

A NOVEL TEACHING PROGRAM FOR INTER-SURFACE RADIATION EXCHANGE

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Abstract

The present article describes a possible pedagogical approach for delivering a module in inter-surface radiation exchange that will encompass the following applications and will also lend to other potential applications which may be identified by the reader: (i) building heating and cooling load, and (ii) energy balance of solar thermal air- and water collectors. The basic cases for inter-surface radiation exchange that are presented here are: (a) surfaces that may share a common edge, i.e. surfaces or extension of surfaces that are at an angle to each other, and (b) parallel surfaces.

This article will not only present solutions to the above-mentioned problems but will also be accompanied by MS-Excel/VBA codes for readers' use.

Key words: radiation exchange, solar energy, engineering and architectural pedagogy, MS-Excel/VBA

1. INTRODUCTION

According to the agreement of Paris Climate Change Conference in November 2015 the governments agreed to aim to limit the increase in the global average temperature to well below 2 °C above pre-industrial levels and to pursue efforts to limit the temperature increase to 1.5 °C above pre-industrial levels, recognizing that this would significantly reduce the risks and impacts of climate change. For the second goal (1.5 °C) developed-country emissions need to be reduced to 85-95% below 1990.

Presently, in the EU buildings consume about 40% of the energy produced and 72% of the electricity. They thus contribute about 30% of EU carbon emissions. The planning of Zero- and nearly Zero Energy Buildings needs advanced engineering knowledge how to manage and control the heat streams within the building and between the building and its environment.

Radiation heat transfer plays an important role in the above knowledge development as it plays an important role in very many engineering applications. An important application in this respect is within the building services sector wherein the radiant exchanges between building surfaces need to be analysed. The CIBSE (CIBSE, 2015) and ASHRAE Guides (ASHRAE, 2013) provide the background physics and the relevant mathematical formulations for radiant energy exchanges between surfaces of different configurations.

Throughout the world a large number of universities offer modules that are related to building services which include heating and cooling load calculations, an important element of which is inter-surface thermal radiation exchange. Furthermore, in response to the societal demand for development of renewable energy training and education there is now on offer a plethora of solar energy related modules. An application of radiation exchange exists in solar thermal collector design. To summarise, therefore, there are at least two areas of pedagogy that are related to radiation energy exchange: (i) building heating and cooling load, and (ii) energy balance of solar thermal air- and water collectors.

The aim of this article is to present pedagogical procedures for the latterly mentioned applications. The general formulations that are presented here are based on a finite-element approach and include (a) surfaces that may share a common edge, i.e. surfaces or extension of surfaces that are at an angle to each other, and (b) parallel surfaces. The justification for such detailed procedures and their applicability within the modern building energy simulation software is also covered.

2. USE OF MICROSOFT-EXCEL

Excel operates with data entered by the user into a spreadsheet. This software recognizes a multitude of engineering, mathematical and trigonometric functions. The most powerful of Excel-based facility is perhaps its ability to incorporate user-written macros (Liengme, 2009). The macros may be written in an in-built Visual basic for Applications (VBA) language which may be taught in the first year of engineering and architecture programs in the United Kingdom, Bulgaria and other EU member states. An introduction to programming languages is indeed initiated in high schools and therefore first year students naturally develop a bent of mind towards programming.

Throughout this article Microsoft Excel–VBA software has been used to demonstrate its applicability to solve complex engineering problems such as those encountered in thermal radiation energy transfer.

3. MATHEMATICAL FORMULATIONS

3.1. Radiation exchange between any two surfaces

For any two black surfaces the thermal radiation exchange is given by,

$$Q_{1-2} = \sigma(T_1^4 - T_2^4)A_1F_{1-2} = \sigma(T_2^4 - T_1^4)A_2F_{2-1} \quad (1)$$

Within thermal radiation heat transfer terminology the term F_{1-2} is known as "configuration factor". There are also other names for the latter such as "view factor", "geometry factor", "angle factor" or "shape factor". For any two elemental surfaces such as those shown in Fig. 1, F_{1-2} is given as,

$$F_{1-2} = \frac{1}{A_1} \int_{A_1} \int_{A_2} \frac{\cos \Phi_1 \cos \Phi_2}{\pi R^2} dA_2 dA_1 \quad (2)$$

where R is the distance between both differential elements dA_1 and dA_2 ; A_1 and A_2 are the faces of both surfaces; Φ_1 and Φ_2 are the angles between the normal vectors to both differential elements and the line between their centres (Fig. 2).

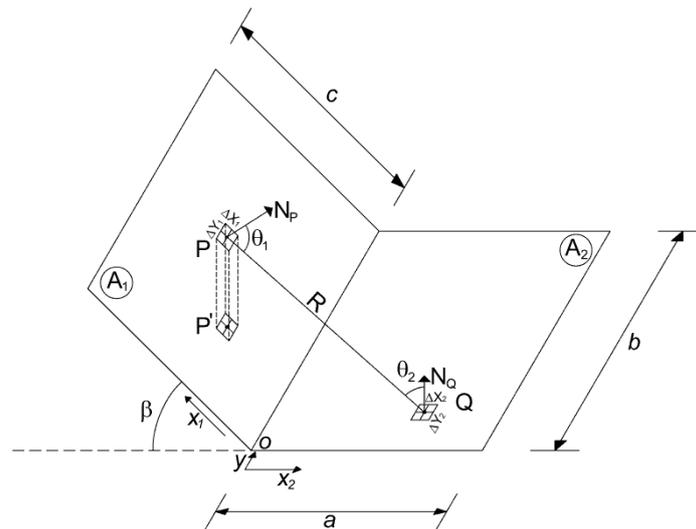


Figure 1. Isometric view of the receiving (A_1) and emitting (A_2) surfaces.

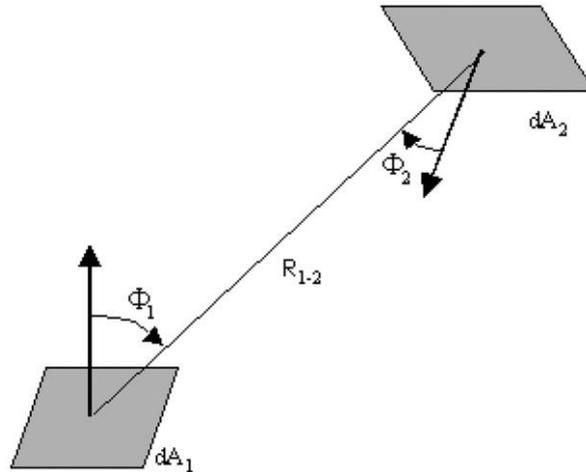


Figure 2. Defining geometry for configuration factor

3.1.1. Orthogonal case

The cases, which find ready application with respect to building services, are two rectangular parallel surfaces and surfaces that are perpendicular to each other (See Fig. 3). The fundamental integral for two rectangular surfaces A_1 with dimensions $a \times b$ and A_2 with dimensions $c \times d$ is Equation (3),

$$F_{1-2} = \frac{1}{ab} \int_{x_1=0}^a \int_{y_1=0}^b \int_{x_2=0}^c \int_{y_2=0}^d \frac{\cos \Phi_1 \cdot \cos \Phi_2}{\pi R^2} dy_2 dx_2 dy_1 dx_1 \tag{3}$$

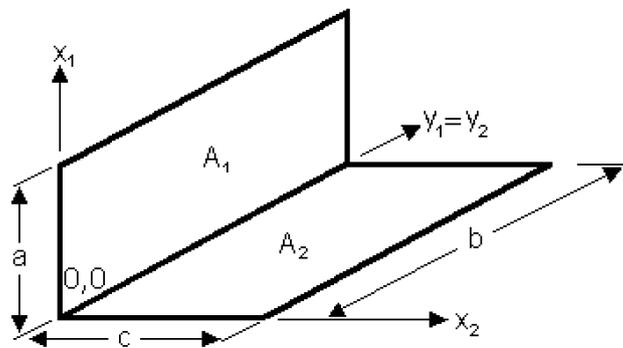


Figure 3. Two orthogonal surfaces with one common edge

For two perpendicular rectangular surfaces with a common edge b (Fig. 3), where $\cos \Phi_1 = x_2 / R$ and $\cos \Phi_2 = x_1 / R$ and $R = \sqrt{x_1^2 + x_2^2 + (y_1 - y_2)^2}$, the resulting integral is Eq. (4):

$$F_{1-2} = \frac{1}{ab} \int_{x_1=0}^a \int_{y_1=0}^b \int_{x_2=0}^c \int_{y_2=0}^b \frac{x_1 x_2}{\pi [x_1^2 + x_2^2 + (y_1 - y_2)^2]^2} dy_2 dx_2 dy_1 dx_1 \tag{4}$$

The configuration factor – solution of this integral, is Eq. (5), where $N = c/b$ and $L = a/b$:

$$F_{1-2} = \frac{1}{\pi L} \left(L \tan^{-1} \left(\frac{1}{L} \right) + N \tan^{-1} \left(\frac{1}{N} \right) - \sqrt{N^2 + L^2} \tan^{-1} \left(\frac{1}{\sqrt{N^2 + L^2}} \right) + \frac{1}{4} \left\{ \ln \left[\frac{(1+L^2)(1+N^2)}{1+L^2+N^2} \right] + L^2 \ln \left[\frac{L^2(1+N^2+L^2)}{(1+L^2)(1+N^2)} \right] + N^2 \ln \left[\frac{N^2(1+N^2+L^2)}{(1+N^2)(N^2+L^2)} \right] \right\} \right) \quad (5)$$

3.1.2. Tilted surface

A more generalised version of the above case is however the one where the two surfaces A_1 and A_2 are not perpendicular to each other. Rather, they are separated by any given angle Φ that may or may not be 90 degrees, as shown in Fig. 4.

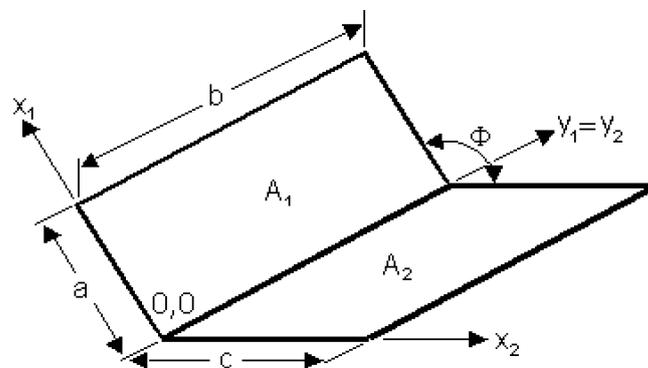


Figure 4. Two rectangular surfaces with one common edge and included angle of Φ

This generalised case, once again, has a number of applications such as solar energy reflected off ground and incident on a sloping roof, solar thermal water or air collectors or indeed photovoltaic modules. Note that for any given situation the ground reflected radiation may emanate from a conglomeration of surfaces of disparate reflectivities such as grass ($\rho=0.24$), tarmac ($\rho=0.15$), soil ($\rho=0.12-0.25$), other roof tops (0.13), pebbles ($\rho=0.14-0.56$) or water bodies ($\rho=0.05-0.2$).

The integration of Equation (2) for the case under discussion is rather involved. It does not lead to an exact solution, as was provided for the special case of $\Phi = 90^\circ$ – see Equation (5). It rather leads to a partial, analytically integrable, one part, and the other part that is only numerically obtained.

If we apply Equation (3) to two rectangular surfaces A_1 with dimensions $a \times b$ and A_2 with dimensions $c \times b$, with angle Φ between them (Fig. 5 and Fig. 6), then the resulting integral is Equation (6):

$$F_{1-2} = \frac{1}{ab} \int_{x_1=0}^a \int_{y_1=0}^b \int_{x_2=0}^c \int_{y_2=0}^b \frac{x_1 x_2 \sin^2 \beta}{\pi [x_1^2 + x_2^2 + 2x_1 x_2 \cos \beta + (y_1 - y_2)^2]} dy_2 dx_2 dy_1 dx_1 \quad (6)$$

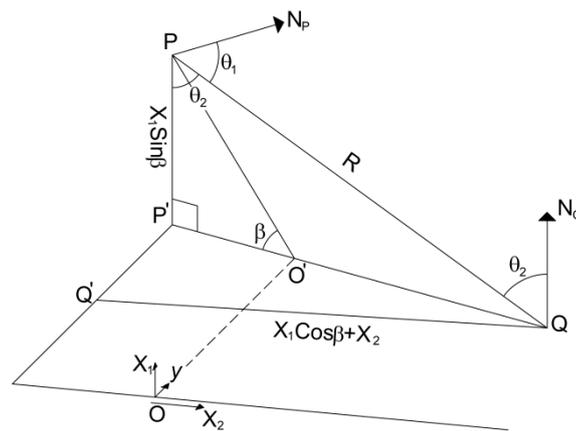


Figure 5. Projection of A₁ and A₂ surfaces on the X₂/Y and X₂/Z planes.

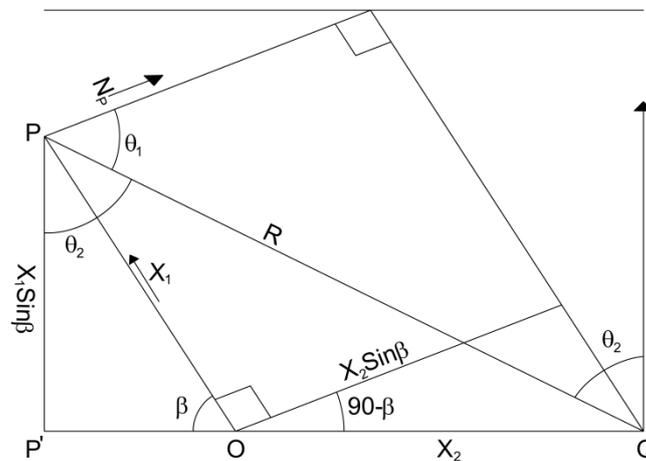


Figure 6. Detail of projection X₂/Z plane

The solution of this integral is Equation (7). The last part of Equation (7) is unsolvable integral. This explains why a complete analytical solution of Equation (6) does not exist. The view factor F_{1-2} can be estimated partially analytically, partially numerically.

$$\begin{aligned}
 F_{1-2} = & -\frac{\sin 2\Phi}{4\pi B} \left[AB \sin \Phi + \left(\frac{\pi}{2} - \Phi\right) (A^2 + B^2) + B^2 \tan^{-1} \left(\frac{A - B \cos \Phi}{B \sin \Phi}\right) + A^2 \tan^{-1} \left(\frac{B - A \cos \Phi}{A \sin \Phi}\right) \right] \\
 & + \frac{\sin^2 \Phi}{4\pi B} \left\{ \left(\frac{2}{\sin^2 \Phi} - 1\right) \ln \left[\frac{(1 + A^2)(1 + B^2)}{1 + C} \right] + B^2 \ln \left[\frac{B^2(1 + C)}{C(1 + B^2)} \right] + A^2 \ln \left[\frac{A^2(1 + A^2)^{\cos 2\Phi}}{C(1 + C)^{\cos 2\Phi}} \right] \right\} \\
 & + \frac{1}{\pi} \tan^{-1} \left(\frac{1}{B}\right) + \frac{A}{\pi B} \tan^{-1} \left(\frac{1}{A}\right) - \frac{\sqrt{C}}{\pi B} \tan^{-1} \left(\frac{1}{\sqrt{C}}\right) \\
 & + \frac{\sin \Phi \sin 2\Phi}{2\pi B} AD \left[\tan^{-1} \left(\frac{A \cos \Phi}{D}\right) + \tan^{-1} \left(\frac{B - A \cos \Phi}{D}\right) \right] \\
 & + \frac{\cos \Phi}{\pi B} \int_0^B \sqrt{1 + z^2 \sin^2 \Phi} \left[\tan^{-1} \left(\frac{z \cos \Phi}{\sqrt{1 + z^2 \sin^2 \Phi}}\right) + \tan^{-1} \left(\frac{A - z \cos \Phi}{\sqrt{1 + z^2 \sin^2 \Phi}}\right) \right] dz
 \end{aligned}
 \tag{7}$$

where $A = c/b$, $B = a/b$, $C = A^2 + B^2 - 2AB \cos \Phi$ and $D = \sqrt{1 + A^2 \sin^2 \Phi}$.

At this stage refer to Fig. 1. By numerically integrating the elemental view factor it is then possible to obtain GVF for surface A_1 . Furthermore, a Visual Basic for Application (VBA) code is presented that would enable the reader to obtain the View Factor (VF) for any given geometry and choice of reflectivities for the foreground (surface A_2).

3.1.3. Derivation of a numerically integrable, general purpose VF: rectangular surfaces A_i and A_j with a common edge

If we consider the rectangular surfaces A_i and A_j with a common edge b as composed of many very small rectangular areas (Fig. 8), we could use numeric integration to receive the same result with a small loss of accuracy:

$$F_{j-i} = \frac{\sin^2 \Phi}{\pi \cdot Na \cdot Nb} \sum_{j_1=1}^{Na} \sum_{j_2=1}^{Nb} \sum_{i_1=1}^{Na} \sum_{i_2=1}^{Nb} \frac{x_i x_j}{\left[x_i^2 + x_j^2 - 2x_i x_j \cos \Phi + (y_i - y_j)^2 \right]^{3/2}} \Delta c \Delta b \quad (8)$$

where $\Delta a = a / Na$, $\Delta b = b / Nb$, $\Delta c = c / Nc$ and Na , Nb , Nc are the numbers of intervals for the numeric integration in each dimension. The coordinates of each fragment's center are: for surface $i - x_i=(i-0.5)\Delta c$; $y_i=(i_2-0.5)\Delta b$; for surface $j - x_j=(j_1-0.5)\Delta a$; $y_j=(j_2-0.5)\Delta b$. Such solution has one main significant advantage – it easily can be adapted for any disposition of both rectangular surfaces (Fig. 8), but also has two serious disadvantages – it gives an approximate result and to avoid this with large numbers of intervals, it needs a lot of computing time.

3.1.4. Derivation of a numerically integrable, general purpose VF: two parallel directly opposed rectangular surfaces A_i and A_j

For two parallel directly opposed rectangular surfaces (Fig. 7), Eq. (2) will have to be modified with these values of $R = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$ and $\cos \Phi_1 = \cos \Phi_2 = c / R$. The resulting integral for the estimation of VF_{1-2} is Eq. (9):

$$F_{1-2} = \frac{c^2}{\pi ab} \int_{x_1=0}^a \int_{y_1=0}^b \int_{x_2=0}^a \int_{y_2=0}^b \frac{1}{\left[(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 \right]^{3/2}} dy_2 dx_2 dy_1 dx_1 \quad (9)$$

Note that the configuration factor – solution of this integral, is Eq. (10), where $X = a / c$ and $Y = b / c$:

$$F_{1-2} = F_{2-1} = \frac{2}{\pi XY} \left(\begin{aligned} & X \sqrt{1+Y^2} \tan^{-1} \left(\frac{X}{\sqrt{1+Y^2}} \right) + Y \sqrt{1+X^2} \tan^{-1} \left(\frac{Y}{\sqrt{1+X^2}} \right) \\ & - X \tan^{-1}(X) - Y \tan^{-1}(Y) + \ln \left[\frac{(1+X^2)(1+Y^2)}{1+X^2+Y^2} \right]^{1/2} \end{aligned} \right) \quad (10)$$

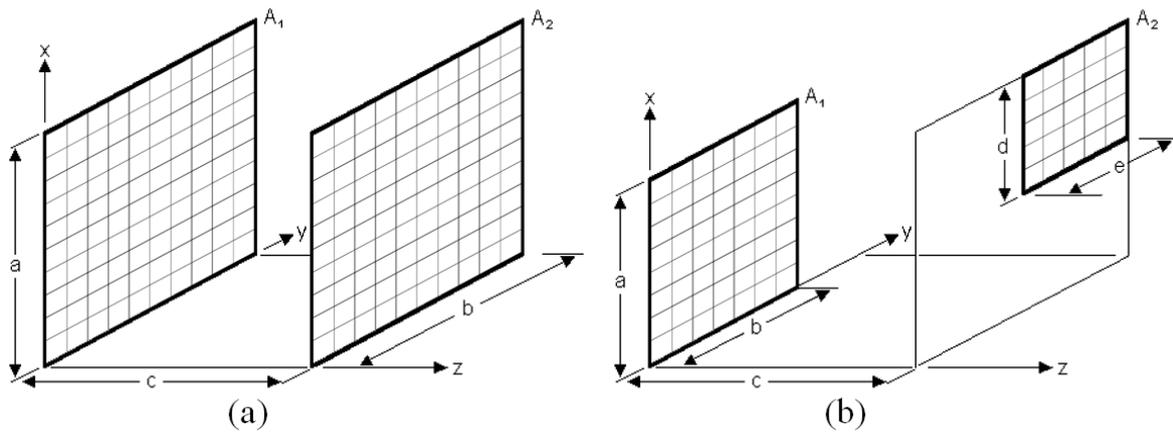


Figure 7. The reflecting and receiving surfaces are divided in two directions to receive a regular perpendicular grid: (a) both surfaces are identical and directly opposite; (b) two parallel surfaces – generalized arrangement

If we consider both parallel and directly opposite rectangular surfaces A_i and A_j as composed of very many small rectangular areas, we could use numerical integration to obtain the same result with only a small loss of accuracy:

$$F_{j-i} = \frac{c^2}{\pi \cdot Na \cdot Nb} \sum_{j_1=1}^{Na} \sum_{j_2=1}^{Nb} \sum_{i_1=1}^{Na} \sum_{i_2=1}^{Nb} \frac{1}{[(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2]^2} \Delta b \Delta a \quad (11)$$

where c is distance between both surfaces, $\Delta a = a / Na$, $\Delta b = b / Nb$ and Na , Nb are the numbers of intervals for the numerical integration in both dimensions.

4. PEDAGOGICAL EXERCISES

Equations (8) and (11) can now form the basis of numerically integrating codes to obtain view factors for the respective cases, i.e. inclined surfaces that share a common edge and parallel-opposed surfaces. The following two cases may be used for evolution of code architecture from simple-most and yet of low efficiency to highly-efficient but more complex. Those cases are:

4.1. Uniform grid

A uniform grid, where all cells within the emitting plane are of same dimension and aspect ratio, is applied on the reflecting surface. Likewise, the cells within the receiving plane have similar properties. The lengths of cells within the emitting and receiving planes may or may not be equal. Square grids for both surfaces show better accuracy in the estimating of VF . This approach can be easily applied as on a combination of two surfaces with one common edge (Fig. 8a), as on a combination of two non-intersecting rectangular surfaces that are inclined to each other (Fig. 8b). For square cells the total number of cells on the receiving surface is $N_{receiving_cells} = (b/a) \cdot Na^2$, and the total number of iterations is $N_{receiving_cells} \cdot N_{emitting_cells}$. This approach does not allow to reach a high accuracy for surfaces, where size 'a' is 10 or more times less than size 'b' and 'c'. A VBA code for this case is provided in Table 1. It is optimized to work faster.

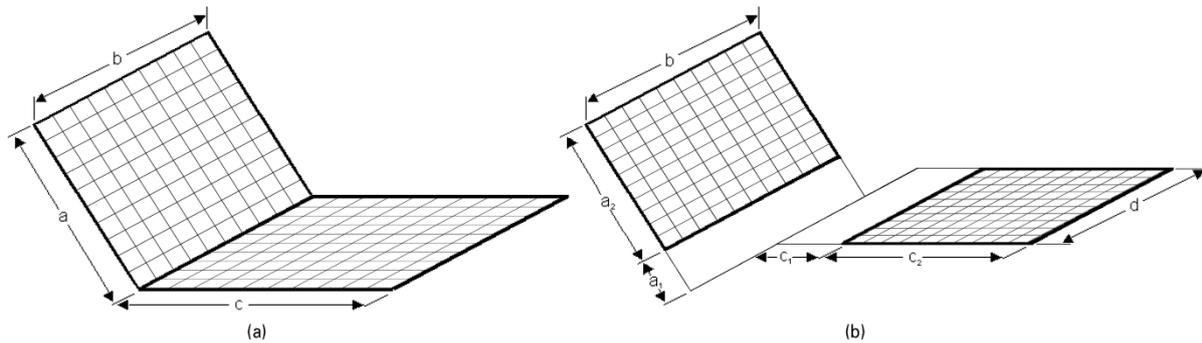


Figure 8. The reflecting and receiving surfaces are divided in two directions to receive a regular perpendicular grid: (a) both surfaces have one common edge and (b) both surfaces are non-intersecting.

Table 1. VBA code for inclined surfaces: uniform grid

```

Sub GVF() ' numerical solution
Dim a As Double, b As Double, c As Double, Fi As Double, Fi_rad As Double, Number%
Dim delx1 As Double, dely1 As Double, delx2 As Double, dely2 As Double
Dim Pi As Double, beta As Double, sinbeta As Double, cosbeta As Double
Dim sinbeta2 As Double, Sum As Double, Wb_name$

Wb_name = "Sheet1"
Number = InputBox("Number of data:")
Pi = Application.Pi()
a = Sheets(Wb_name).Cells(Number + 1, 2).Value
b = Sheets(Wb_name).Cells(Number + 1, 3).Value
c = Sheets(Wb_name).Cells(Number + 1, 4).Value
Fi = Sheets(Wb_name).Cells(Number + 1, 5).Value
Fi_rad = Fi * Pi / 180
delx1 = Sheets(Wb_name).Cells(Number + 1, 6).Value
dely1 = Sheets(Wb_name).Cells(Number + 1, 7).Value
delx2 = Sheets(Wb_name).Cells(Number + 1, 8).Value
dely2 = Sheets(Wb_name).Cells(Number + 1, 9).Value

beta = Pi - Fi_rad
sinbeta = Sin(beta): sinbeta2 = sinbeta * sinbeta
cosbeta = Cos(beta)

Dim xx1 As Long, yy1 As Long, xx2 As Long, yy2 As Long,
Dim x1 As Double, y1 As Double, x2 As Double, y2 As Double
Dim SumVF As Double, Na As Long, Nb As Long, Nc As Long, Nd As Long,
Dim R1 As Double, R2 As Double, x12 As Double, x1beta As Double
Dim dx1 As Double, dy1 As Double, dx2 As Double, dy2 As Double, GVF As Double

SumVF = 0

Na = a / delx1: Nb = b / dely1: Nc = c / delx2: Nd = b / dely2
dx1 = -delx1 / 2: dy1 = -dely1 / 2: dx2 = -delx2 / 2: dy2 = -dely2 / 2

Sum = 0
x1 = dx1
For xx1 = 1 To Na
    x1 = x1 + delx1: x12 = x1 * x1: x1beta = 2 * x1 * cosbeta
    x2 = dx2
    For xx2 = 1 To Nc
        x2 = x2 + delx2
        SumVF = 0
        R1 = x12 + x2 * x2 + x1beta * x2
        y1 = dy1
        For yy1 = 1 To Nd
            y1 = y1 + dely1
            y2 = dy2
            For yy2 = 1 To Nb
                y2 = y2 + dely2
                R2 = R1 + (y1 - y2) ^ 2
                SumVF = SumVF + 1 / (R2 * R2)
            Next yy2
        Next yy1
        Sum = Sum + SumVF * x1 * x2
    Next xx2
Next xx1

GVF = delx2 * dely2 * Sum / (Pi * Na * Nb) * sinbeta2
Sheets(Wb_name).Cells(Number + 1, 10).Value = GVF
End Sub

```

4.2. Arithmetic Progression

This case is applicable for inclined surfaces. A non-uniform grid in which the cell dimensions increase in an arithmetic progression as one moves from the common edge (Fig. 9). This development may be undertaken once the nature of influence of cells receding from the common edge is systematically studied. The shape of each cell is as close as possible to a square. This is especially important for the cells in the rows that are closer to the common line, because any other proportion of these cells generates significant errors in the result. The size of cell in first row of both surfaces is equal to the step in the arithmetic progression. The algorithm is the same for a composition of two surfaces with common edge and for a composition of non-intersecting rectangular surfaces that are inclined to each other. The number of square cells on the receiving surface as shown in Fig. 9a and b is $N_{receiving_cells} = (b/a) \cdot N_a \cdot (N_a + 1) \cdot (1 + 1/2 + 1/3 + \dots + 1/N_a) / 2$, the number of square cells on the receiving surface as shown in Fig. 9c and d is $N_{receiving_cells} = (b/a^2) \cdot N_a \cdot (N_a + 1) \cdot (1 + 1/2 + 1/3 + \dots + 1/N_a) / 2$. The number of square cells on the emitting surface can be estimated by analogy. It may be shown that the total number of iterations is $N_{receiving_cells} \cdot N_{emitting_cells}$. Codes for this and other cases are provided at this website: <https://www.dropbox.com/sh/8eehqf5szu1u68x/AAD4z7GFYkztzf-VgUqvHg7ea?dl=0>

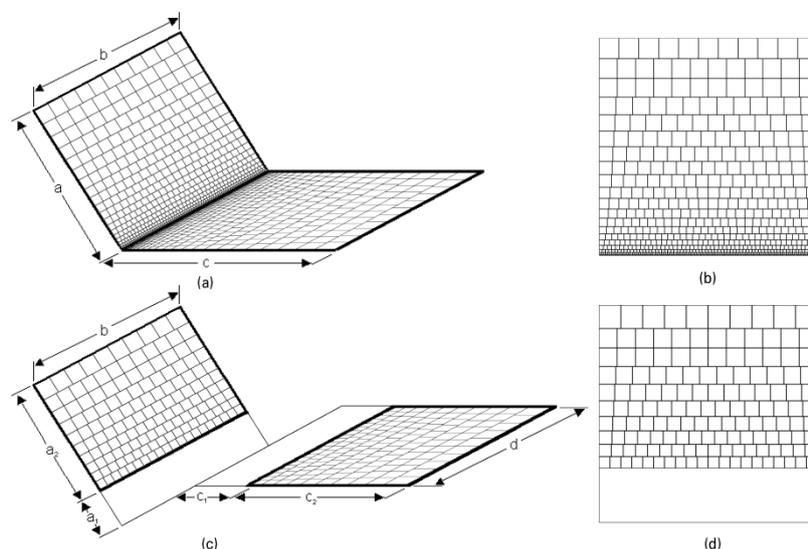


Figure 9. A non-uniform grid, where cell sizes increase in arithmetic progression, could be applied on: (a, b) two rectangular surfaces with one common edge; (c, d) two non-intersecting rectangular surfaces that are inclined to each other

Some of the tutorial material that may now be developed for providing a taught module in radiation exchange is presented below:

- Identify the practical engineering and architectural applications for radiation exchange for the configurations presented in Figs. 3, 4 and 7.
- Consider Fig. 3. For a given parametric values of $a=3$, $b=6$ and $c=6$ units obtain the view factor F_{12} using Eq. 5. Note: you may obtain the solution using your calculator. You may then progress to using Microsoft-Excel worksheet, keying in the functions in a step-wise manner. In each case keep account of the time taken for your entire activity and the accuracy of the solution obtained. Note that the precise answer for this case is 0.292373.
- Refer to Fig. 4. Repeat Exercise 'b' for the case of an inclination angle $\Phi=120^\circ$. Note that the precise answer for this case is 0.129731.
- Refer to Fig. 7. Repeat Exercise 'b' for the case of parallel-opposed surfaces using identical parametric values. Note that the precise answer for this case is 0.116657.

- e) Refer to Fig. 7. Write a Microsoft Excel-VBA code to obtain F_{12} using a suitable uniform mesh size. Compare your answer and execution time with that obtained in Exercise 'd'.
- f) Refer to Fig. 8. Write a Microsoft Excel-VBA code to obtain F_{12} using a suitable uniform mesh size. Compare your answer and execution time with that obtained in Exercise 'c'.
- g) Refer to Fig. 9. Write a Microsoft Excel-VBA code to obtain F_{12} using a non-uniform grid in which the cell dimensions increase in an arithmetic progression. Compare your answer and execution time with that obtained in Exercise 'c'.

5. CONCLUSIONS

Excel can easily be integrated into a thermal radiation exchange teaching module. Most students are already familiar with the basic operation of Excel. Further, the learning curve for Excel is very gentle and easy going. Help is available in the form of well-written text, study guides, and training videos.

An introduction to application of Excel in solving heat transfer problem takes 30 to 45 minutes of class time to demonstrate how to enter formulas into cells of Excel worksheet.

The advantage of using a spreadsheet-based computing environment such as Microsoft Excel is that the training times are of the order of, at most, a few hours to include finite element analysis (FEA) to obtain complex analysis such as those presented in the article.

A set of seven tutorial exercises were presented that will enable the pupils to gain a thorough understanding of not only the science of radiation exchange but also the use of a powerful computing medium such as Microsoft Excel-VBA to analyse complex engineering problems.

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