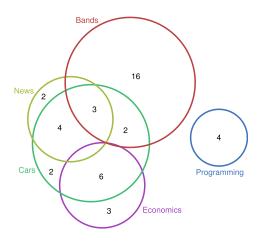
• ED-P



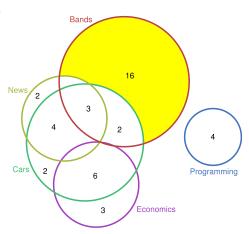
• ED-P:Explanation



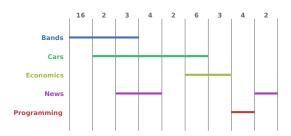
• ED-P&N



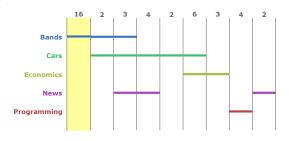
• ED-P&N:Explanation



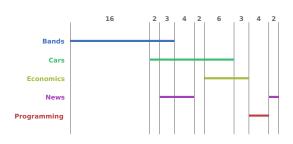
• LD-N



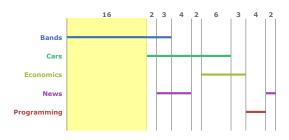
• LD-N:Explanation



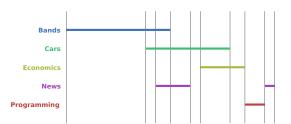
• LD-P



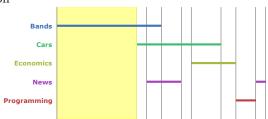
• LD-P:Explanation



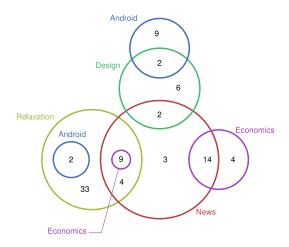
• LD-P&N



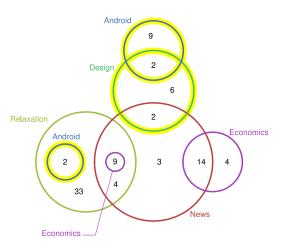
• LD-P&N:Explanation



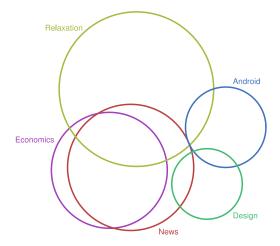
- Question: Tick the check boxes where the total number of people interested in that topic is less than the total number of people interested in Economics.
- Check boxes to be ticked: Android, Design
- Task Type: Set comparison Less than.
- SNAP Data Set Used: 16652550
- ED-N



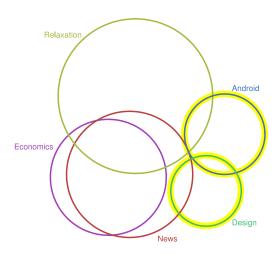
• ED-N:Explanation



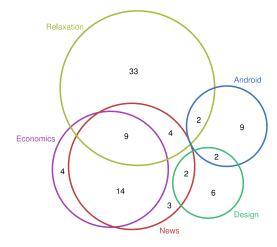
• ED-P



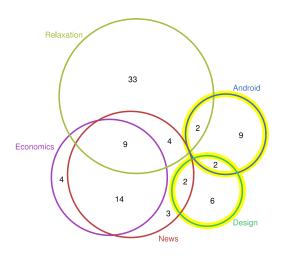
• ED-P:Explanation



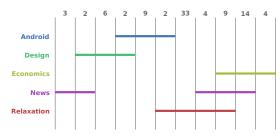
• ED-P&N



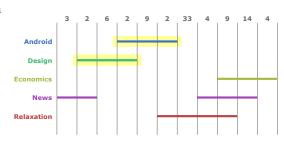
• ED-P&N:Explanation



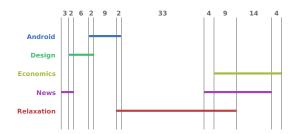
• LD-N



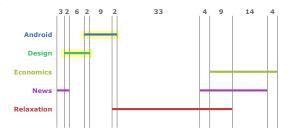
• LD-N:Explanation



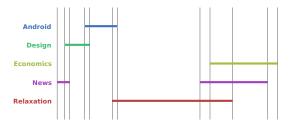
• LD-P



• LD-P:Explanation

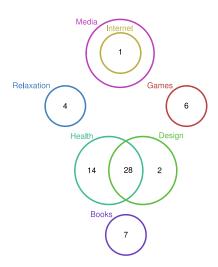


• LD-P&N

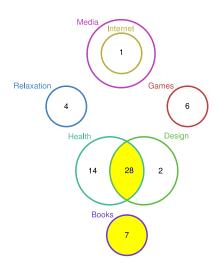




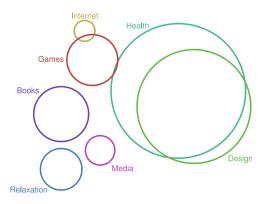
- Question: Tick the check boxes where more people have exactly that combination of interests than Games only.
- Check boxes to be ticked: Books, Design and Health
- SNAP Data Set Used: 19492215
- ED-N



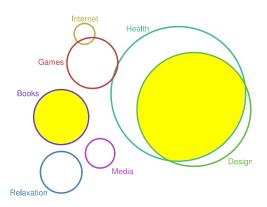
• ED-N:Explanation



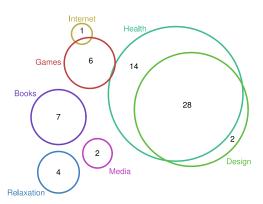
• ED-P

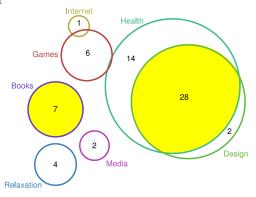


• ED-P:Explanation

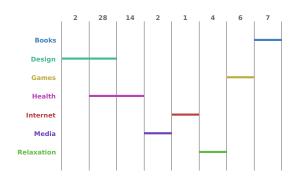


• ED-P&N

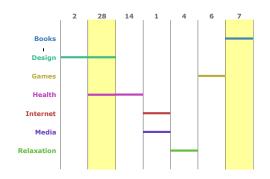




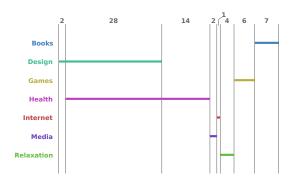
• LD-N



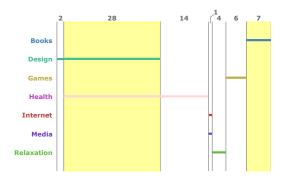
• LD-N:Explanation

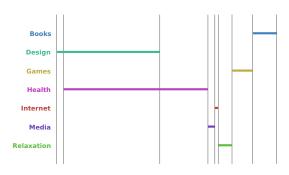


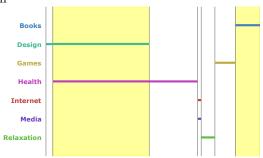
• LD-P



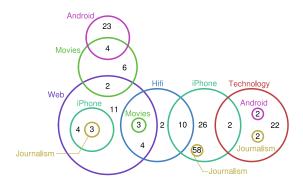
• LD-P:Explanation



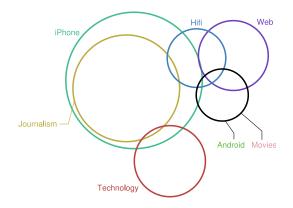




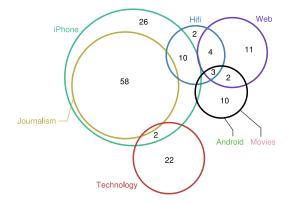
- Question: Tick the check boxes where the total number of people interested in that topic is **greater than** the total number of people interested in **iPhone**.
- Check boxes to be ticked: None of the above
- Task Type: Set comparison More than.
- SNAP Data Set Used: 105918870
- ED-N



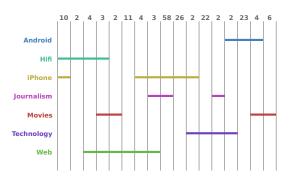
• ED-P



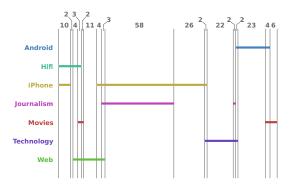
• ED-P&N

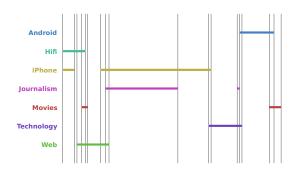


• LD-N



• LD-P

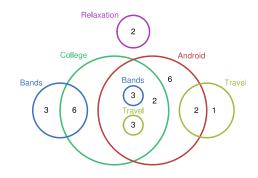




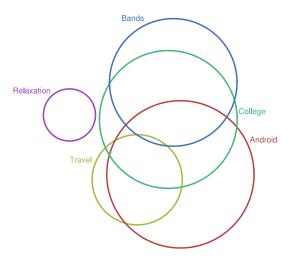
C - Studies A, B & C

Question 1

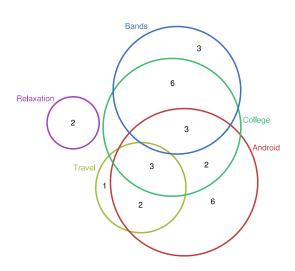
- Question: Tick the check boxes where the total number of people interested in that topic is greater than the total number of people interested in Relaxation.
- Check boxes to be ticked: Android, Bands, College, Travel
- Task Type: Set comparison More than.
- SNAP Data Set Used: 73024519
- ED-N



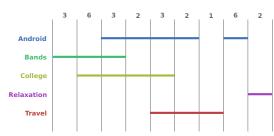
• ED-P



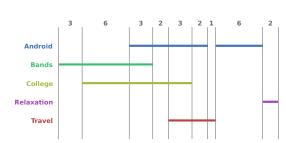
• ED-P&N

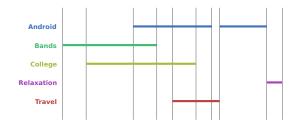


• LD-N

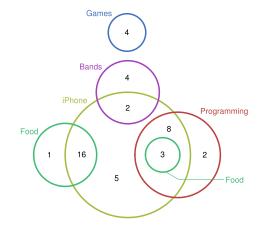


• LD-P

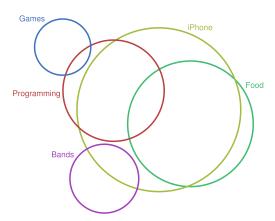




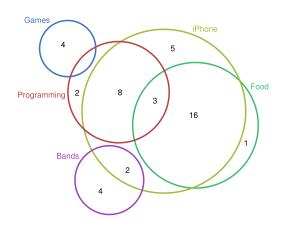
- Question: Tick the check boxes where the total number of people interested in that topic is less than the total number of people interested in **Programming**.
- Check boxes to be ticked: Bands, Games
- Task Type: Set comparison Less than.
- SNAP Data Set Used: 73738774
- ED-N



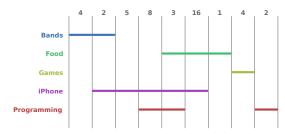
• ED-P



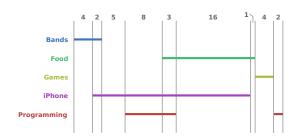
• ED-P&N

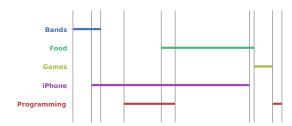


• LD-N

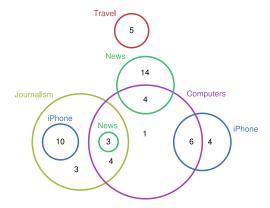


• LD-P

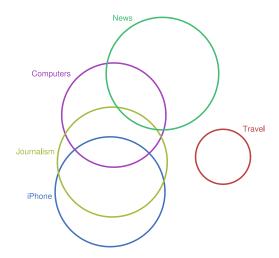




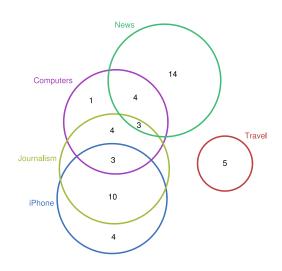
- Question: Tick the check boxes where more people have exactly that combination of interests than **News only**.
- Check boxes to be ticked: None of the above
- SNAP Data Set Used: 14957252
- ED-N



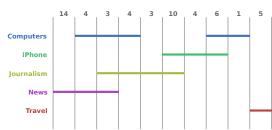
• ED-P



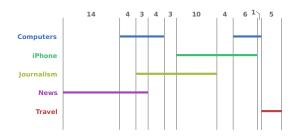
• ED-P&N

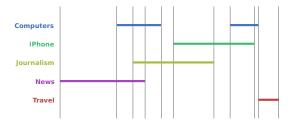


• LD-N

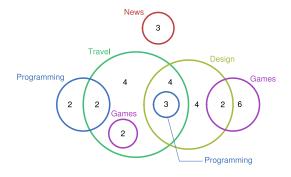


• LD-P

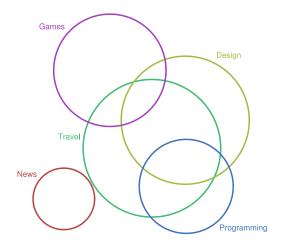




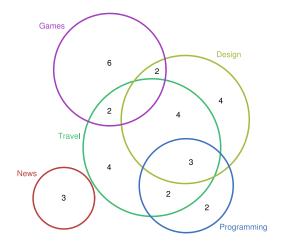
- Question: Tick the check boxes where more people have exactly that combination of interests than **Design and Games only**.
- Check boxes to be ticked: None of the above
- SNAP Data Set Used: 18687625
- ED-N



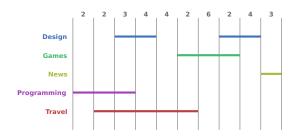
• ED-P



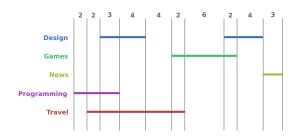
• ED-P&N

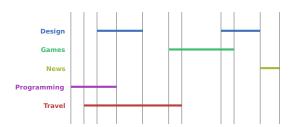




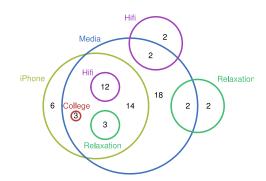


• LD-P

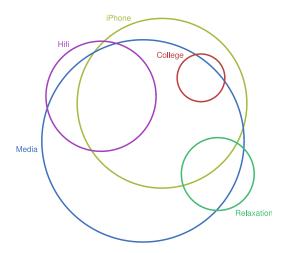




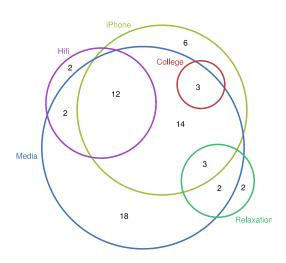
- Question: Tick the check boxes where the total number of people interested in that topic is greater than the total number of people interested in Relaxation.
- Check boxes to be ticked: Hifi, iPhone, Media
- SNAP Data Set Used: 234169190
- ED-N



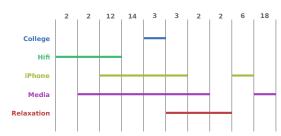
• ED-P



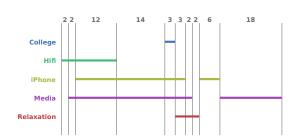
• ED-P&N

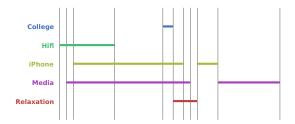


• LD-N

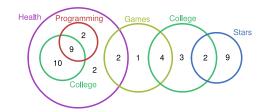


• LD-P

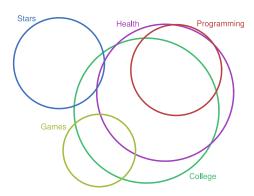




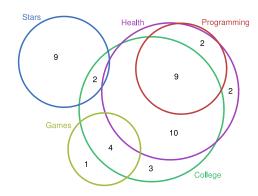
- Question: Tick the check boxes where the total number of people interested in that topic is less than the total number of people interested in Stars.
- Check boxes to be ticked: Games
- SNAP Data Set Used: 46160500
- ED-N



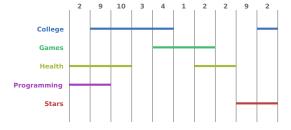
• ED-P



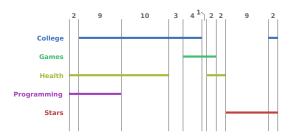
• ED-P&N

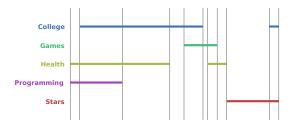


• LD-N

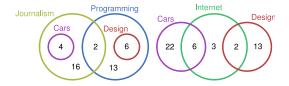




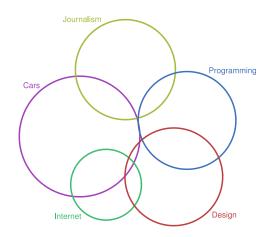




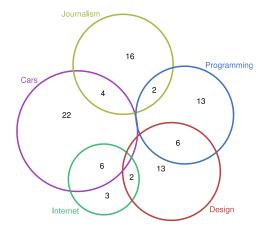
- Question: Tick the check boxes where more people have exactly that combination of interests than Internet only.
- Check boxes to be ticked: Cars, Cars and Internet, Design and Programming, Programming
- Task Type: Intersection comparison More than.
- SNAP Data Set Used: 87004822
- ED-N



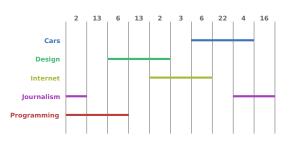
• ED-P



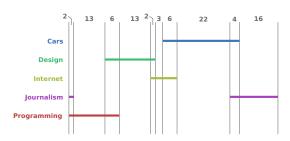
• ED-P&N

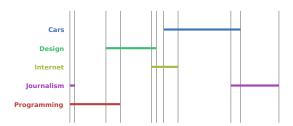


• LD-N

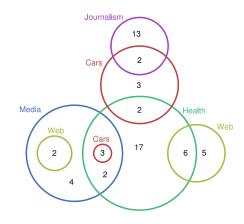


• LD-P

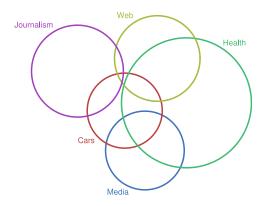




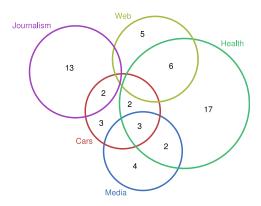
- Question: Tick the check boxes where more people have exactly that combination of interests than Web only.
- Check boxes to be ticked: Cars, Health and Media, Media
- SNAP Data Set Used: 14528221
- ED-N



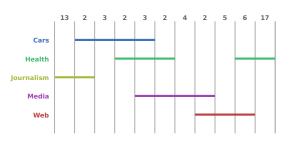
• ED-P



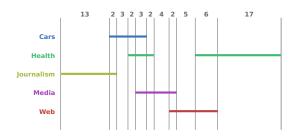
• ED-P&N

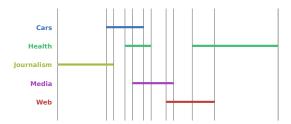


• LD-N

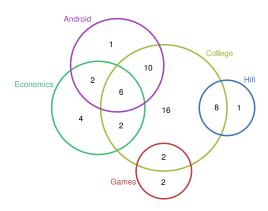


• LD-P

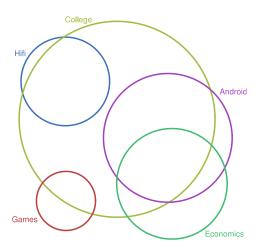




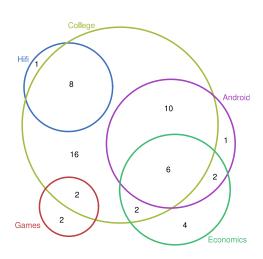
- Question: Tick the check boxes where the total number of people interested in that topic is **greater than** the total number of people interested in **Games**.
- Check boxes to be ticked: Android, College, Economics, Hifi
- SNAP Data Set Used: 17561785
- ED-N



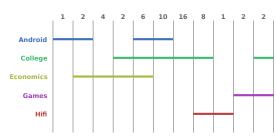
• ED-P



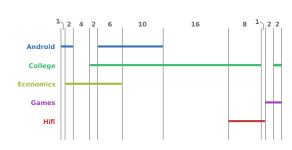
• ED-P&N

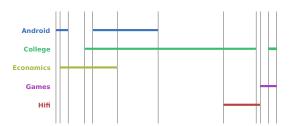


• LD-N

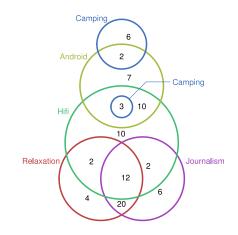


• LD-P

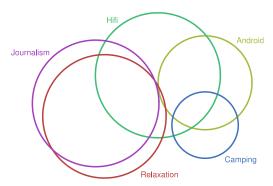




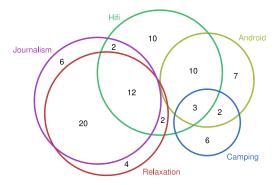
- Question: Tick the check boxes where the total number of people interested in that topic is less than the total number of people interested in Camping.
- Check boxes to be ticked: None of the above
- SNAP Data Set Used: 105150583
- ED-N



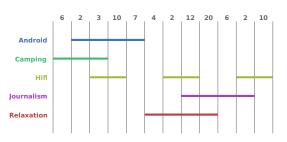
• ED-P



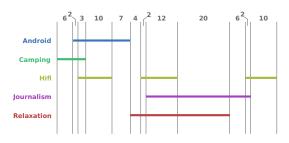
• ED-P&N

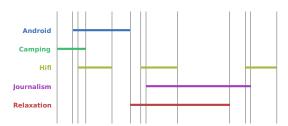


• LD-N

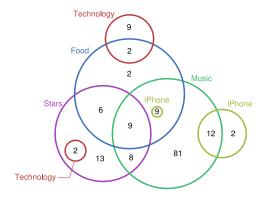


• LD-P

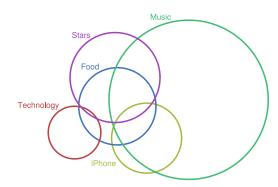




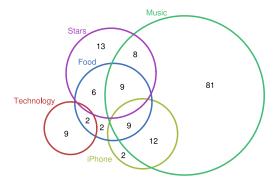
- Question: Tick the check boxes where more people have exactly that combination of interests than **Technology only**.
- Check boxes to be ticked: None of the above
- SNAP Data Set Used: 18534908
- ED-N



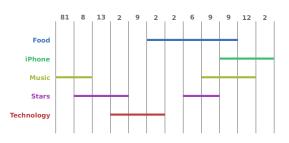
• ED-P

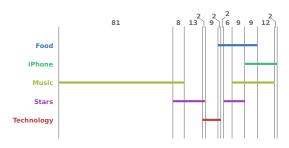


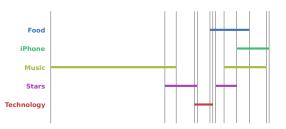
• ED-P&N



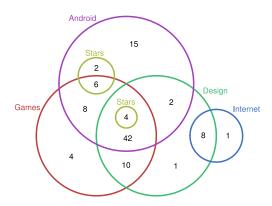
• LD-N



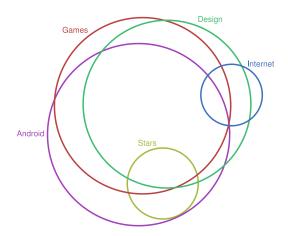


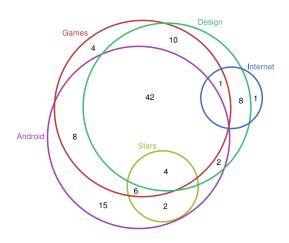


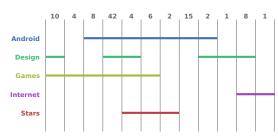
- Question: Tick the check boxes where more people have exactly that combination of interests than Android and Games only.
- Check boxes to be ticked: Android and Design, Android and Stars
- SNAP Data Set Used: 59588845
- ED-N



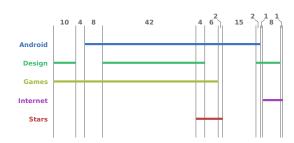
• ED-P

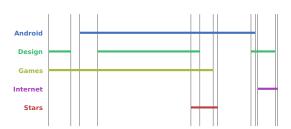




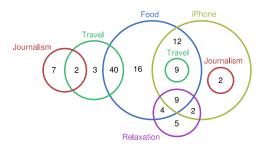


• LD-P

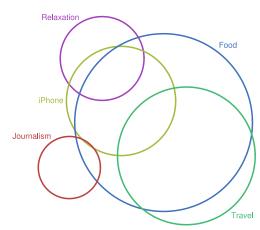




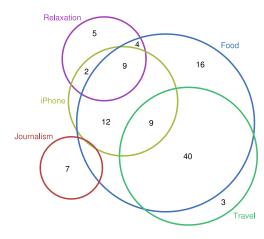
- Question: Tick the check boxes where the total number of people interested in that topic is greater than the total number of people interested in Relaxation.
- Check boxes to be ticked: Food, iPhone, Travel
- Task Type: Set comparison More than.
- SNAP Data Set Used: 64441390
- ED-N



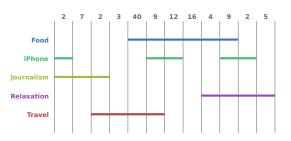
• ED-P

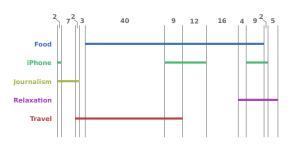


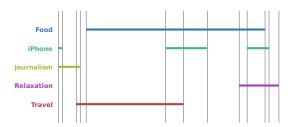
• ED-P&N



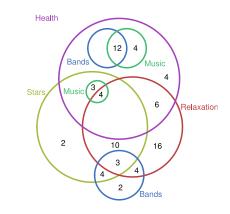
 \bullet LD-N



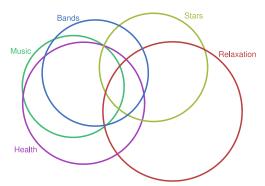




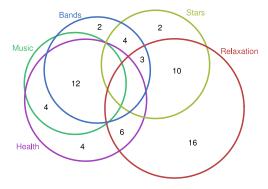
- Question: Tick the check boxes where the total number of people interested in that topic is less than the total number of people interested in Stars.
- Check boxes to be ticked: Music
- SNAP Data Set Used: 190696559
- ED-N



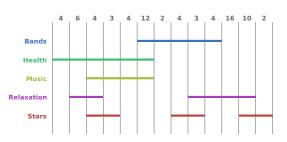
• ED-P

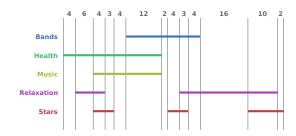


• ED-P&N



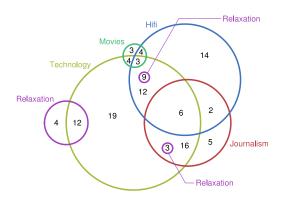
• LD-N



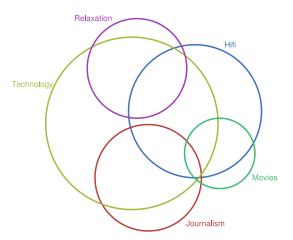


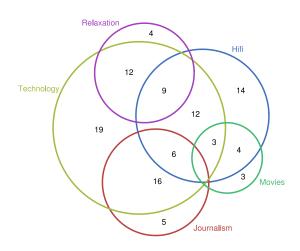


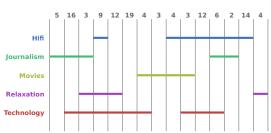
- Question: Tick the check boxes where more people have exactly that combination of interests than Hifi, Movies and Technology only.
- Check boxes to be ticked: Hifi, Journalism and Technology, Hifi and Journalism and Technology, Hifi and Relaxation and Technology
- Task Type: Intersection comparison More than.
- SNAP Data Set Used: 22863638
- ED-N



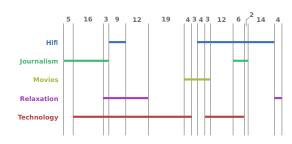
• ED-P

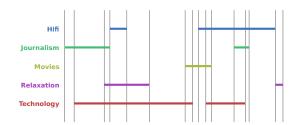




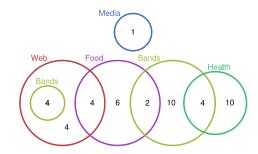


• LD-P

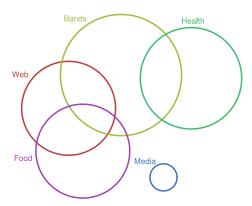




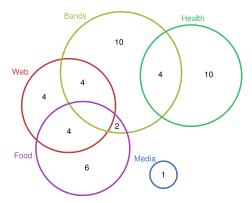
- Question: Tick the check boxes where more people have exactly that combination of interests than Bands and Web only.
- Check boxes to be ticked: Bands and Food
- SNAP Data Set Used: 15864197
- ED-N



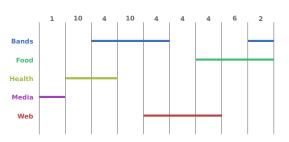
• ED-P

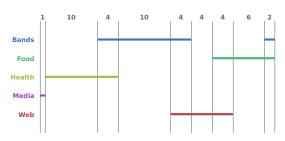


• ED-P&N



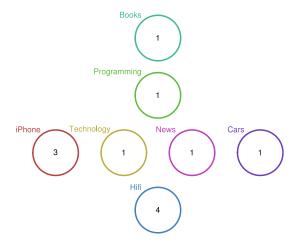
• LD-N



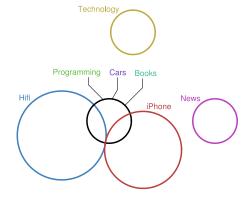




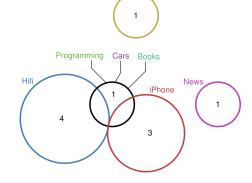
- Question: Tick the check boxes where the total number of people interested in that topic is **greater than** the total number of people interested in **Books**.
- Check boxes to be ticked: Hifi, iPhone
- SNAP Data Set Used: 1046661
- ED-N



• ED-P

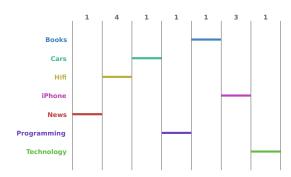


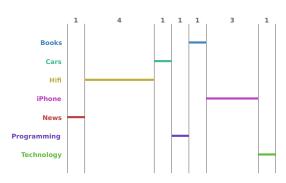
• ED-P&N

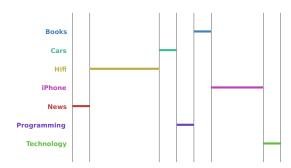


Technology

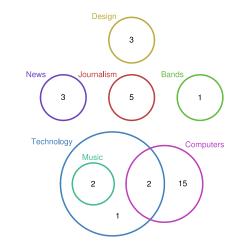




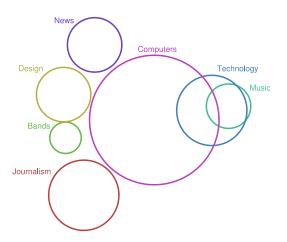


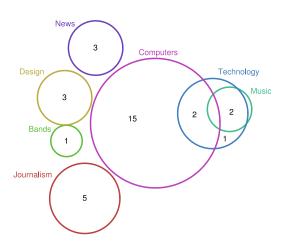


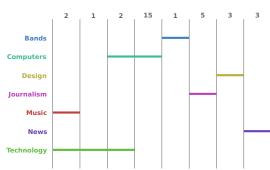
- Question: Tick the check boxes where the total number of people interested in that topic is less than the total number of people interested in Journalism.
- Check boxes to be ticked: Bands, Design, Music, News
- SNAP Data Set Used: 124296976
- ED-N



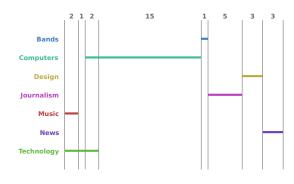
• ED-P

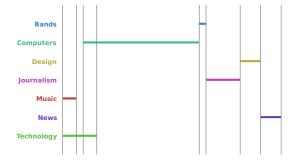




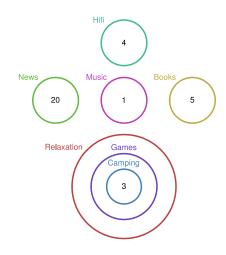


• LD-P

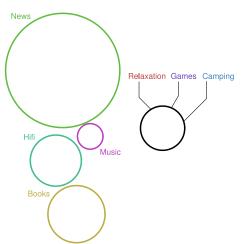


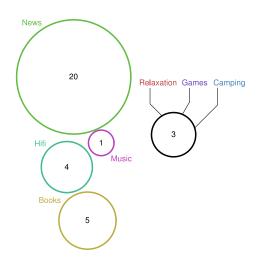


- Question: Tick the check boxes where more people have exactly that combination of interests than Camping, Games and Relaxation only.
- Check boxes to be ticked: News, Hifi, Books
- SNAP Data Set Used: 13747362
- ED-N

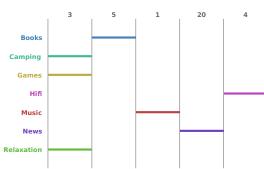


• ED-P

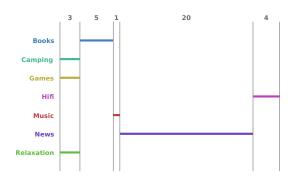


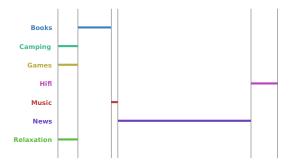


\bullet LD-N

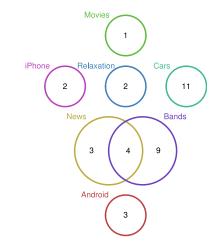


• LD-P

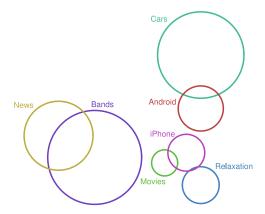


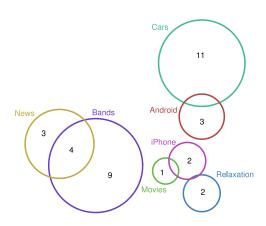


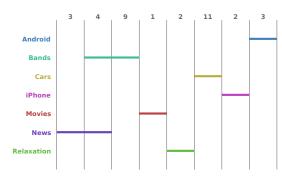
- Question: Tick the check boxes where more people have exactly that combination of interests than **iPhone only**.
- Check boxes to be ticked: Movies
- SNAP Data Set Used: 23790454
- ED-N



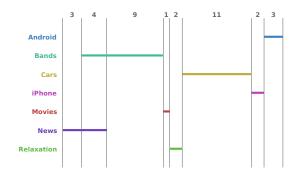
• ED-P

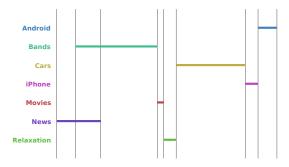




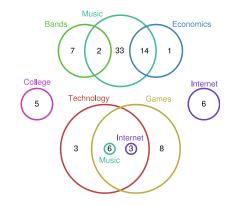


• LD-P

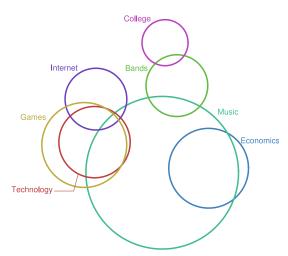


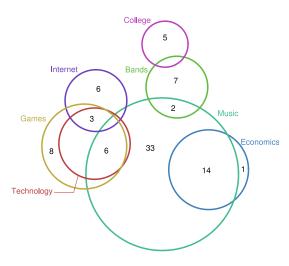


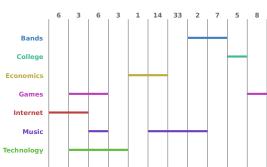
- Question: Tick the check boxes where the total number of people interested in that topic is **greater than** the total number of people interested in **Bands**.
- Check boxes to be ticked: Games, Music
- SNAP Data Set Used: 13274152
- ED-N



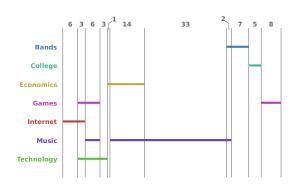
• ED-P

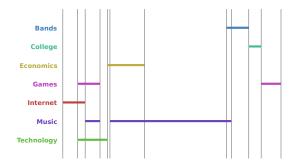




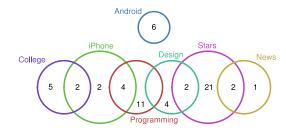


• LD-P

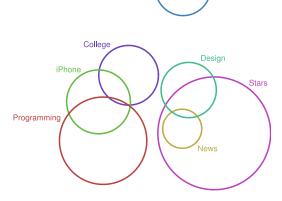




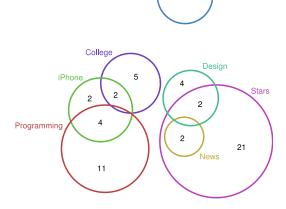
- Question: Tick the check boxes where the total number of people interested in that topic is less than the total number of people interested in **Design**.
- Check boxes to be ticked: News
- SNAP Data Set Used: 16834201
- ED-N



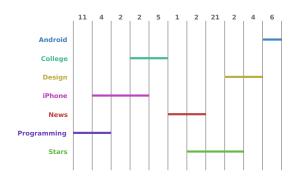
• ED-P

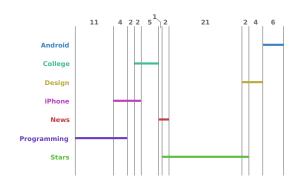


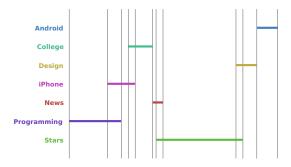
Android



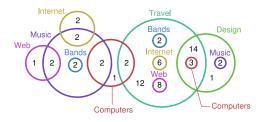




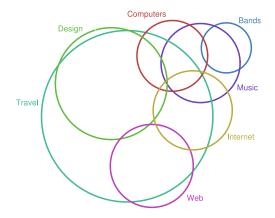




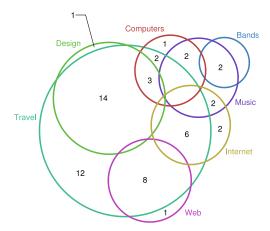
- Question: Tick the check boxes where more people have exactly that combination of interests than Computers, Design and Travel only.
- Check boxes to be ticked: Internet and Travel
- Task Type: Intersection comparison More than.
- SNAP Data Set Used: 29514951
- ED-N



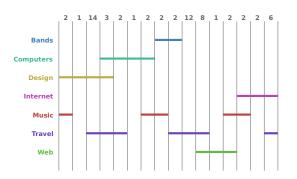
• ED-P

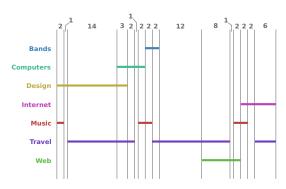


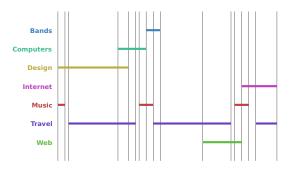
• ED-P&N



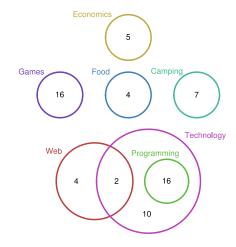
• LD-N



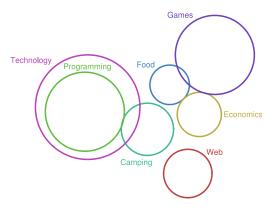


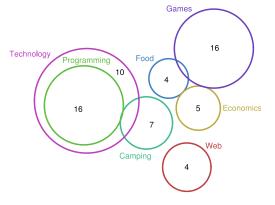


- Question: Tick the check boxes where more people have exactly that combination of interests than Camping only.
- Check boxes to be ticked: Camping, Food, Economics
- SNAP Data Set Used: 17767841
- ED-N

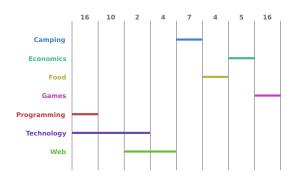


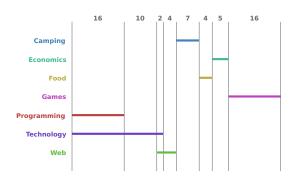
• ED-P

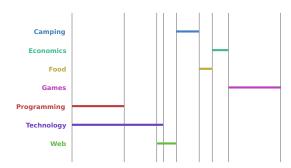




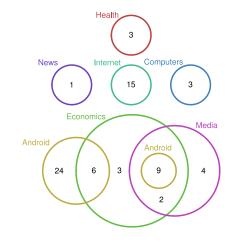




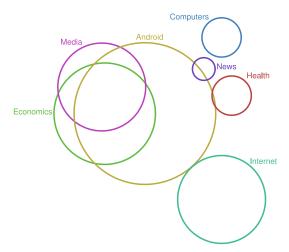


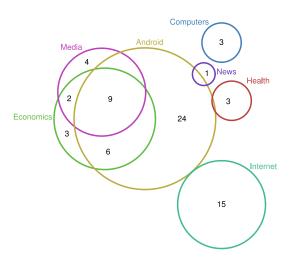


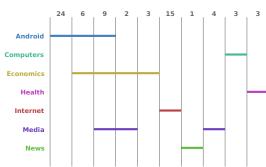
- Question: Tick the check boxes where the total number of people interested in that topic is greater than the total number of people interested in **Health**.
- Check boxes to be ticked: Economics, Internet, Media
- SNAP Data Set Used: 22121627
- ED-N



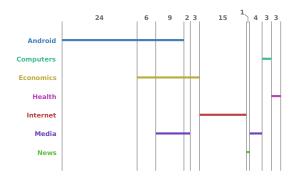
• ED-P

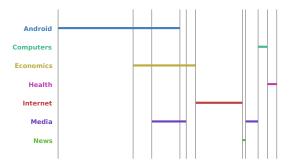




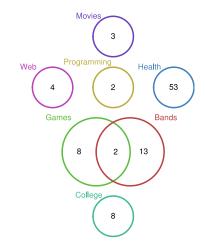


• LD-P

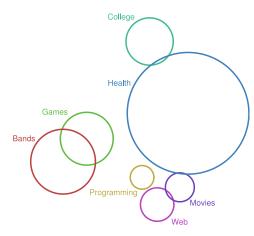


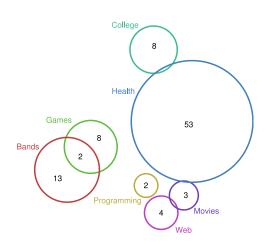


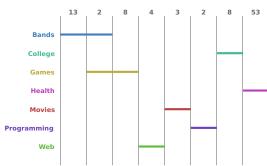
- Question: Tick the check boxes where the total number of people interested in that topic is less than the total number of people interested in Health.
- Check boxes to be ticked: Bands, Games, Movies, Programming
- SNAP Data Set Used: 2029971
- ED-N



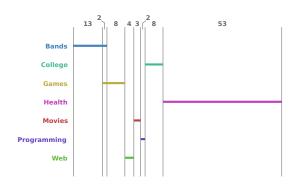
• ED-P

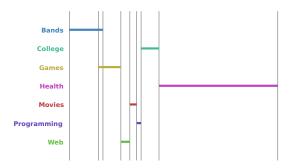




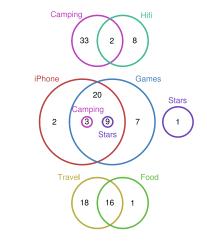


• LD-P

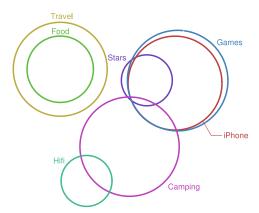


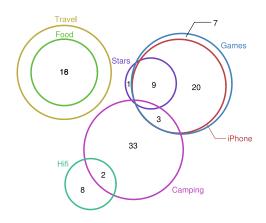


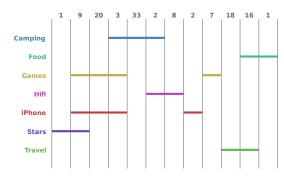
- Question: Tick the check boxes where more people have exactly that combination of interests than **Hifi only**.
- Check boxes to be ticked: Games and iPhone, Food and Travel
- SNAP Data Set Used: 629863
- ED-N



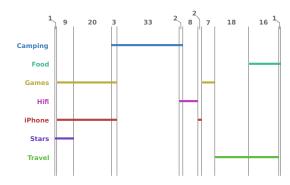
• ED-P

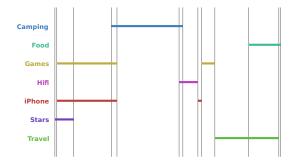




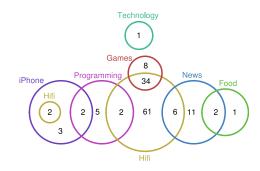


• LD-P

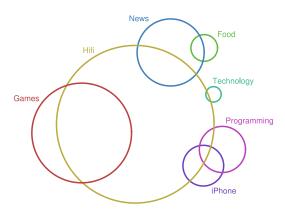




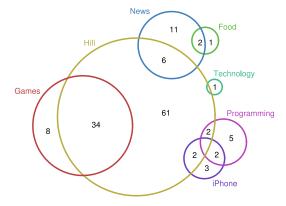
- Question: Tick the check boxes where more people have exactly that combination of interests than Food only.
- Check boxes to be ticked: None of the above
- SNAP Data Set Used: 30031265
- ED-N



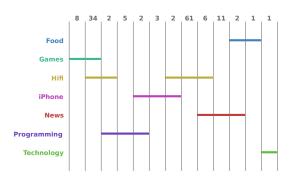
• ED-P

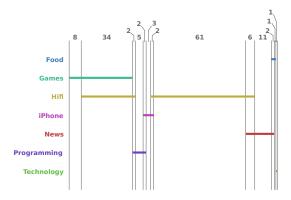


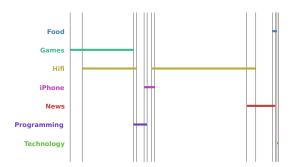
• ED-P&N



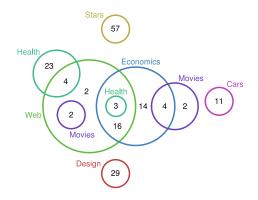
• LD-N



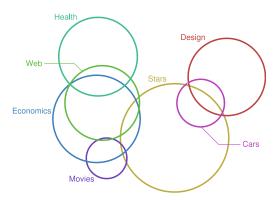




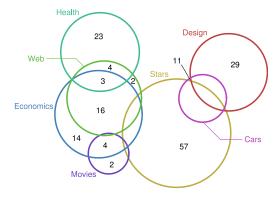
- Question: Tick the check boxes where the total number of people interested in that topic is **greater than** the total number of people interested in **Web**.
- Check boxes to be ticked: Design, Health, Stars
- Task Type: Set comparison More than.
- SNAP Data Set Used: 20436059
- ED-N



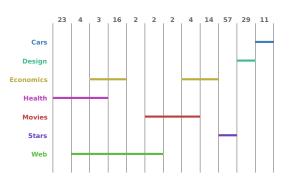
• ED-P



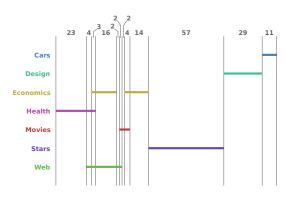
• ED-P&N

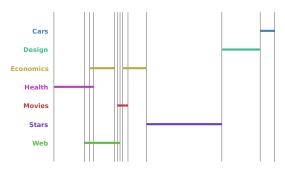


• LD-N



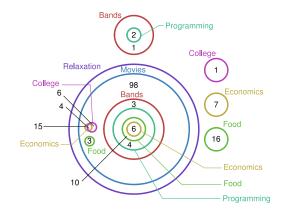
• LD-P



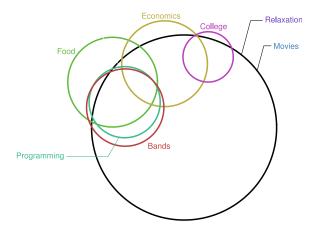


Question 30

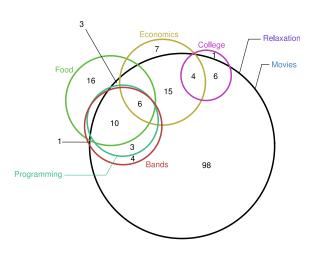
- Question: Tick the check boxes where the total number of people interested in that topic is less than the total number of people interested in Economics.
- Check boxes to be ticked: College, Programming
- Task Type: Set comparison Less than.
- SNAP Data Set Used: 30207757
- ED-N



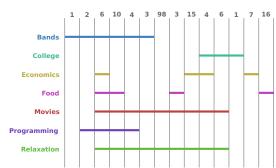
• ED-P



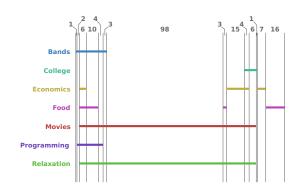
• ED-P&N

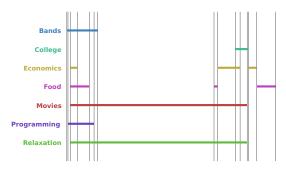


• LD-N



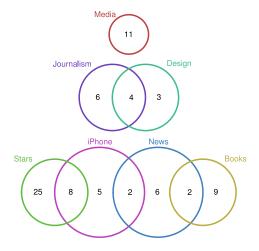
• LD-P



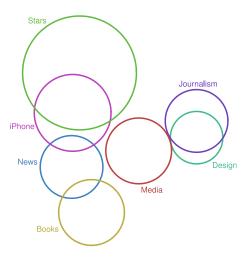


Question 31

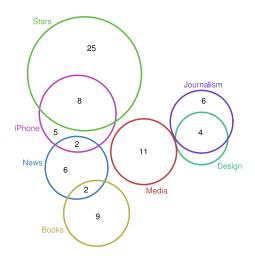
- Question: Tick the check boxes where more people have exactly that combination of interests than Stars only.
- Check boxes to be ticked: None of the above
- SNAP Data Set Used: 14060856
- ED-N



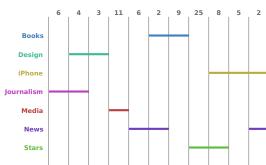
• ED-P



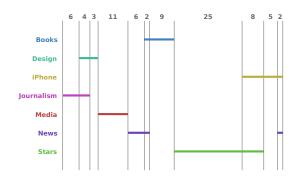
• ED-P&N

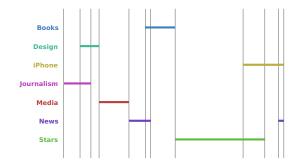


• LD-N



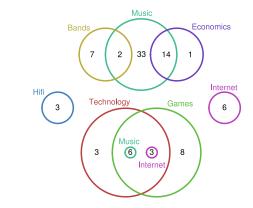
• LD-P



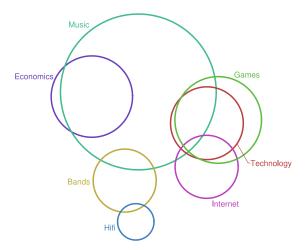


Question 32

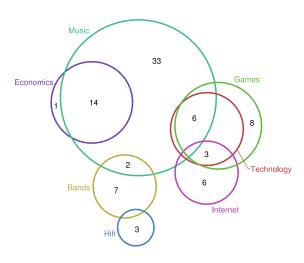
- Question: Tick the check boxes where more people have exactly that combination of interests than Games, Internet and Technology only.
- Check boxes to be ticked: None of the above
- SNAP Data Set Used: 13274152
- ED-N



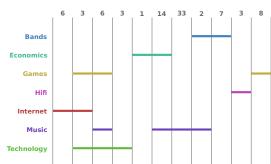
• ED-P



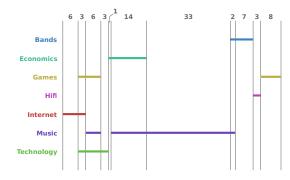
• ED-P&N

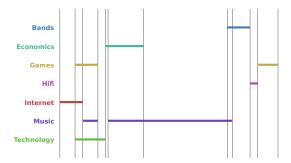


• LD-N



• LD-P





D - Statistical Methods

Here we present in detail the statistical models used for the first two studies, each of which included three treatment groups. The models used for the last study were simpler, as there were only two treatments.

For each of the three empirical studies, we employed two local odds ratios generalized estimating equations models [2] to analyse the accuracy data. An ANOVA calculation was not appropriate for these data as they violated the normality assumption of an ANOVA test. The non-parametric version of ANOVA, Kruskal-Wallis, was also not appropriate, as the responses for each individual are correlated, and thus not independent. The first model, for the first two studies, was employed to answer RQ1, RQ2 and RQ3, compared the visualization types overall irrespective of task category:

$$\log \left[\frac{\Pr(Y_{ik} \le j)}{1 - \Pr(Y_{ik} \le j)} \right] = \beta_{0j} + \beta_1 x_{ik1} + \beta_2 x_{ik2}$$

where

- $Pr(Y_{ik} \leq j)$ is the probability for subject i to provide at most j-1 correct answers to question k and where $j=1,...,5^1$.
- x_{ik1} is the indicator that the diagram given to subject i for answering question k was proportional, and
- x_{ik2} is the indicator that the diagram given to subject i for answering question k was both proportional and numerical

for i = 1, ..., n, given n participants, and k = 1, ..., 32. With this model, we could determine whether the odds of providing j or fewer correct answers for one of the visualization types (proportional, numerical, or both) was significantly different from others while taking into account the expected correlation among the responses provided by each individual participant.

The second model, for the first two studies, was employed to see whether the answers to RQ1, RQ2 and RQ3 still held when we take into account task category (set or intersection):

$$\log \left[\frac{\Pr(Y_{ik} \le j)}{1 - \Pr(Y_{ik} \le j)} \right] = \beta_{0j} + \beta_1 x_{ik1} + \beta_2 x_{ik2} + \beta_3 x_{ik3} + \beta_4 (x_{ik1} \times x_{it3}) + \beta_5 (x_{ik2} \times x_{ik3})$$

where the variables are as above and, in addition, x_{ik3} is the indicator that the diagram given to subject i for answering question k was of type I. This model allowed us to estimate the odds of providing j or fewer correct answers with one combination of visualization type and task category compared to other combinations and determine whether significant differences existed.

For the time data, we used two generalized estimation models [1] that allowed us to estimate whether the time taken to provide answers was significantly different. Again we present details of the two models used for the first two studies. Following a similar approach for the accuracy data, the first of the two models directly addressed RQ1, RQ2 and RQ3 and the more complex model delved deeper into the data to see whether task category was important. The more complex model is given here, which

 $^{^{1}}j$ takes values from 1 to 5 since, for each question, there are 5 checkboxes which need to be correctly checked or not.

accounts for the different combinations of visualization type (proportional, numerical, or both) and task category (set or intersection):

$$\log (t_{ik}) = \delta_0 + \delta_1 x_{ik1} + \delta_2 x_{ik2} + \delta_3 x_{ik3} + \delta_4 (x_{ik1} \times x_{ik3}) + \delta_5 (x_{ik2} \times x_{ik3})$$

where

- t_{ik} is the time needed for subject i to answer question k, and
- x_{ik1} , x_{ik2} and x_{it3} are defined in the previous model

for i = 1, ..., n, given n participants, and k = 1, ..., 32. This model allowed us to estimate the ratio of times of answering a question, given a pair of combinations of visualization type and task category. The simpler model excluded all terms involving x_{ik3} .

Statistical output is included in the submitted supplementary material. We report on the main findings in the associated paper.

E - Study A (ED) - Statistical Analysis

Here we present the statistical models used specifically for study 1 and the associated output.

Models

For the overall comparison, the following model was fitted to the accuracy data

$$\log \left[\frac{\Pr(Y_{ik} \le j)}{1 - \Pr(Y_{ik} \le j)} \right] = \beta_{0j} + \beta_1 x_{ik1} + \beta_2 x_{ik2}$$

where

- $Pr(Y_{ik} \leq j)$ is the probability for participant i to provide at most j-1 correct answers to question k and where j=1,2,3,4,5.
- x_{ik1} is the indicator that the diagram given to participant i for answering question k was ED-P,
- x_{ik2} is the indicator that the diagram given to participant i for answering question k was ED-PEN

for
$$i = 1, ..., 277$$
 and $j = 1, ..., 32$.

For the overall comparison, the following model was fitted to the time data

$$\log(t_{ik}) = \delta_0 + \delta_1 x_{ik1} + \delta_2 x_{ik2}$$

where

- t_{ik} is the time needed for participant i to answer question k,
- x_{ik1} and x_{ik2} are defined in the previous model

for i = 1, ..., 277 and k = 1, ..., 32.

For the set and intersection comparisons, following model was fitted to the accuracy data

$$\log \left[\frac{\Pr(Y_{ik} \le j)}{1 - \Pr(Y_{ik} \le j)} \right] = \beta_{0j} + \beta_1 x_{ik1} + \beta_2 x_{ik2} + \beta_3 x_{ik3} + \beta_4 (x_{ik1} \times x_{ik3}) + \beta_5 (x_{ik2} \times x_{ik3})$$

where

- $Pr(Y_{ik} \leq j)$ is the probability for participant i to provide at most j-1 correct answers to question k and where j=1,2,3,4,5.
- x_{ik1} is the indicator that the diagram given to participant i for answering question k was ED-P,
- x_{ik2} is the indicator that the diagram given to participant i for answering question k was ED-P $\mathcal{E}N$
- x_{ik3} is the indicator that the diagram given to participant i for answering question k was of type I

for
$$i = 1, ..., 277$$
 and $j = 1, ..., 32$.

For the set and intersection comparisons, the following model was fitted to the time data

$$\log(t_{ik}) = \delta_0 + \delta_1 x_{ik1} + \delta_2 x_{ik2} + \delta_3 x_{ik3} + \delta_4 (x_{ik1} \times x_{ik3}) + \delta_5 (x_{ik2} \times x_{ik3})$$

where

- t_{ik} is the time needed for participant i to answer question k,
- x_{ik1} , x_{ik2} and x_{ik3} are defined in the previous model

for i = 1, ..., 277 and k = 1, ..., 32.

95% LB

1.483904 1.183448 1.860640 0.000600

Overall results

Estimate

The accuracy analysis:

```
# ED-P vs ED-N
ComparisonStats(fitmodel,c(0, 0, 0, 0, 0, 1, 0))

## Estimate 95% LB 95% UB p-value
## 1.315777 1.058239 1.635991 0.013500

# ED-P&N vs ED-N
ComparisonStats(fitmodel,c(0, 0, 0, 0, 0, 0, 1))

## Estimate 95% LB 95% UB p-value
## 0.8866999 0.6888363 1.1413985 0.3506000

# ED-P vs ED-P&N
ComparisonStats(fitmodel,c(0, 0, 0, 0, 0, 1, -1))
```

The inference will be based on the model based estimated odds ratios (the value under Estimate). The results will be declared statistically significant only if the corresponding p-value is less than 0.05 and/or if the corresponding 95% confidence interval does not contain 1. The two middle columns correspond to the lower and the upper bound of the 95% confidence interval for the odds ratios. The interpretation of

95% UB p-value

the above analysis is the following: The estimated odds of having j or less correct answers with ED-P diagrams is 1.3158 times that with ED-N diagrams (p-value = 0.0135). The estimated odds of having j or less correct answers with ED-P diagrams is 1.4839 times that with ED-P diagrams (p-value = 0.0006). In other words, ED-N and ED-P diagrams are more likely to produce a higher ammount of correct answers than ED-P diagrams.

The fitted probabilities for j for each group are

```
X1 \leftarrow matrix(fitted(fitmodel)[c(1000,8000,3000),],3,6)
rownames(X1) <- dataset1$diagram[c(1000,8000,3000)]</pre>
colnames(X1) <- 0:5
X1
##
                     0
## ED-P
           0.02431216 0.03763469 0.07304745 0.1789546 0.1394637
           0.01651485 0.02609171 0.05255504 0.1405389 0.1228714
## ED-P&N
## ED-N
           0.01858585 0.02920467 0.05824127 0.1520142 0.1286269
##
## ED-P
          0.5465874
## ED-P&N 0.6414281
## ED-N
          0.6133271
The time analysis:
# ED-P vs ED-N
ComparisonStats(fittimemodel,c(0, 1, 0))
    Estimate
                95% LB
                           95% UB
                                    p-value
## 0.7809330 0.7081761 0.8611648 0.0000000
# ED-P&N vs ED-N
ComparisonStats(fittimemodel,c(0, 0, 1))
    Estimate
                95% LB
                           95% UB
                                    p-value
## 0.9125743 0.8188579 1.0170163 0.0980000
# ED-P vs ED-P&N
ComparisonStats(fittimemodel,c(0, 1, -1))
    Estimate
                95% LB
                           95% UB
                                    p-value
```

The inference will be based on the model based estimated ratios (the value under Estimate). The results will be declared statistically significant only if the corresponding p-value is less than 0.05 and/or if the corresponding 95% confidence interval does not contain 1. The p-values and the 95% confidence intervals suggest that participants needed less time with ED-P diagrams than with ED-P $\mathcal{E}N$ or ED-N diagrams and that there is no significance difference between ED-P $\mathcal{E}N$ and ED-N diagrams.

Set cardinalities results

0.8557473 0.7741852 0.9459021 0.0023000

Accuracy comparisons:

```
# (ED-P and type S) vs (ED-N and type S)
ComparisonStats(fitmodel,c(0, 0, 0, 0, 0, 1, 0, 0, 0, 0))
```

```
##
  Estimate
                95% LB
                           95% UB
                                    p-value
## 0.8069548 0.6083496 1.0703979 0.1367000
# (ED-P\ThetaN and type S) vs (ED-N and type S)
ComparisonStats(fitmodel, c(0, 0, 0, 0, 0, 0, 1, 0, 0, 0))
  Estimate
                95% LB
                           95% UB
                                    p-value
## 0.5059211 0.3544472 0.7221277 0.0002000
# (ED-P and type S) vs (ED-P\ThetaN and type S)
ComparisonStats(fitmodel, c(0, 0, 0, 0, 0, 1, -1, 0, 0, 0))
              95% LB
                        95% UB p-value
## Estimate
## 1.595021 1.094169 2.325136 0.015200
```

The inference will be based on the model based estimated odds ratios (the value under Estimate). The results will be declared statistically significant only if the corresponding p-value is less than 0.05 and/or if the corresponding 95% confidence interval does not contain 1. The two middle columns correspond to the lower and the upper bound of the 95% confidence interval for the odds ratios. The interpretation of the above analysis is the following: For type S diagrams, the estimated odds of having j or less correct answers with ED-P diagrams is 0.8070 times that with ED-N diagrams (p-value = 0.1367). The estimated odds of having j or less correct answers with ED-P diagrams is 1.5950 times that with ED-P6N0 diagrams (p-value = 0.0152). In other words, for type S, ED-N and ED-P6N0 diagrams are more likely to produce a higher ammount of correct answers than ED-P diagrams and there is no difference between ED-N and ED-P6N0 diagrams.

Time comparisons:

```
# (ED-P \text{ and type } S) vs (ED-N \text{ and type } S)
ComparisonStats(fittimemodel, c(0, 1, 0, 0, 0, 0))
    Estimate
                  95% LB
                             95% UB
                                       p-value
## 0.6839369 0.6223542 0.7516133 0.0000000
# (ED-P\ThetaN and type S) vs (ED-N and type S)
ComparisonStats(fittimemodel, c(0, 0, 1, 0, 0, 0))
                  95% LB
                             95% UB
                                       p-value
    Estimate
## 0.8750651 0.7878886 0.9718872 0.0127000
# (ED-P \text{ and type } S) vs (ED-P \mathcal{E}N \text{ and type } S)
ComparisonStats(fittimemodel,c(0, 1,-1, 0, 0, 0))
                                       p-value
    Estimate
                  95% LB
                             95% UB
## 0.7815840 0.7073398 0.8636211 0.0000000
```

The inference will be based on the model based estimated ratios (the value under Estimate). The results will be declared statistically significant only if the corresponding p-value is less than 0.05 and/or if the corresponding 95% confidence interval does not contain 1. For type I, the p-values and the 95% confidence intervals suggest that participants needed less time with ED-P diagrams than with ED-P%N or ED-N diagrams, and that participants needed less time with ED-P%N than ED-N diagrams.

Intersection cardinalities results

Accuracy analysis:

```
# (ED-P \text{ and } type I) vs (ED-N \text{ and } type I)
ComparisonStats(fitmodel,c(0, 0, 0, 0, 0, 1, 0, 0, 1, 0))
                        95% UB p-value
## Estimate
               95% LB
## 1.862365 1.445066 2.400170 0.000000
# (ED-PSN \text{ and type } I) vs (ED-N \text{ and type } I)
ComparisonStats(fitmodel,c(0, 0, 0, 0, 0, 0, 1, 0, 0, 1))
## Estimate
                        95% UB p-value
              95% LB
## 1.266412 0.944948 1.697235 0.113900
# (ED-P and type I) vs (ED-P&N and type I)
ComparisonStats(fitmodel, c(0, 0, 0, 0, 0, 1, -1, 0, 1, -1))
Time analysis:
# (ED-P and type I) vs (ED-N and type I)
ComparisonStats(fittimemodel,c(0, 1, 0, 0, 1, 0))
## Estimate
                 95% LB
                           95% UB
                                     p-value
## 0.8916851 0.7972190 0.9973450 0.0448000
# (ED-P&N and type I) vs (ED-N and type I)
ComparisonStats(fittimemodel,c(0, 0, 1, 0, 0, 1))
  Estimate
                 95% LB
                            95% UB
                                     p-value
## 0.9516914 0.8449521 1.0719146 0.4146000
# (ED-P \ and \ type \ I) vs (ED-P \&N \ and \ type \ I)
ComparisonStats(fittimemodel,c(0, 1,-1, 0, 1,-1))
## Estimate
                 95% LB
                           95% UB
                                     p-value
## 0.9369478 0.8377156 1.0479345 0.2542000
```

F - Study B (LD) - Statistical models and output

Here we present the statistical models used specifically for study 2 and the associated output.

Models

For the overall comparison, the following model was fitted to the accuracy data

$$\log \left[\frac{\Pr(Y_{ik} \le j)}{1 - \Pr(Y_{ik} \le j)} \right] = \beta_{0j} + \beta_1 x_{ik1} + \beta_2 x_{ik2}$$

where

• $Pr(Y_{ik} \leq j)$ is the probability for participant i to provide at most j-1 correct answers to question k and where j=1,2,3,4,5.

- x_{ik1} is the indicator that the diagram given to participant i for answering question k was LD-P,
- x_{ik2} is the indicator that the diagram given to participant i for answering question k was LD-P $\mathcal{E}N$

for i = 1, ..., 272 and k = 1, ..., 32.

For the overall comparison, the following model was fitted to the time data

$$\log(t_{ik}) = \delta_0 + \delta_1 x_{ik1} + \delta_2 x_{ik2}$$

where

- t_{ik} is the time needed for participant i to answer question k,
- x_{ik1} and x_{ik2} are defined in the previous model

for
$$i = 1, ..., 272$$
 and $k = 1, ..., 32$.

For the set and intersection comparisons, the following model was fitted to the accuracy data

$$\log \left[\frac{\Pr(Y_{ik} \le j)}{1 - \Pr(Y_{ik} \le j)} \right] = \beta_{0j} + \beta_1 x_{ik1} + \beta_2 x_{ik2} + \beta_3 x_{ik3} + \beta_4 (x_{ik1} \times x_{ik3}) + \beta_5 (x_{ik2} \times x_{ik3})$$

where

- $Pr(Y_{ik} \leq j)$ is the probability for participant i to provide at most j-1 correct answers to question k and where j=1,2,3,4,5.
- x_{ik1} is the indicator that the diagram given to participant i for answering question k was LD-P,
- x_{ik2} is the indicator that the diagram given to participant i for answering question k was LD-P $\mathcal{E}N$
- x_{ik3} is the indicator that the diagram given to participant i for answering question k was of type I

for i = 1, ..., 272 and k = 1, ..., 32.

For the set and intersection comparisons, the following model was fitted to the time data

$$\log(t_{ik}) = \delta_0 + \delta_1 x_{ik1} + \delta_2 x_{ik2} + \delta_3 x_{ik3} + \delta_4 (x_{ik1} \times x_{ik3}) + \delta_5 (x_{ik2} \times x_{ik3})$$

where

- t_{ik} is the time needed for participant i to answer question k,
- x_{ik1} , x_{ik2} and x_{ik3} are defined in the previous model

for i = 1, ..., 272 and k = 1, ..., 32.

Overall results

Accuracy analysis:

```
# LD-P vs LD-N
ComparisonStats(fitmodel,c(0, 0, 0, 0, 0, 1, 0))
```

```
## Estimate 95% LB 95% UB p-value
## 1.1348457 0.9399509 1.3701512 0.1882000
```

```
# LD-P&N vs LD-N
ComparisonStats(fitmodel,c(0, 0, 0, 0, 0, 0, 1))
## Estimate
                95% LB
                          95% UB
                                   p-value
## 1.1018091 0.8897432 1.3644198 0.3741000
# LD-P vs LD-P&N
ComparisonStats(fitmodel,c(0, 0, 0, 0, 0, 1, -1))
## Estimate
                95% LB
                          95% UB p-value
## 1.0299840 0.8456953 1.2544318 0.7690000
There is no difference between the three types of diagrams.
Time analysis:
# LD-P vs LD-N
ComparisonStats(fittimemodel,c(0, 1, 0))
## Estimate
                95% LB
                          95% UB
                                   p-value
## 0.8134194 0.7435824 0.8898155 0.0000000
# LD-P&N vs LD-N
ComparisonStats(fittimemodel, c(0, 0, 1))
## Estimate
                95% LB
                          95% UB
                                   p-value
## 0.9797777 0.8980481 1.0689453 0.6457000
# LD-P vs LD-P&N
ComparisonStats(fittimemodel,c(0, 1, -1))
## Estimate
                95% LB
                          95% UB
                                   p-value
## 0.8302082 0.7636182 0.9026049 0.0000000
```

Participants needed less time with LD-P diagrams than with LD-P $\mathcal{E}N$ or LD-N diagrams and that there is no significance difference between LD-P $\mathcal{E}N$ and LD-N diagrams.

Set cardinalities results

Accuracy analysis:

```
# (LD-P and type S) vs (LD-N and type S)
ComparisonStats(fitmodel, c(0, 0, 0, 0, 0, 1, 0, 0, 0))
## Estimate
                95% LB
                          95% UB p-value
## 1.0263526 0.7901575 1.3331515 0.8454000
# (LD-P\ThetaN and type S) vs (LD-N and type S)
ComparisonStats(fitmodel, c(0, 0, 0, 0, 0, 0, 1, 0, 0, 0))
## Estimate
                95% LB
                          95% UB
                                   p-value
## 1.2253349 0.9403471 1.5966931 0.1324000
# (LD-P and type S) vs (LD-P&N and type S)
ComparisonStats(fitmodel, c(0, 0, 0, 0, 0, 1, -1, 0, 0, 0))
## Estimate
                95% LB
                          95% UB
                                   p-value
```

```
## 0.8376098 0.6365894 1.1021079 0.2057000
```

For type S, there is no difference among the three types of diagrams.

Time analysis:

```
# (LD-P and type S) vs (LD-N and type S)
ComparisonStats(fittimemodel,c(0, 1, 0, 0, 0, 0))
## Estimate
                 95% LB
                            95% UB
                                     p-value
## 0.7846409 0.7208330 0.8540971 0.0000000
# (LD-P&N and type S) vs (LD-N and type S)
ComparisonStats(fittimemodel,c(0, 0, 1, 0, 0, 0))
## Estimate
                 95% LB
                            95% UB
                                     p-value
## 0.9970524 0.9150658 1.0863846 0.9462000
# (LD-P \text{ and type } S) vs (LD-P \mathcal{E} N \text{ and type } S)
ComparisonStats(fittimemodel,c(0, 1,-1, 0, 0, 0))
## Estimate
                 95% LB
                           95% UB
                                     p-value
## 0.7869606 0.7257799 0.8532986 0.0000000
```

For type S, participants needed less time with LD-P diagrams than with LD-P $\mathcal{E}N$ or LD-N and there is no significance difference between LD-P $\mathcal{E}N$ and LD-N diagrams.

Intersection cardinalities results

Accuracy analysis:

```
# (LD-P and type I) vs (LD-N and type I)
ComparisonStats(fitmodel,c(0, 0, 0, 0, 0, 1, 0, 0, 1, 0))
## Estimate
                95% LB
                          95% UB p-value
## 1.2038819 0.9449141 1.5338235 0.1332000
# (LD-P&N and type I) vs (LD-N and type I)
ComparisonStats(fitmodel, c(0, 0, 0, 0, 0, 0, 1, 0, 0, 1))
## Estimate
                95% LB
                          95% UB p-value
## 1.0341693 0.7835026 1.3650321 0.8125000
# (LD-P and type I) vs (LD-P&N and type I)
ComparisonStats(fitmodel,c(0, 0, 0, 0, 0, 1,-1, 0, 1,-1))
## Estimate
                95% LB
                           95% UB p-value
## 1.1641052 0.9025326 1.5014869 0.2419000
Time analysis:
# (LD-P \text{ and type } Z) vs (LD-N \text{ and type } Z)
ComparisonStats(fittimemodel,c(0, 1, 0, 0, 1, 0))
## Estimate
                95% LB
                           95% UB
                                    p-value
## 0.8432534 0.7600814 0.9355265 0.0013000
# (LD-P\ThetaN and type Z) vs (LD-N and type Z)
ComparisonStats(fittimemodel, c(0, 0, 1, 0, 0, 1))
```

```
## Estimate 95% LB 95% UB p-value
## 0.9628023 0.8730566 1.0617733 0.4477000

# (LD-P and type Z) vs (LD-P&N and type Z)
ComparisonStats(fittimemodel,c(0, 1,-1, 0, 1,-1))
```

```
## Estimate 95% LB 95% UB p-value
## 0.8758324 0.7958562 0.9638455 0.0067000
```

G - Study C (ED vs LD) - Statistical models and output

Here we present the statistical models used specifically for study 3 and the associated output.

Models

For the overall comparison, the following regression model was fitted to the accuracy data

$$\log \left[\frac{\Pr(Y_{ik} \le j)}{1 - \Pr(Y_{ik} \le j)} \right] = \beta_{0j} + \beta_1 x_{ik1}$$

where

- $Pr(Y_{ik} \leq j)$ is the probability for participant i to provide at most j-1 correct answers to question k and where j=1,2,3,4,5.
- x_{ik1} is the indicator that an *ED-P&N* diagram was given to participant i for answering question k,

for i = 1, ..., 185 and k = 1, ..., 32.

For the overall comparison, the following regression model was fitted to the time data

$$\log\left(t_{ik}\right) = \delta_0 + \delta_1 x_{ik1}$$

where

- t_{ik} is the time needed for participant i to answer question t,
- x_{ik1} is defined as in the model for the accuracy data

for i = 1, ..., 185 and k = 1, ..., 32.

For the set and intersection comparisons, the following regression model was fitted to the accuracy data

$$\log \left[\frac{\Pr(Y_{ik} \le j)}{1 - \Pr(Y_{ik} \le j)} \right] = \beta_{0j} + \beta_1 x_{ik1} + \beta_2 x_{ik2} + \beta_3 (x_{ik1} \times x_{ik2})$$

where

- $\Pr(Y_{ik} \leq j)$ is the probability for participant i to provide at most j-1 correct answers to question k and where j=1,2,3,4,5.
- x_{ik1} is the indicator that an *ED-P&N* diagram was given to participant i for answering question k,
- x_{ik2} is the indicator that the k-th question given to participant i was type I

```
for i = 1, ..., 185 and k = 1, ..., 32.
```

For the set and intersection comparisons, the following regression model was fitted to the time data

$$\log(t_{ik}) = \delta_0 + \delta_1 x_{ik1} + \delta_2 x_{ik2} + \delta_3 (x_{ik1} \times x_{ik2})$$

where

- t_{ik} is the time needed for participant i to answer question k,
- x_{ik1} and x_{ik2} are defined as in the model for the accuracy data

for i = 1, ..., 185 and k = 1, ..., 32.

Overall results

Accuracy analysis:

```
# (ED-PEN) vs (LD-P)
ComparisonStats(fitmodel, c(0, 0, 0, 0, 0, 1))

## Estimate 95% LB 95% UB p-value
## 0.7814704 0.6249702 0.9771602 0.0306000
```

The ED-P&N diagrams are better than LD-P when the questions type is ignored.

Time analysis:

```
# (ED-P&N) vs (LD-P)
ComparisonStats(fittimemodel, c(0, 1))

## Estimate 95% LB 95% UB p-value
## 1.159591 1.037295 1.296306 0.009200
```

The time needed with a ED-P@N diagram is 1.15 times that needed with a LD-P diagram when the question type was ignored.

Set cardinalities results

Accuracy analysis:

```
# (ED-P&N and typeS) vs (LD-P and typeS)
ComparisonStats(fitmodel, c(0, 0, 0, 0, 0, 1, 0, 0))

## Estimate 95% LB 95% UB p-value
## 0.9628247 0.6884480 1.3465527 0.8248000
```

There is no difference between $ED\text{-}P\mathcal{C}N$ and LD-P diagrams with for questions of type S.

Time analysis:

```
# (ED-PEN and typeS) vs (LD-P and typeS)
ComparisonStats(fittimemodel, c(0, 1, 0, 0))

## Estimate 95% LB 95% UB p-value
```

1.255564 1.130982 1.393870 0.000000

To answer questions of type S, the time needed with a ED-P $\mathcal{E}N$ diagram is 1.26 times that needed with a LD-P diagram.

Intersection cardinalities results

Accuracy analysis:

```
# (ED-P&N and typeI) vs (LD-P and typeI)
ComparisonStats(fitmodel, c(0, 0, 0, 0, 1, 0, 1))
```

```
## Estimate 95% LB 95% UB p-value ## 0.6571699 0.5113266 0.8446114 0.0010000
```

For questions of type I, ED-P & N diagrams are more likely to increase the number of correct answers when comprared to LD-P diagrams.

Time analysis:

```
# (ED-PEN and typeI) vs (LD-P and typeI)
ComparisonStats(fittimemodel, c(0, 1, 0, 1))

## Estimate 95% LB 95% UB p-value
## 1.0709541 0.9448343 1.2139088 0.2836000
```

References

- K. Y. Liang and S. L. Zeger. Longitudinal data analysis using generalized linear models. *Biometrika*, 73:13–22, 1986.
- 2. A. Touloumis, A. Agresti, and M. Kateri. Generalized estimating equations for multinomial responses using a local odds ratios parameterization. *Biometrics*, 69(3):633–640, 2013.